

## Contents

Preface xv

### I Essentials 1

#### 1 Key Features of Quantum Mechanics 3

- 1.1 Linearity of the Equations of Motion 4
- 1.2 Complex Numbers Are Essential 7
- 1.3 Loss of Determinism 10
- 1.4 Quantum Superpositions 14
- 1.5 Entanglement 22
- 1.6 Making Atoms Possible 25
- Problems 27

#### 2 Light, Particles, and Waves 31

- 2.1 Mach-Zehnder Interferometer 31
- 2.2 Elitzur-Vaidman Bombs 37
- 2.3 Toward Perfect Bomb Detection 39
- 2.4 Photoelectric Effect 44
- 2.5 Compton Scattering 47
- 2.6 Matter Waves 51
- 2.7 De Broglie Wavelength and Galilean Transformations 53
- 2.8 Stationary Phase and Group Velocity 58
- Problems 60

#### 3 Schrödinger Equation 67

- 3.1 The Wave Function for a Free Particle 67
- 3.2 Equations for a Wave Function 69
- 3.3 Schrödinger Equation for a Particle in a Potential 73
- 3.4 Interpreting the Wave Function 78
- 3.5 Normalization and Time Evolution 79
- 3.6 The Wave Function as a Probability Amplitude 81
- 3.7 The Probability Current 83

3.8 Probability Current in Three Dimensions and Current Conservation	85
Problems	87
<b>4 Wave Packets, Uncertainty, and Momentum Space</b>	93
4.1 Wave Packets and Uncertainty	93
4.2 Wave Packet Shape Changes	98
4.3 Time Evolution of a Free Wave Packet	100
4.4 Uncovering Momentum Space	101
Problems	106
<b>5 Expectation Values and Hermitian Operators</b>	111
5.1 Expectation Values of Operators	111
5.2 Time Dependence of Expectation Values	114
5.3 Hermitian Operators and Axioms of Quantum Mechanics	117
5.4 Free Particle on a Circle—a First Look	123
5.5 Uncertainty	125
Problems	128
<b>6 Stationary States I: Special Potentials</b>	133
6.1 Stationary States	133
6.2 Solving for Energy Eigenstates	137
6.3 Free Particle on a Circle—a Second Look	140
6.4 The Infinite Square Well	144
6.5 The Finite Square Well	148
6.6 The Delta Function Potential	156
6.7 The Linear Potential	161
Problems	165
<b>7 Stationary States II: General Features</b>	171
7.1 General Properties	171
7.2 Bound States in Slowly Varying Potentials	175
7.3 Sketching Wave Function Behavior	180
7.4 The Node Theorem	185
7.5 Shooting Method	187
7.6 Removing Units from the Schrödinger Equation	189
7.7 Virial Theorem	191
7.8 Variational Principle	194
7.9 Hellmann-Feynman Lemma	198
Problems	201
<b>8 Stationary States III: Scattering</b>	211
8.1 The Step Potential	211
8.2 Wave Packets in the Step Potential	218
8.3 Resonant Transmission in a Square Well	223
Problems	227

<b>9 Harmonic Oscillator</b>	233
9.1 Harmonic Oscillator	233
9.2 Solving the Harmonic Oscillator Differential Equation	236
9.3 Algebraic Solution for the Spectrum	242
9.4 Excited States of the Oscillator	246
Problems	251
<b>10 Angular Momentum and Central Potentials</b>	255
10.1 Angular Momentum in Quantum Mechanics	255
10.2 Schrödinger Equation in Three Dimensions and Angular Momentum	259
10.3 The Angular Momentum Operator	261
10.4 Commuting Operators and Rotations	264
10.5 Eigenstates of Angular Momentum	267
10.6 The Radial Equation	272
Problems	277
<b>11 Hydrogen Atom</b>	281
11.1 The Two-Body Problem	281
11.2 Hydrogen Atom: Potential and Scales	285
11.3 Hydrogen Atom: Bound State Spectrum	287
11.4 Rydberg Atoms	295
11.5 Degeneracies and Semiclassical Electron Orbits	298
Problems	302
<b>12 The Simplest Quantum System: Spin One-Half</b>	307
12.1 A System with Two States	307
12.2 The Stern-Gerlach Experiment	314
12.3 Spin States	322
12.4 Quantum Key Distribution	326
Problems	330
<b>II Theory</b>	335
<b>13 Vector Spaces and Operators</b>	337
13.1 Vector Spaces	337
13.2 Subspaces, Direct Sums, and Dimensionality	344
13.3 Linear Operators	349
13.4 Null Space, Range, and Inverses of Operators	354
13.5 Matrix Representation of Operators	360
13.6 Eigenvalues and Eigenvectors	366
13.7 Functions of Linear Operators and Key Identities	370
Problems	377
<b>14 Inner Products, Adjoints, and Bra-kets</b>	381
14.1 Inner Products	381
14.2 Orthonormal Bases	387

14.3	Orthogonal Projectors	390
14.4	Linear functionals and Adjoint Operators	395
14.5	Hermitian and Unitary Operators	398
14.6	Remarks on Complex Vector Spaces	403
14.7	Rotation Operators for Spin States	404
14.8	From Inner Products to Bra-kets	407
14.9	Operators, Projectors, and Adjoints	411
14.10	Nondenumerable Basis States	417
	Problems	421

## 15 Uncertainty Principle and Compatible Operators 427

15.1	Uncertainty Defined	427
15.2	The Uncertainty Principle	431
15.3	Energy-Time Uncertainty	435
15.4	Lower Bounds for Ground State Energies	438
15.5	Diagonalization of Operators	440
15.6	The Spectral Theorem	442
15.7	Simultaneous Diagonalization of Hermitian Operators	447
15.8	Complete Set of Commuting Observables	451
	Problems	454

## 16 Pictures of Quantum Mechanics 459

16.1	Schrödinger Picture and Unitary Time Evolution	459
16.2	Deriving the Schrödinger Equation	461
16.3	Calculating the Time Evolution Operator	464
16.4	Heisenberg Operators	467
16.5	Heisenberg Equations of Motion	469
16.6	Axioms of Quantum Mechanics	474
	Problems	478

## 17 Dynamics of Quantum Systems 481

17.1	Basics of Coherent States	481
17.2	Heisenberg Picture for Coherent States	484
17.3	General Coherent States	489
17.4	Photon States	492
17.5	Spin Precession in a Magnetic Field	494
17.6	Nuclear Magnetic Resonance	498
17.7	Two-State System Viewed as a Spin System	503
17.8	The Factorization Method	504
	Problems	511

## 18 Multiparticle States and Tensor Products 519

18.1	Introduction to the Tensor Product	519
18.2	Operators on the Tensor Product Space	522
18.3	Inner Products for Tensor Spaces	526
18.4	Matrix Representations and Traces	527

18.5 Entangled States	530
18.6 Bell Basis States	532
18.7 Quantum Teleportation	536
18.8 EPR and Bell Inequalities	538
18.9 No-Cloning Property	545
Problems	547
<b>19 Angular Momentum and Central Potentials: Part II</b>	<b>555</b>
19.1 Angular Momentum and Quantum Vector Identities	555
19.2 Properties of Angular Momentum	559
19.3 Multiplets of Angular Momentum	562
19.4 Central Potentials and Radial Equation	573
19.5 Free Particle and Spherical Waves	578
19.6 Rayleigh's Formula	581
19.7 The Three-Dimensional Isotropic Oscillator	584
19.8 The Runge-Lenz Vector	589
Problems	593
<b>20 Addition of Angular Momentum</b>	<b>599</b>
20.1 Adding Apples to Oranges?	599
20.2 Adding Two Spin One-Half Angular Momenta	601
20.3 A Primer in Perturbation Theory	606
20.4 Hyperfine Splitting	607
20.5 Computation of $\mathbf{1} \otimes \frac{1}{2}$	612
20.6 Spin-Orbit Coupling	615
20.7 General Aspects of Addition of Angular Momentum	619
20.8 Hydrogen Atom and Hidden Symmetry	625
Problems	631
<b>21 Identical Particles</b>	<b>639</b>
21.1 Identical Particles and Exchange Degeneracy	640
21.2 Permutation Operators	643
21.3 Complete Symmetrizer and Antisymmetrizer	648
21.4 The Symmetrization Postulate	652
21.5 Building Symmetrized States and Probabilities	659
21.6 Particles with Two Sets of Degrees of Freedom	665
21.7 States of Two-Electron Systems	667
21.8 Occupation Numbers	671
Problems	677
<b>III Applications</b>	<b>685</b>
<b>22 Density Matrix and Decoherence</b>	<b>687</b>
22.1 Ensembles and Mixed States	687
22.2 The Density Matrix	692
22.3 Dynamics of Density Matrices	700

22.4 Subsystems and Schmidt Decomposition	701
22.5 Open Systems and Decoherence	709
22.6 The Lindblad Equation	717
22.7 A Theory of Measurement?	721
Problems	730
<b>23 Quantum Computation</b>	<b>737</b>
23.1 Qubits and Gates	739
23.2 Deutsch's Computation	747
23.3 Grover's Algorithm	749
Problems	756
<b>24 Charged Particles in Electromagnetic Fields</b>	<b>761</b>
24.1 Electromagnetic Potentials	761
24.2 Schrödinger Equation with Electromagnetic Potentials	763
24.3 Heisenberg Picture	767
24.4 Magnetic Fields on a Torus	770
24.5 Particles in Uniform Magnetic Field: Landau Levels	774
24.6 The Pauli Equation	779
24.7 The Dirac Equation	781
Problems	783
<b>25 Time-Independent Perturbation Theory</b>	<b>793</b>
25.1 Time-Independent Perturbations	793
25.2 Nondegenerate Perturbation Theory	796
25.3 The Anharmonic Oscillator	804
25.4 Degenerate Perturbation Theory	807
25.5 Degeneracy Lifted at Second Order	814
25.6 Review of Hydrogen Atom	817
25.7 Fine Structure of Hydrogen	821
25.8 Zeeman Effect	833
Problems	838
<b>26 WKB and Semiclassical Approximation</b>	<b>849</b>
26.1 The Classical Limit	849
26.2 WKB Approximation Scheme	852
26.3 Using Connection Formulae	860
26.4 Airy Functions and Their Expansions	863
26.5 Connection Formulae Derived	868
26.6 Tunneling through a Barrier	872
26.7 Double-Well Potentials	876
Problems	884
<b>27 Time-Dependent Perturbation Theory</b>	<b>891</b>
27.1 Time-Dependent Hamiltonians	891
27.2 The Interaction Picture	893

27.3	Perturbative Solution in the Interaction Picture	899
27.4	Constant Perturbations	904
27.5	Harmonic Perturbations	907
27.6	Fermi's Golden Rule	911
27.7	Helium Atom and Autoionization	919
27.8	Modeling the Decay of a Discrete State to the Continuum	921
27.9	Ionization of Hydrogen	928
27.10	Atoms and Light	935
27.11	Atom-Light Dipole Interaction	939
27.12	Selection Rules	943
	Problems	945
<b>28</b>	<b>Adiabatic Approximation</b>	953
28.1	Adiabatic Changes and Adiabatic Invariants	953
28.2	From Classical to Quantum Adiabatic Invariants	955
28.3	Instantaneous Energy Eigenstates	960
28.4	Quantum Adiabatic Theorem	964
28.5	Landau-Zener Transitions	969
28.6	Berry's Phase	975
28.7	Born-Oppenheimer Approximation	981
28.8	The Hydrogen Molecule Ion	987
	Problems	993
<b>29</b>	<b>Scattering in One Dimension</b>	999
29.1	Scattering on the Half Line	999
29.2	Time Delay	1003
29.3	Levinson's Theorem	1009
29.4	Resonances	1011
29.5	Modeling Resonances	1015
	Problems	1020
<b>30</b>	<b>Scattering in Three Dimensions</b>	1025
30.1	Energy Eigenstates for Scattering	1026
30.2	Cross Sections from Scattering Amplitudes	1029
30.3	Scattering Amplitude in Terms of Phase Shifts	1031
30.4	Computation of Phase Shifts	1036
30.5	Integral Equation for Scattering	1042
30.6	The Born Approximation	1046
	Problems	1050
	References	1057
	Index	1065