

Contents

Preface	ix
1 Introduction	1
1.1 Categories	1
1.2 Monoidal Categories	4
1.3 The Examples and the Mini-Courses	5
1.4 Mini-Course: Sets and Categorical Thinking	8
2 Categories	33
2.1 Basic Definitions and Examples	33
2.2 Basic Properties	58
2.3 Related Categories	64
2.4 Mini-Course: Basic Linear Algebra	67
3 Structures within Categories	83
3.1 Products and Coproducts	83
3.2 Limits and Colimits	102
3.3 Slices and Coslices	110
3.4 Mini-Course: Self-Referential Paradoxes	111
4 Relationships between Categories	141
4.1 Functors	141
4.2 Natural Transformations	159
4.3 Equivalences	168
4.4 Adjunctions	171
4.5 Exponentiation and Comma Categories	188
4.6 Limits and Colimits Revisited	196
4.7 The Yoneda Lemma	201
4.8 Mini-Course: Basic Categorical Logic	208
5 Monoidal Categories	225
5.1 Strict Monoidal Categories	226
5.2 Cartesian Categories	232
5.3 Monoidal Categories	237
5.4 Coherence Theory	254

5.5	String Diagrams	262
5.6	Mini-Course: Advanced Linear Algebra	266
6	Relationships between Monoidal Categories	281
6.1	Monoidal Functors and Natural Transformations	282
6.2	Coherence Theorems	294
6.3	When Coherence Fails	308
6.4	Mini-Course: Duality Theory	313
7	Variations of Monoidal Categories	331
7.1	Braided Monoidal Categories	332
7.2	Closed Categories	341
7.3	Ribbon Categories	354
7.4	Mini-Course: Quantum Groups	360
8	Describing Structures	381
8.1	Algebraic Theories	381
8.2	Operads	395
8.3	Monads	409
8.4	Algebraic 2-Theories	427
8.5	Mini-Course: Databases and Schedules	433
9	Advanced Topics	447
9.1	Enriched Category Theory	447
9.2	Kan Extensions	454
9.3	Homotopy Theory	466
9.4	Higher Category Theory	500
9.5	Topos Theory	511
9.6	Mini-Course: Homotopy Type Theory	522
10	More Mini-Courses	533
10.1	Mini-Course: Knot Theory	534
10.2	Mini-Course: Basic Quantum Theory	543
10.3	Mini-Course: Quantum Computing	557
	Appendix A: Venn Diagrams	591
	Appendix B: Index of Categories	597
	Appendix C: Suggestions for Further Study	603
	Appendix D: Answers to Selected Problems	607
	Bibliography	619
	Index	639