Algorithms for Decision Making
Mykel J. Kochenderfer, Tim A. Wheeler & Kyle H. Wray

A comprehensive introduction to algorithms for decision making under uncertainty that clearly breaks down representations of decision making problems and provides objective overviews of strategies for solving them.

From an author team situated at the cutting-edge of industry and research, this book presents the most actionable and impactful new ideas alongside an interdisciplinary, high-level view of the field.

• Written with a distinct point of view by field leaders  
• Code-focused: provides real code implementations for algorithms in Julia  
• Multidisciplinary approaches and applications span AI, operations research, control theory, psychology, neuroscience, engineering, and economics  
• Accessible treatment of Markov Decision Processes (MDPs) for industry use in robotics, automation, and research models

"Its remarkable clarity, range, and depth make this a magnificent book both to learn from and to teach. It opens the door to so many modern techniques while firmly grounding them in the statistical and mathematical theory given to us by the founders. Truly exceptional.”  
—Thomas J. Sargent,  
Department of Economics, New York University; Senior Fellow, Hoover Institution, Stanford University

For a deep dive on what lies behind decision making under uncertainty, check out these previously published books from the MIT Press:

Decision Making Under Uncertainty  
Theory & Application  
Mykel J. Kochenderfer  
A penetrating study of decision making under uncertainty from a computational perspective.

Algorithms for Optimization  
Kochenderfer and Wheeler  
A broad introduction with a focus on practical algorithms for the design of engineering systems.

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