Music, Mind, and Mathetics

Learning to Learn Through Programming and Piano Playing

I would like to tell a story of what might have been, had Seymour Papert and Marvin Minsky extended their collaboration into the domain of music learning. Throughout my doctoral research, I noticed many powerful parallels between Seymour's ideas about learning programming and Marvin's personal approach to learning music. The convergence of Seymour and Marvin has had a profound influence on my ideas about music as a vehicle through which to think about thinking, as well as my research in technologies for music learning.

But I can speak for neither Seymour nor Marvin, much less reinvent the past. I will, however, share my own thoughts on this subject, with guidance from the Seymour and Marvin that exist in my mind. Though I never had a chance to make Seymour's acquaintance, I still carry with me a vivid sense of his ideas and personality, from reading his words, watching his videos, and conversations with his friends, students, and colleagues. My mental Marvin comes not only from his writings and conversations with family and friends but also from many personal memories, almost always near a piano, for music was what had brought us together. I was a weekly guest at the Minsky home in the last year of Marvin's life to play the piano with and for him.

The core of this essay extends and elaborates, within the domain of music, Seymour's notion of *mathetics*, a term he coined for the art of learning (S. Papert, 1993b). It reframes Marvin's approach of learning to play the piano as a form of learning through construction and discusses how the particular demands of playing music can add new facets to six mathetic principles distilled from Seymour's writings. The first portion of this essay sets the stage for these discussions, beginning with a brief history of Seymour and Marvin's collaboration, then summarizing Seymour's work with Logo and the mathetic principles he articulated in the context of programming and mathematics.

I. Seymour & Marvin

Figure 1: Marvin Minsky (left) and Seymour Papert (right) in 1971. Photo by Cynthia Solomon.

Seymour and Marvin first met at a conference in London, where they presented almost identical papers (Minsky & Selfridge, 1961). Marvin had a hand in Seymour's arrival to MIT in 1963, and for almost 20 years the two became inseparable intellectual partners, collaborating on nearly everything to the point of finishing each other's sentences (Rifkin, 2016). Official work together included co-writing the book *Perceptrons* and co-directing the Artificial Intelligence group at MIT, which yielded many co-written memos and co-advised students (Minsky & Papert, 1988). But even when their names did not appear together on paper, the two still had a hand and a say in each other's work. Marvin certainly gave his input to Seymour's Logo, and Seymour was instrumental to Marvin's formulation of the Society of Mind (Minsky, 1986; Papert, 1993a).

Seymour and Marvin both had roots in mathematics and shared a deep longing to understand the mind, not just cognition in the abstract but diverse dimensions of thinking, like perception, action, and emotion (Hillis et al., 2007; Minsky, 2006; Papert, 1966). From Seymour, Marvin learned about Piaget and became interested in children as a source of insight on the genesis of cognition (Minsky & Papert, 1971). Marvin brought to the table a love for studying creature intelligence, and the theories of Tinbergen and Lorenz (Minsky, 2006).

Given the breadth of their collaboration, it surprised me to hear that Seymour and Marvin never delved into music together. I was told by many who knew him that Seymour was by nature "not very musical", but at his Media Lab memorial event in January 2017, I learned from Tod Machover that Seymour became very interested in music later in life. He tried learning to sing and proposed a new school where everything was taught in connection with music, likely inspired by his earlier experiences with samba schools in Brazil (Papert, 1993a; Papert & Machover, 2004). Sadly, this was not long before Seymour's accident, and by that time, he and Marvin had long ceased their collaboration.

II. Seymour & Mathetics

One reason Seymour and Marvin went their separate ways was due to their divergent ultimate goals. Marvin wished to create computers that could learn and think like people; Seymour wished to use computers to empower people, particularly children, with better ways to learn and to think.

Seymour was motivated by an intense dissatisfaction with how children were taught in schools, especially in math. "School math", as Seymour called it, is generally presented as an arbitrary sequence of topics with little connection to children's interests or to everyday life. Students are assigned endless repetitive, rote exercises like drawing graphs and adding numbers, which are graded only for correctness. Mistakes are penalized, resulting in a fear of math among many children, and among some even a fear of learning.

"School math" stood in sharp contrast to the way that Seymour himself had learned to learn (and learned to love) the subject called mathematics. As a child, Seymour would entertain himself for hours with a set of gears, exploring relationships between numbers not as rote exercises divorced from meaning but through hands-on construction, experimentation, and invention. As an adult mathematician, Seymour noticed that the way he—and other mathematicians—thought, learned, and solved problems had more in common with his childhood experiences playing with gears than how most children learned to "do math" in school. He wished to give more children the experience of meaningfully and inventively engaging with mathematics and saw in the interactivity of the computer a unique opportunity.

Thus Logo was born, which allowed children to program the movement of a "turtle"—living in the computer screen or as a robot in the world—by giving it a series of instructions like FORWARD and

Figure 2: Screenshots of a square, a star, and a game programmed in Logo (left). A boy inspecting a turtle robot (right).

RIGHT (Papert, 1993a). Logo not only offered children a fun and engaging environment to learn computer programming, but also provided them a powerful vehicle to learn about their own learning and thinking.

Seymour saw programming as a way for children to externalize and concretize trains of thought, which can lead to the discovery of powerful mathetic principles. One such principle is how something big and complex can be more easily tamed when approached as smaller modules. Using Logo, children can encode complex behaviors into the computer one step at a time by building sub-procedures, each responsible for a part of the whole. In larger programs, sub-procedures themselves can be further divided into successively smaller procedures of their own. Through building such modular programs to make their own drawings, stories, and games, children experimented with different ways of breaking down problems. Doing so, they learned through experience that there are often many ways to solve the same problem and that decisions of how to break down a problem can greatly affect its difficulty.

Figure 3: Seymour and a child in a classroom in 1982

“Bugs”, or unexpected behaviors in a program set the stage for other mathetic lessons. Instead of seeing them as mistakes to be penalized, Seymour celebrated the occurrence of bugs. On one hand, bugs provided opportunities for children to systematically reason through their programs—and their thinking. Commonly known as “debugging”, this process involves isolating the unexpected behavior, formulating theories about its cause, inventing a solution, and testing it, trying different solutions until the bug is fixed. However, not all bugs need to be fixed. Through Logo, children quickly learned that the unexpected and unplanned are often the greatest source of insight and delight. Knowing how to explore the creative potential of a bug is just as important as knowing how to eliminate it.

Programming can also introduce two other mathetic principles, which Seymour connected to the psychologist Jean Piaget’s observations of early childhood (Papert, 1993; Piaget & Inhelder, 1969). One is to take what is new and relate it to something you already know, known as “assimilation” in Piagetian lingo. The other is to take what is new and to make it your own, a concept at the heart of Piaget’s Constructivist theory of learning. Logo gave children a sandbox in which to learn through constructing their own computer programs. When writing a new program, children were encouraged to repurpose sub-procedures from previously written code. This not only enabled building more interesting programs in less time but also instilled the habit of looking for common features across seemingly different subjects.

Logo also employed these two principles when used to teach "advanced" topics in mathematics such as differential geometry and Newtonian physics. For differential geometry, Seymour and his colleague Cynthia Solomon developed classroom activities where they asked children to "play turtle", by imagining themselves moving in the place of the turtle to discover how different shapes are can be procedurally drawn. The idea of mobilizing the sense for one's own body—called *body syntonicity* by Seymour—repurposes children's existing familiarity with their own bodies to help them understand otherwise abstract concepts. In the case of Newton's laws of motion, which cannot be easily observed in daily life, the microworld of the turtle helped children build a concrete understanding for the concepts by playing with the movements of the turtles. This experience can then help children ground the equations they learn from school.

Mathetic principles introduced through Logo programming have direct analogues with problem-solving techniques employed by "real" mathematicians. For example, mathematician and educator George Polya recommended students to run through a mental checklist of problem solving heuristics whenever they encounter a new problem (Polya, 1957). These heuristics included whether the problem can be subdivided into simpler problems and whether it can be related to a previously solved problem. The visual nature of Logo and the body syntonic classroom activities bring to mind Albert Einstein's descriptions of his own thinking. In the absence of clearly defined problems, Einstein relied on "combinatory play" in "visual and muscular modes of thought" and only later translated these thoughts into "logical constructions in words" (Hadamard, 1954).

One final mathetic principle advocated by Seymour is the concept of "giving yourself time." The reminder to take time is especially important in light of the aforementioned mathetic principles. While dividing a problem, debugging, transforming bugs to features, relating to existing knowledge, and making something your own can help us learn more efficiently and effectively, none of these strategies can be "applied" in a formulaic way. Knowing when and how to use each principle comes from experience, which necessarily takes time to accumulate. That said, the time it takes to build these ways of thinking can be far more rewarding than the time spent doing rote exercises.

III. Marvin & Music

From my time learning to play the piano as a child, I have experienced the analog of "school math" in how music is typically taught. The prevailing view about music education is that children (and adults) are to learn pieces by a set of composers (mostly long dead) on a traditional instrument (often the piano). Learning a piece means first learning to correctly play the notes of the piece, a process that necessarily involves long hours of repetition in order to cement "muscle memory." Expression is generally seen as secondary to the correctness of the notes, and when considered, is often reduced to following the dynamic and articulation markings as indicated in the score.

Figure 4: The author as a child, slaving over finger exercises

Only after many years of diligent study are students introduced to "advanced" ideas like how a piece is put together, how to evoke emotions, or how to craft an original interpretation, composition, or improvisation. By that time, most who begin to study music will have long quit. Some may still maintain a love for the art and a lifelong appreciation for the creation of others, but many avoid making music themselves. They admire their favorite musicians for their "talent", "gifts", or "genius", and blame their own failure to learn on the lack of something innate.

Marvin was often lauded as a "genius" by those around him, but he himself disliked the term, for it obscures the means by which people's extraordinary abilities come to be (Minsky 1986). Music was one domain in which Marvin exhibited extraordinary abilities, for he was capable of improvising on the piano fugues in the style of Bach and variations inspired by Beethoven. Looking beyond the common view that these abilities were simply another facet of Marvin's genius, let us examine how Marvin went about learning music.

Aside from composition studies with Irving Fine while an undergraduate at Harvard, Marvin received very little formal musical training. Fueled by his own *passion*, Marvin's musical knowledge was largely constructed through self-guided *projects*, interactions with *peers*, and creative *play*. According to Mitch Resnick, Seymour's student and successor at MIT, passion, projects, peers, and play are hallmarks of the sort of learning Seymour strived to promote (Resnick, 2017).

Figure 5: Marvin improvising on the piano, August 2015

Like Seymour, who grew up tinkering with his gears, Marvin grew up tinkering with his family's player piano, "programming" melodies by punching holes in the spare paper at the ends of piano rolls and "erasing" mistakes with pieces of sticky tape. As an adult, Marvin created his own vocabulary of musical building blocks, some borrowed from his favorite composers, and constantly experimented with different combinations and variations of how they can come together. Beyond his improvisations, Marvin's musical projects included both the study of classical repertoire and the creation of original compositions. His family remembers the many hours he spent at the piano to refine his technique and interpretation for Beethoven's *Appassionata* sonata, and the iterations over several years on his own *Nursery Rhyme Suite*, which reimagines familiar childhood tunes through Modern classical sounds. Marvin's peers included a circle of musicians across genres and disciplines, from the classical composer Frederic Rzewski to the singer-songwriter Bono. Along with his wife Gloria, Marvin hosted salon-style gatherings at his home and seized every opportunity to converse with others who are musically inclined.

The varied ways in which Marvin engaged with music speaks to his famous saying that "you don't understand anything until you learn it more than one way", but one may ask what exactly Marvin was trying to understand about music. To learn how music is put together, it suffices to study rules of composition and existing pieces, and to try writing one's own music by following rules and examples. The laborious task of learning to physically play an instrument is not strictly necessary to attain this understanding, and researchers from the days of Logo to the present have envisioned ways of using the computer to help youngsters play with the building blocks of music without the overhead of learning a traditional instrument (Bamberger, 2013; Rosenbaum, 2015). In fact, it was Marvin who built the Logo music box, a 4-voice analog synthesizer that enabled computers of the 1970s to output melodies and rhythms specified by code.

Despite knowing how to write music and to make machines that play music, Marvin did spend considerable time and effort working out how to play the piano and how to improvise on it; he never stopped playing until the end of his life. I have a theory that Marvin's piano playing was not only another way to understand music but also a way to understand his own mind at work. Almost anyone can "compose" little pieces by following rules of Western harmony and by some trial and error. But when improvising the way Marvin did, there is no time to "reason" through rules, much less try possibilities for what to play next. Improvising music requires connecting and coordinating many different parts of the mind to act in real time (Jones, 1981). Playing existing repertoire can also be seen in similar terms.

I like to think of learning to play music as building a set of "mental infrastructures." In broad strokes, we need "mental infrastructures" to listen to sound, to recall patterns, to control our movements, to make adjustments, and to express emotion. Like the insides of a computer, these "infrastructures" are invisible. To make sure that they are properly built and properly attuned, one must "output" music through playing and make theories based on what one hears and feels. Effective practice for both a performer and an improviser is not about rote repetition but is guided by the construction of one's own internal state.

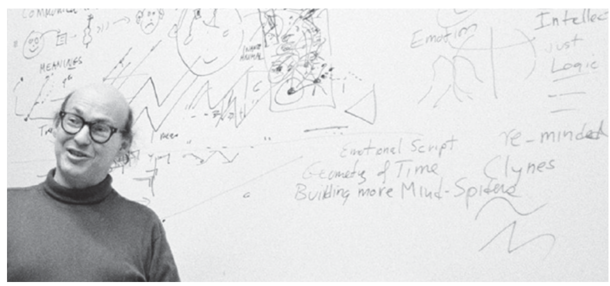

Figure 6: A talk Marvin gave at IRCAM in Paris at the "Composer and the Computer" conference organized by Tod Machover. Note the "Building more mind spiders" statement on the whiteboard.

Seymour thought of computer programming as one way to externalize thought. From Marvin's example, we can see that playing music is another. For Marvin, playing the piano, and especially improvising, was a special "microworld" in which the building blocks are not just the building blocks of music but also the parts and processes of his own mind. And let us not forget that "mind" to Marvin included not only human's ability for abstract reasoning but also involved the body (perception and action), the emotions, and the imagination (Minsky 2006)!

IV. Music & Mathetics

If Marvin derived insights about the mind from music, we may adopt Seymour's perspective and ask what children (and adults) can learn from music about their own learning and thinking. Our discussion centers on how Seymour's mathetic principles can be discovered through music. However, just as "school math" does not lead to the discovery of higher-level learning strategies, we must look beyond how music is typically taught and examine how “real musicians” think. Marvin was able to work out for himself the thinking of “real musicians”, which included not only the logical structure of music but also how to play music through the body. A central theme of this discussion is how the embodied aspects of playing music add new dimensions to Seymour’s mathetic principles discussed earlier, which included dividing something into parts, debugging, taking inspiration from the unexpected, relating to what you know, making what is new your own, and taking time.

In the context of programming, the first principle of how to divide something big into smaller parts is not so much about the obvious fact that lines of code make up a program as it is about how to organize the code into procedures, which can then be used to build larger procedures. Similarly, while it is true that any piece of music can be divided into measures of notes, there are more useful methods of subdivision. We can identify patterns of rhythm, melody, and harmony that form building blocks across individual notes, which can then be combined to form larger musical structures. Knowing how to organize the building blocks of music is not only reserved for composers but also gives insights on playing.

For the performer and improviser, how music is put together is not only about the building blocks themselves but also about expressing them on an instrument. In fact, the techniques to play musical patterns can be seen as another kind of building blocks. While technical exercises such as scales, arpeggios, and octaves are common in a performer’s training, they are often treated as repetitive drills divorced from musical context. Seeing techniques as building blocks means not only practicing their physical coordination but also recognizing their musical function, identifying what purpose they serve in a piece, and creating one’s own pieces. To discover the musical function of a technique means experimenting with different ways of playing it, varying its articulation, dynamics, and temporal feel in order to discover a diverse palette of colors, textures, and moods.

Even the process of building the physical coordination for techniques needs not to be mindless. Contrary to popular opinion, learning a new technique is less about securing “muscle memory” and more about solving the problem of how to perform a new movement efficiently and fluently. Whether practicing or performing, good musicians never stop listening to the sounds they produce. Guided by the ear, they are constantly making subtle adjustments to the movements of their body. While practicing a technique may look repetitive “from the outside”, the repetition involved is never rote. Rather, the performer is always experimenting to discover the movements to create the desired sound.

Seeing music in terms of its building blocks enables musicians to engage in a process of “debugging.” To debug playing music, we must first identify what is a “bug.” Wrong notes are the most obvious type of “bug”, but musical “bugs” also include shortcomings in a piece’s expression, such as shaky rhythm, harsh tone, or a muddled melody. Learning to recognizing “bugs” in expression means sensitizing the ear, not only for the notes but for the quality of sound. Just as we form theories about what is causing bugs in our code, we must also theorize about the underlying cause of musical “bugs.” Sometimes, the “bug” may be physical in nature, such as an awkward fingering sequence, a technique that has not yet been worked out, or unnecessary tension somewhere in the body. Other times, the "bug" could be more "intellectual", such as not recognizing the melodic line or not understanding the harmonic progression. Determining the nature of a "bug" allows us to target our practice for it. We can design exercises for ourselves that focus on a specific technique or help us to see the musical building blocks of the passage. If an exercise does not improve our sound, we can always come up with a new hypothesis and try again.

While musical bugs encompass more than just wrong notes, not all wrong notes are "bugs" or mistakes. As jazz legend Miles Davis famously said, "Do not fear mistakes; there are none" (Arca, 2008). Just as bugs can inspire new creative directions in code, wrong notes, especially for the improvising performer, can become a source of musical invention.

Recognizing the building blocks of music also facilitates "reusing what you already know." When learning any new piece, we can recognize patterns and techniques within the piece from previous pieces we have already learned. Making connections between old and new material significantly speeds up the learning of new repertoire, but it is not usually automatic. Even if we recognize a familiar pattern, it is often necessary to "transfer" the sense of familiarity and ease by alternately practicing the familiar and the new passage.


Figure 7: Glenn Gould singing while performing on the piano

Another example of "transfer" is using one part of a body to teach another. Movement knowledge of sufficient dexterity and complexity seems to be specific to the part of the body where it was learned. For example, most people can only write with one hand. In music, a common practice among experts is singing while playing, as exemplified by pianist Glenn Gould. While many view Gould's singing as an eccentric quirk, it is actually an essential expressive tool. Even when not trained in proper singing technique, most people's voices already "know" how to channel expression from years of communication. Fingers, on the other hand, are not usually so experienced with producing expressive sounds. By singing and playing together, pianists like Glenn Gould are using their voices to "teach" their fingers how to channel emotion through sound.

Our discussion thus far has already encountered several examples of "making what is new your own" in music. The discussion of musical building blocks focused on the example of learning a new technique, where varying how we play helps us work out the physical mechanism. Practicing variations of a pattern is not just useful for technical training. In fact, one of the best ways to "own" any musical pattern is to play it in many different ways. When learning to improvise, Marvin borrowed patterns from Bach or Beethoven, practiced them in every key, and experimented with different ways of putting them together. This "combinatoric" approach is one way of "owning" a musical pattern.

Another approach focuses on the "style" of expressing patterns. Though Marvin was best known for emulating Bach and Beethoven in his improvisations, he actually experimented with many different styles in his prime, from Gilbert & Sullivan to ragtime. Playing a musical pattern within different styles helps us "hear" the pattern in different ways and sensitizes the ear to recognize the pattern in different contexts. It also helps us understand how various elements of expression, such as articulation, dynamics, rhythmic feel, tempo, and mood, come together to evoke a specific sound.

Lastly, let us revisit the notion of "giving yourself time." Most musical laymen and novices believe that playing music takes time to learn because of the many hours of repetition required to encode the "muscle memory." This essay argues for a different reason why learning music takes time—because it is a language. But music is more akin to a language for humans (like English) than a language for the computer (like Logo) (Wooten, 2012). Just as communication in English is not only about stringing words together to form grammatically correct sentences, communication in music is not only about manipulating logical patterns. Learning to speak a human language means learning to use our bodies; English uses not only the vocal chords but also engages the whole body to convey emotion. Likewise, playing music not only uses the parts of the body in contact with the instrument but also channels expression through the whole body. And just as learning English means learning to have a conversation, to read, to write, and to recite, learning music should also encompass reading, writing (composing), reciting (performing), conversing (improvising), and above all, communicating.

Thinking of music as a language, it is obvious that learning it ought to take time, even with the aid of technology. Infants spend the first few years of life learning to speak their native tongue, and eloquence in both speaking and writing is an art that takes years to master. But even though it takes time to truly understand music, the process needs not be unpleasant or dull. At every stage, we can and should express ourselves to the extent of our abilities, and at every stage there is always a new puzzle to solve—to understand how building blocks come together, to work out a technique, or to debug a problem in our playing. Solving these problems gives us the invaluable opportunity to think about our own thinking, not only of the rational mind but also of the embodied and emotional minds.

V. Conclusion

Through stories of Seymour Papert and Marvin Minsky, this essay traced the connections between programming the computer and playing the piano. In reaction to rote repetition in mainstream mathematics education ("school math"), Logo offered programming as a new way to learn mathematics that also imparted valuable lessons about the art of learning (*mathetics).* While this work has inspired connections between mathematics, programming, and music *theory*, ideas about rote repetition still dominate the mainstream conception of *playing* music. Drawing from Marvin's approach to music as well as my personal knowledge, this essay explored how learning to *play* music shares many ways of thinking and problem solving with programming. Doing so, I extended Seymour's ideas about *mathetics* to the domain of music, bringing new facets to these ideas in regards to "thinking" through the body. By way of a conclusion, I offer thoughts on three open questions to invite further contemplation and exploration.

First, just as Logo used new technologies to enable a new way of learning mathematics, we may ask how new interactive technologies can support new ways of learning to play music. My PhD research explored two projects on the piano that combined projection mapping with the actuated keys of a player piano. One project, Andante, featured miniature silhouettes of figures that appeared to walk and dance on the piano keys. It encouraged children to understand musical building blocks and expression in terms of movements of their own body, inspired by the body syntonicity of Turtle Geometry (Xiao et al., 2016). The other, MirrorFugue, gave the illusion of a virtual reflection playing the physical piano, thus inviting anyone to sit down at the keyboard in place of the virtual pianist, feel the music in the body, and learn to play through imitation (Xiao, Pereira, & Ishii, 2013). Both these projects emphasize the embodied aspects of playing and understanding music, a rich topic for future research.

Figure 8: A girl plays with her own "reflection" on MirrorFugue (left). An Andante figure tiptoes across the piano (right).

Another question is how we may better communicate the "infrastructures" that we build in our minds. One reason why ideas about rote learning have persisted for so long may be due to deficient models of what is happening in the mind when learning occurs. The idea that "knowledge is constructed" from Seymour by way of Piaget is a useful view, but Seymour rarely discussed what exactly is "constructed" inside, instead focusing on the external conditions that promote knowledge construction. When Seymour did talk about the mind, his language is often metaphorical; in fact, the idea of "constructing knowledge" itself is a metaphor. Marvin, on the other hand did try to formulate theories that precisely describe the workings of the mind, in terms of high-level abstractions for mental activity such as *agents* and *k-lines* (Minsky, 1986). Because Marvin's goal was to build artificial minds, his language may not be suitable as an everyday vocabulary for learners. Still, many of Marvin's concepts, in combination with Seymour's metaphors could help us improve the way we discuss, and debug, our own thinking and learning.

Finally, though this essay focused on piano playing, neither the piano in particular nor music in general is the only artistic discipline rich in mathetic insights. Learning in all forms of art involves developing and connecting different ways of "thinking", such as attuning the senses, training the body's movements, and expanding the imagination. Unfortunately, many counterproductive attitudes about artists and the arts are still widespread among the public. Too often, we admire artists for their "genius" without questioning how they got to become the way they are. At the same time, the arts are constantly dismissed as less rigorous, and less valuable, than technical subjects. In closing, I strongly recommend anyone interested in the art of learning to take a closer look at artistic practices, and the yet unexplored connections in ways of thinking across technical and artistic domains.

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