Preface and Navigation Guide

This book is about the semantics and pragmatics of natural language sentence connectives and about the properties of and relations between analogous devices in the formal languages of numerous systems of propositional (or sentential) logic. And the intended readership is: all who find connectives, and the conceptual issues arising in thinking about them, to be a source of fascination. Such readers are most likely to have come from one or more of the traditional breeding grounds for logicians: philosophy, mathematics, computer science, and linguistics. These different backgrounds will of course give rise to different interests and priorities. The best way – or at least one reasonable way – to read the book is to browse through to a topic of interest, or use the index to locate such topics, and then follow the references forward and backward within it on the topic in question, or those to the extensive literature outside it.

In more detail: after Chapter 0, in which some minor mathematical preliminaries are sorted out (and which can be skipped for later consultation as necessary), Chapters 1–4 and Chapter 9 contain general material on sentence connectives in formal logic, such as truth-functionality, unique characterization by rules, etc., while Chapters 5–8 concern specific connectives (conjunction, disjunction, and so on), considering their pragmatic and semantic properties in natural languages as well as various attempts to simulate the latter properties in the formal languages of various systems of propositional logic. (The word "the" in the title *The Connectives* picks up on this aspect of the development; an equally good title, reflecting the more general concerns of Chapters 1–4 and 9, would have been simply *Connectives*.) Chapter 2 surveys various different logics (typically seen as extensions of or alternatives to classical propositional logic) to provide a background for later discussion. This means the treatment is selective and many popular themes not closely related to the behaviour of specific connectives are not touched on – in particular, issues of decidability, and computational complexity more generally, are completely ignored. Similarly at the more general level, while most of the connectives attended to here are what people would happily classify as *logical* connectives, no attention will be given to the philosophical question of what makes something an item of logical vocabulary (a 'logical constant', as it is put – and of course this is a status that not only sentence connectives but also quantifiers and other expressions may merit). As with other topics not gone into in detail but bearing of matters under discussion, the notes and references at the end of each section provide pointers to the literature; if a section covers numerous topics separately, the notes and references are divided into parts, each with its own topic-indicating heading. Let us turn to matters of navigational assistance.

Chapters are divided into sections, with Chapter 4, for example, consisting of three sections: §4.1, §4.2, and §4.3. Each section ends with notes and references for material covered in that section. The sections themselves are divided into subsections, labelled by dropping the "§" and adding a further digit to indicate order within the section. Thus the first and fourth subsections of §4.2 are numbered 4.21 and 4.24 respectively. We can safely write "4.21", rather than something along the lines of "4.2.1", to denote a subsection, as no section has more than nine subsections. References such as "Humberstone [2010*b*]" are to the bibliography at the end. (The work just cited contains some highlights from Chapters 1–4 and 9, leaving out proofs and most of the discussion – though also touching briefly on some topics not covered here.)

Within each subsection, the discussion is punctuated by numbered items of two kinds: results and non-results. Items in the result category – Lemma, Theorem, Corollary, and Observation – are set in italics and usually have proofs terminated with an end-of-proof marker: "□". This enables them to stand out from the surrounding text. (A similar symbol, □, is used for a necessity-like operator in modal logic, discussed in §2.2 and elsewhere, but this will cause no confusion.) As usual, Theorems are principal results, with Lemmas leading up to them and Corollaries drawn from them. ("Observation" is used for results not naturally falling into any of these categories, this term being chosen in place of the more usual "Proposition", simply because we already need several different notions of *proposition* in connection with the subject matter itself, as opposed to the presentation of results about that subject matter.) The non-result categories are Example, Exercise, and Remark, are not set in italics, so are they indented from the left margin to make it easy to see here they begin and end. The category *Remark* is included to make it possible to refer precisely, as opposed to merely citing a page number, to items in this category from elsewhere in the text. All numbered items, whether results or non-results, take their numbering successively within a subsection, to make them easy to locate. Thus Theorem 1.12.3 is the third numbered item – rather than the third *theorem* – in subsection 1.12 (both 1.12.1 and 1.12.2 being Exercises, as it happens). Similarly, Example 2.32.18 is the eighteenth numbered item in Subsection 2.32, which happens to be an Example; and so on. The presence of *Exercise* as a category indicates that this material can be used selectively for teaching purposes, but instructors should note that in a few cases – usually explicitly indicated – questions asked in the exercises are addressed in the subsequent main discussion. Also, while the exercises are given to allow for practice with concepts and to ask for (mostly) routine proofs, there are also a few discussion questions to be set to one side by instructors using some of this material for purely formal courses. Definitions are not numbered, and neither are Digressions; the latter can be omitted on a casual reading but do contain material referred to elsewhere. Diagrams and tables are labelled alphabetically within a subsection. Thus the first three in subsection 2.13 would be called Figure 2.13*a*, Figure 2.13*b*, and Figure 2.13*c*, in order of appearance.

The following abbreviations have been used with sufficient frequency to merit listing together in one place at the outset; unfamiliar terms on this list will be explained as they come up:

Coro.	for	*Corollary*
Def.	for	*(by) definition*
esp.	for	*especially*
Fig.	for	*Figure*
gcr	for	*generalized consequence relation*
g.l.b.	for	*greatest lower bound*
iff	for	*if and only if*
lhs	for	*left-hand side*
l.u.b.	for	*least upper bound*

Obs.	for	*Observation*
resp.	for	*respectively*
rhs	for	*right-hand side*
Thm.	for	*Theorem*
w.r.t.	for	*with respect to*

We use "gcr's" as the plural of "gcr", sacrificing apostrophic propriety to avoid the hard-to-read "gcrs". Other abbreviations will be explained as they arise (as indeed most of the above are) and can also be found in the index. But here it may help to mention that "CL", "IL", and "ML" abbreviate "Classical Logic", "Intuitionistic Logic" and "Minimal Logic"; additional similar abbreviations are also used (see the index entry for *logics and consequence relations*), and the abbreviations in question may also appear subscripted to a turnstile, such as, in particular, "$\vdash$". (As the preceding sentence illustrates, we use what is sometimes called 'logical' punctuation as opposed to the traditional convention, when it comes to ordering quotation marks and commas. The comma is inside the quotation marks only if it is part of what is quoted.) Thus $\vdash_{CL}$ is the consequence relation associated with classical propositional logic, sometimes also called the relation of tautological or truth-functional consequence. We also use the "$\vdash$"-notation for generalized (*alias* multiple-conclusion) consequence relations, so as to be able to write down in a single way formal conditions applying to consequence relations proper and to gcr's alike, rather than having to write them down twice, using a separate notation (such as "$\Vdash$") in the latter case. Similarly, the neutral verbal formulation "(generalized) consequence relation" is sometimes used, meaning: consequence relation or gcr.

The set of natural numbers (taken as non-negative integers here) is denoted by $\mathbb{N}$ or, when convenient, by ω (a lower case omega), the latter also understood as denoting the smallest infinite ordinal. This use of "$\mathbb{N}$" is standard mathematical practice, but the corresponding use of "$\mathbb{R}$" and "$\mathbb{Q}$" for the sets of real and of rational numbers respectively is not followed here; instead "$\mathbb{R}$" denotes either of a pair of structural rules introduced in 1.22 (as do "$\mathbb{M}$" and "$\mathbb{T}$"), while "$\mathbb{Q}$" is pressed into service for a piece of semantic apparatus in 8.33. The mathematical prerequisites for following the discussion in general are minimal: a preparedness for abstract symbolically assisted thought and a passing familiarity with proof by (mathematical) induction will suffice. Some prior acquaintance, for example in an introductory course, with formal logic would be desirable. Many topics are treated here in a way suitable for those with no prior acquaintance, but perhaps rather too many for those without some such background to assimilate (or, more importantly, to enjoy).

Some remarks on notation are in order. As schematic letters for formulas the following are used: A, B, C,... rather than A, B, C,..., so that A and B can be used to denote the universes (or carrier-sets) of algebras respectively denoted by $\mathbf{A}$ and $\mathbf{B}$, without confusion. (Similarly, propositional variables have been set as p, q, r... rather than p, q, r,...) Thus in general notation is highly font sensitive. An exception is the use of labels along the lines of "CL", "IL",..., for classical logic, intuitionistic logic,..., which appear in roman except when used as subscripts ("$\vdash_{CL}$" etc.), in which case they appear in italics. Otherwise the font is significant. For example, "T" is the name of the truth-value *True*, while T and $\top$ are a sentential constants which are associated in ways explained in the text with this truth value (as also is t); on the other hand "(T)" is the name of a condition on consequence relations and "($\mathbb{T}$)" – already alluded to above – is the name of a corresponding rule, while "**T**" refers to a certain modal principle, as well as to an unrelated system of relevant logic. Names of logics, whether this is understood to mean proof systems or at a more abstract level (on which see the Appendix to §1.2: p. 180), mostly attempt to follow the most widespread practice in the area concerned. Thus although we use IL for intuitionistic logic, a particular natural deduction system and a particular sequent calculus for this logic are called *INat* and *IGen* respectively ("Gen" for Gentzen), while (the core of) Girard's system of intuitionistic linear logic is called **ILL**; boldface is also used for normal modal logics (**K**, **S4**, etc.) and relevant (or 'relevance') logics: **R**, **RM**, **E**, **T**.

One usage over which there is an abstract possibility of confusion, in practice resolved by context, is the double use of "V", in the first place for sets of valuations (bivalent truth-value assignments) and in the second to denote the component of a Kripke model (or similar) which specifies at which points the propositional variables are true; this component we call a Valuation (with a capital "V"). Also used is "BV", as an unstructured label for the class of boolean valuations for whatever language is under consideration. When only a single candidate conjunction, disjunction, negation, implication (conditional) or equivalence (biconditional) connective is in play, this is written as $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$, respectively; metalinguistically, we sometimes use & and $\Rightarrow$ for conjunction and implication (though sometimes these too are used as object language connectives). For an ordered n-tuple we use the notation $\langle a_1, \ldots, a_n \rangle$. If the objects concerned (here $a_1, \ldots, a_n$) are of different types and the tuples under consideration only allow a given type in a given position, we tend to use "(" and ")" in place of angle brackets, thus instead of writing "$\langle W, R, V \rangle$" to denote a Kripke model, considered as a certain kind of ordered triple, with W a non-empty set, R a binary relation on W, and V – a Valuation in the sense alluded to above – is a function from propositional variables to subsets of W, the alternative notation "(W, R, V)" is used. Sometimes to indicate the mention rather than the use of a formal symbol quotation marks are used, and sometimes the symbol appears without quotation marks, simply as a name for itself. To refer to natural language expressions, quotation marks are again employed, with the use of italics as a variant. I have tried to follow a policy, in using quotation marks, of using double quotes when mentioning linguistic expressions (though, as already noted, they may be dropped in favour of italics or dropped altogether when the expressions come from a formal language) and single quotes when using expressions in order to draw attention to the fact that those expressions are being used (scare quotes, shudder quotes), as well as to refer to headings and article titles – though this attempt may not have been completely successful, especially as the distinction just drawn is not always as clear in practice as it sounds, and the policy has not been imposed in citing passages from other authors (or in index entries).

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