1 Introduction

Radim Belohlavek and George J. Klir

1.1 Concepts and Fuzzy Logic

To avoid any confusion about the terms *concepts* and *fuzzy logic* in the title of this book, let us explain at the very outset what we mean by these terms and how we use them throughout the whole book. We use the term *concepts* as it is commonly used in the literature on the psychology of concepts. Other aspects of concepts, such as philosophical or logical aspects, are not of primary interest in this book. The principal issues involved in the psychology of concepts are presented in chapter 2. We use the term *fuzzy logic* to refer to all aspects of representing and manipulating knowledge that employ intermediary truth-values. This general, commonsense meaning of the term *fuzzy logic* encompasses, in particular, fuzzy sets, fuzzy relations, and formal deductive systems that admit intermediary truth-values, as well as the various methods based on them. An overview of basic ideas of fuzzy logic is presented in the form of a tutorial in chapter 3.

1.2 From Classical Logic to Fuzzy Logic

As is well known, classical logic is based on the assumptions that there are exactly two truth-values, *false* and *true*, and that the truth-value of any logical formula is uniquely defined by the truth-values of its components. These assumptions are usually called *bivalence* and *truth functionality*, respectively. The various many-valued logics, which have been of interest and under investigation since the beginning of the twentieth century (Rescher 1969; Gottwald 2000), abandon bivalence while adhering to truth functionality. This means that additional truth-values are recognized in each many-valued logic. Even though it is not obvious how to interpret these additional truth-values, they are usually viewed as intermediary truth-values between *false* and *true* and interpreted as degrees of truth. Many-valued logics differ from one another in the sets of truth-values they employ and in the definitions they use for basic logical operations, that is, negation, conjunction, disjunction, implication, and equivalence.

Classical logic is closely connected with classical set theory. Each predicate is uniquely associated with a classical set. In other words, for any given object, a proposition formed by the predicate is true for this object if and only if the object is a member of the associated set. The associated set plays the role of the extension of the predicate. For example, the predicate *prime*(x) is true for a particular number n if and only if n is a member of the set of all prime numbers, that is, the set associated in this case with the predicate. Therefore, the set of prime numbers represents the extension of the predicate *prime*(x). Moreover, each logical operation on predicates has a unique counterpart—an operation on the associated classical sets. For example, the counterparts of negation, conjunction, and disjunction on predicates are the operations of complement, intersection, and union on the associated sets, respectively.

When the assumption of bivalence was abandoned in the various proposed many-valued logics, the connection between predicates and sets was lost. Classical sets were simply not able to play the role of extensions of many-valued predicates, that is, predicates that apply to objects to intermediary degrees. The connection was eventually renewed when Lotfi Zadeh introduced the concept of a fuzzy set in his seminal paper (Zadeh 1965).

The connection of fuzzy sets with many-valued logics was recognized by Zadeh in his seminal paper only in a one-sentence remark in a footnote. However, it is worth noting that, independently of Zadeh, set theory for many-valued logics was also investigated in the 1960s by Klaua (1966), as documented by Gottwald (2000).

Zadeh returned to the connection between fuzzy sets and many-valued logics ten years later after his seminal paper, and began to use the term *fuzzy logic* (introduced first by Goguen [1968–69]) in the following sense (Zadeh 1975, 409): "A fuzzy logic, FL, may be viewed, in part, as a fuzzy extension of a multi-valued logic which constitutes a *base* logic for FL." However, he also attempted to expand the notion of fuzzy logics in this sense (usually referred to as fuzzy logics in the narrow sense) with the aim of developing approximate reasoning that would ultimately be able to emulate commonsense human reasoning in natural language. To this end, he introduced appropriate fuzzy sets for representing certain types of linguistic terms employed in human reasoning. For example, fuzzy truth-values are fuzzy sets defined on the set of recognized truth-values (usually the interval [0,1]) that represent linguistic terms such as *true*, *false*, *very true*, *more or less true*, *very false*, and the like; fuzzy probabilities are fuzzy sets defined on [0,1] that represent linguistic terms such as *likely*, *unlikely*, *very likely*, *highly unlikely*, and so on; and fuzzy quantifiers are fuzzy sets defined on appropriate sets of numbers that represent linguistic terms such as *many*, *most*, *almost all*, *very few*, and so forth. This expanded notion of any of the fuzzy logics in the narrow sense is usually called a fuzzy logic in the broad sense.

It is interesting that Zadeh recognized the need for fuzzy logic a few years before he published his seminal paper on fuzzy sets. In a paper discussing developments in the area of system theory (Zadeh 1962, 858), he writes:

> [T]here is a wide gap between what might be regarded as "animate" system theorists and "inanimate" system theorists at the present time, and it is not at all certain that this gap will be narrowed, much less closed, in the near future. There are some who feel this gap reflects the fundamental inadequacy of the conventional mathematics—the mathematics of precisely-defined points, functions, sets, probability measures, etc.—for coping with the analysis of biological systems, and that to deal effectively with such systems, which are generally orders of magnitude more complex than man-made systems, we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities.

Then, he begins his seminal paper as follows:

> More often than not, the classes of objects encountered in the real physical world do not have precisely defined criteria of membership. For example, the class of animals clearly includes dogs, horses, birds, etc. as its members, and clearly excludes objects as rocks, fluids, plants, etc. However, such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the "class" of all real numbers which are much greater than 1.

Clearly, the "class of all real numbers that are much greater than 1," or "the class of beautiful women," or "the class of tall men" do not constitute classes or sets in the usual mathematical sense of these terms. Yet, the fact remains that such imprecisely defined "classes" play an important role in human thinking. . . . The purpose of this note is to explore in a preliminary way some of the basic properties and implications of a concept which may be of use in dealing with "classes" of the type cited above. The concept in question is that of a *fuzzy set*, that is a "class" with a continuum of grades of membership. (Zadeh 1965, 338)

To represent and deal with classes of objects that are not precisely defined was thus the principal motivation for introducing fuzzy sets. Since such classes are pervasive in all human activities involving natural language, fuzzy sets opened new and potentially useful ways of looking at human cognition, reasoning, communication, decision making, and the like. Perhaps the most important of these was a new way of looking at knowledge expressed by statements in natural language. Such knowledge assumed a new significance owing to the possibility of representing it and dealing with it in a mathematically rigorous way. Its utility in science, engineering, and other areas of human affairs has been increasingly recognized, especially since the early 1990s, as is briefly surveyed in section 3.8 of chapter 3. In the next section, we examine how this utility has been viewed in the psychology of concepts.

1.3 Fuzzy Logic in the Psychology of Concepts

Shortly after Zadeh introduced fuzzy sets, Joseph Goguen, a mathematician and computer scientist, published an important paper entitled "The Logic of Inexact Concepts," where he writes:

The "hard" sciences, such as physics and chemistry, construct exact mathematical models of empirical phenomena, and then use these models to make predictions. Certain aspects of reality always escape such models, and we look hopefully to future refinements. But sometimes there is an elusive fuzziness, a readjustment to context, or an effect of observer upon observed. These phenomena are particularly indigenous to natural language, and are common in the "soft" sciences, such as biology and psychology. . . . "Exact concepts" are the sort envisaged in pure mathematics, while "inexact concepts" are rampant in everyday life. . . . Ordinary logic is much used in mathematics, but applications to everyday life have been criticized because our normal language habits seem so different. Various modifications of orthodox logic have been suggested as remedies. . . . Without a semantic representation for inexact concepts it is hard to see that one modification of traditional logic really provides a more satisfactory syntactic theory of inexact concepts than another. However, such a representation is now available (Zadeh 1965). (Goguen 1968–69, 325)

It is interesting that Goguen refers in this quote specifically to biology and psychology as areas of science in which fuzziness is common. However, these two areas of science have been, paradoxically and for different reasons, the slowest ones to harness the capabilities of fuzzy logic. In the following, we focus on the psychology of concepts and show how positive attitudes toward fuzzy logic in the 1970s in this area, revealed by occasional remarks in the literature, changed abruptly to strongly negative attitudes in the 1980s.

Prior to the 1970s, it had been taken for granted in the psychology of concepts that concept categories are classical sets. This generally accepted view of concepts, referred to as the *classical view* (see chapter 2), was seriously challenged in the 1970s, primarily as a result of experimental work by Eleanor Rosch (see chapter 4). Rosch designed and performed a series of psychological experiments that consistently demonstrated (among other things) that concept categories are graded and, as a consequence, that they cannot be adequately represented by classical sets. This led to a virtual deposition of the classical view in the psychology of concepts.

Recognizing Rosch (1973) for her experimental demonstration that concept categories are graded, Lakoff (1972)[1] argued that this was also the case for statements in natural language:

Logicians have, by and large, engaged in the convenient fiction that sentences of natural languages (at least declarative sentences) are either true or false or, at worst, lack a truth value, or have a third value often interpreted as "nonsense," . . . Yet students of language, especially psychologists and linguistic philosophers, have long been attuned to the fact that natural language concepts have vague boundaries and fuzzy edges and that, consequently, natural language sentences will very often be neither true, nor false, nor nonsensical, but rather *true to a certain extent and false to a certain extent, true in certain respects and false in other respects.* (Lakoff 1972, 458, italics added)

Lakoff further argued that fuzzy set theory, as suggested by Zadeh (1965), was potentially capable of dealing with degrees of membership, and hence, with categories that do not have sharp boundaries:

Fuzzy concepts have had a bad press among logicians, especially in this century when the formal analysis of axiomatic and semantic systems reached a high degree of sophistication. It has been generally assumed that such concepts were not amenable to serious formal study. I believe that the development of fuzzy set theory . . . makes such serious study possible. (Ibid., 491)

The potential role of fuzzy set theory in dealing formally with vagueness in natural language was also recognized by Hersh and Caramazza (1976):

Recently, there has been considerable interest on the part of linguists in such problems as the role of vagueness in language and the quantification of meaning. Much of this interest has been the result of the development of fuzzy set theory, a *generalization of the traditional theory of sets*. (255; italics added)

Another author who recognized a potentially fruitful connection between fuzzy sets and concepts was Gregg C. Oden. He addressed this connection in two papers, both published in 1977. In his first paper (Oden 1977a), he builds on previous psychological studies, which "have shown that many subjective categories are fuzzy sets," by studying how proper rules (or operations) of conjunction and disjunction of statements that are true to some degree can be determined experimentally within each given context. His overall conclusion is that "it is not unreasonable for different rules to be used under various situations" (Oden 1977a, 572). This, of course, is well known in fuzzy set theory, where classes of conjunctions, disjunctions, and other types of operations on fuzzy propositions are well delimited and have been extensively researched (see chapter 3 of this volume).

In his second paper (Oden 1977b), he addresses the issue of the capability of human beings to make consistent judgments regarding degrees of membership (or degrees of truth). His conclusion, based on experiments he performed, is positive:

Recent research indicates that class membership may subjectively be a continuous type of relationship. The processing of information about the degree to which items belong to a particular class was investigated in an experiment in which subjects compared two statements describing class membership relationships. The results strongly supported a simple model which describes the judgment process as directly involving subjective degree-of-truthfulness values. The success of the model indicates that the subjects were able to process this kind of fuzzy information in a consistent and systematic manner. (Oden 1977b, 198)

Needless to say, experiments of this kind are considerably more attuned to the spirit of fuzzy logic than the more traditional experiments. Moreover, they also seem to be more meaningful from the psychological point of view. A subject is not required to make a choice between two extremes, neither of which he or she may consider appropriate, but is explicitly allowed to respond in a continuous manner.

The two types of experiments were compared by an experimental study performed by McCloskey and Glucksberg (1978). The study demonstrated that results of experiments that allowed subjects to judge category memberships in terms of degrees showed a significantly higher consistency, especially for intermediate-typicality items, than those in which membership degrees were not allowed.

By and large, these positive attitudes toward fuzzy logic by some psychologists and linguists drastically changed in the psychology of concepts in the early 1980s, and fuzzy logic started to be portrayed as useless for representing and dealing with concepts. It was virtually abandoned in the psychology of concepts as a viable generalization of classical logic. This situation contrasts sharply with numerous other areas, where the expressive power of fuzzy logic has been increasingly recognized and utilized, sometimes in quite profound ways (see chapter 3, sec. 3.8, of this volume).

It is certainly possible that fuzzy logic is completely useless in the psychology of concepts. However, such a conclusion would have to be supported by convincing arguments. As a matter of fact, no such convincing arguments have ever been presented in the literature on the psychology of concepts. As is shown in detail in chapter 5, all the arguments that have actually been presented are, for various reasons, fallacious.

It is clear that this undesirable situation in the psychology of concepts can be resolved in one of two possible ways. One is to find a sufficiently convincing argument that fuzzy logic is not applicable, at least in its current state of development, to the issues of concern in the psychology of concepts. The other way is to demonstrate that fuzzy logic is essential or at least better than classical logic for dealing with at least some of the issues. We believe that neither of these ways of revising the situation can be successful without the cooperation of the two communities involved—psychologists specializing in concepts and mathematicians specializing in fuzzy logic.

In this cooperation, psychologists should explain to the mathematicians those problems regarding concepts for which no mathematical treatment is currently available and challenge them to find the solutions. In a narrower sense, they should also challenge mathematicians to scrutinize any possible new arguments against the use of fuzzy logic they want to pursue. On the other hand, mathematicians should suggest to psychologists some applications of fuzzy logic in the psychology of concepts and challenge them to critically examine their psychological significance.

We are convinced that the time is ripe for such a mutually beneficial cooperation between the two communities, and the main purpose of this book is to stimulate such cooperation.

Cooperation between researchers working in different areas is always difficult, but it is usually very fruitful. The challenges and benefits involved in such cooperation are well captured by Norbert Wiener in his famous book on cybernetics (Wiener 1948). The following quote from the book is based on Wiener's own experience. Wiener, a mathematician, collaborated with a Mexican physiologist, Arthuro Rosenblueth, at the end of World War II, and this collaboration led to profound results in physiology. We took the liberty to add a few words to the quote (all bracketed and in italics) to make the quote more explicitly related to the purpose of this book:

> For many years Dr. Rosenblueth and I had shared the conviction that the most fruitful areas for the growth of the sciences were those which have been neglected as a no-man's land between the various established fields. . . . It is these boundary regions of science which offer the richest opportunities to the qualified investigator. They are at the same time the most refractory to the accepted techniques of mass attack and the division of labor. If the difficulty of a physiological [*or psychological*] problem is mathematical in essence, ten physiologists [*or psychologists*] ignorant of mathematics will get precisely as far as one physiologist [*or psychologist*] ignorant of mathematics. If a physiologist [*or psychologist*], who knows no mathematics, works with a mathematician who knows no physiology [*or psychology*], the one will be unable to state his problem in terms that the other can manipulate and the second will be unable to put the answers in any form that the first can understand. . . . The mathematician need not have the skill to conduct a physiological [*or psychological*] experiment, but he must have the skill to understand one, to criticize one, and to suggest one. The physiologist [*or psychologist*] need not be able to prove a certain mathematical theorem, but he must be able to grasp its physiological [*or psychological*] significance and to tell the mathematician for what he should look. (Wiener 1948, 8–9)

1.4 Summary of the Book

To stimulate cooperation between psychologists of concepts and mathematicians devoted to fuzzy logic, the book contains two tutorials, one on concepts (chapter 2, by Edouard Machery) and one on fuzzy logic (chapter

3, by Radim Belohlavek and George J. Klir). The aim of these tutorials is to help readers who are not psychologists to understand, at least to some degree, experimental and theoretical issues that are relevant to the psychology of concepts, and also to help psychologists to understand the current capabilities of fuzzy logic.

In chapter 4, Eleanor H. Rosch describes her experiments that led to the rejection of the classical view of concepts in the 1970s (as mentioned above in section 1.3). She also describes some peculiar events associated with these experiments. Although she was not aware of fuzzy logic when she designed and performed these experiments and was solely interested at that time in understanding the structure of concept categories from the psychological point of view and not in the issue of how to formalize this structure, she reflects now in chapter 4 on the prospective role of fuzzy logic in the psychology of concepts and makes some valuable suggestions in this regard.

Chapter 5 (again by Belohlavek and Klir) is devoted to a careful analysis of arguments against the use of fuzzy logic in the psychology of concepts that were presented in the early 1980s. First, it is shown that these arguments were actually advanced in a single paper by Osherson and Smith (1981) and that this paper has tremendously influenced attitudes toward fuzzy logic in the psychology of concepts ever since. Second, it is shown in detail that all the arguments presented in this paper are fallacious and that, in spite of this, they were by and large uncritically accepted as sound by those in the field of the psychology of concepts.

The problem of constructing fuzzy sets for representing concepts is discussed in detail in chapter 6, by Jay Verkuilen, Rogier Kievit, and Annemarie Zand Scholten. The authors argue that it is important to look at the various issues involved from the point of view of measurement theory. They show how measurement theory applies to these issues, and they present basic methods for constructing fuzzy sets. They illustrate the general principles by describing two examples in specific detail.

Chapter 7, by Belohlavek, deals with a particular data analysis method in which concepts play a crucial role. This method, called *formal concept analysis*, is based on a rigorous theory of concepts that is inspired by traditional logic. The chapter provides the reader with an overview of basic notions of classical formal concept analysis, its extension to data with fuzzy attributes, and appropriate illustrative examples. Belohlavek also discusses the relationship between formal concept analysis and the psychology of concepts, as well as possible interactive research in these two areas.

Chapter 8 deals with an important issue in the psychology of concepts—the issue of conceptual combinations. It is written by one of the leading psychologists pursuing experimental research on conceptual combinations, James A. Hampton. He describes the outcomes of psychological experiments (many of them performed by himself) that pertain to this issue and discusses the possibilities, as well as difficulties, in using fuzzy logic for formalizing conceptual combinations. The difficulties described here are genuine and can be viewed as challenges to the relevance of fuzzy logic to concepts.

In chapter 9, Hampton examines a particularly important type of concept, namely concepts in natural language—or *lexical concepts*. Special attention is given to the issue of vagueness in meaning and the capability of fuzzy logic to represent and deal with vagueness in natural language.

Chapter 10, the title of which is "Epilogue," is a kind of overall reflection by the editors on the purpose of this book. After examining important distinctions between theories of concepts and mathematical theories, which have often been blurred in the psychology of concepts (as discussed in chapter 5), we outline some challenges for fuzzy logic and some challenges for the psychology of concepts that emerged from this book. Finally, we discuss the conditions for effective cooperation between psychologists working on concepts and mathematicians working on fuzzy logic in the future.

Note

1. Lakoff actually refers to a preliminary version of the cited paper, which was released by the Psychology Department of University of California at Berkeley in 1971 under Rosch's former name of Heider.

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