1 Leverage

We use a very simple model of credit rationing as the basic building block for our liquidity analysis. An entrepreneurial firm has an investment opportunity with a known outcome, but only part of the return is pledgeable to investors. When the pledgeable income is insufficient to cover the full investment cost, the firm has to cover the gap with funds it has accumulated from the past. As a result the firm's investment is constrained by the firm's net worth (unlike in classic theory). We start with a version of the model where the investment scale is fixed. We then introduce a constant-returns-to-scale version, which allows us to study (in chapter 3) the critical trade-off between investment in scale versus investment in liquidity that credit rationed firms inevitably face.

There are many ways to rationalize the assumption that not all of a firm's income is pledgeable. We present a simple moral hazard model with limited liability as an illustration and a reference point for later discussion.

1.1 A Simple Model of Credit Rationing with Fixed Investment Scale

Consider a risk-neutral entrepreneur with an investment opportunity that is worth Z_1 but only $Z_0 < Z_1$ to outside investors. We assume that the initial investment I satisfies $Z_1 > I > Z_0$ (see figure 1.1). The investment has a *positive net present value*, $Z_1 > I$, but it is *not self-financing* because the most that investors can be promised is less than the investment, $Z_0 < I$. The shortfall $I - Z_0 > 0$ must be paid by the entrepreneur (or covered by claims on the market value of the firm's existing assets).

There are a variety of reasons why the full returns of a project cannot be paid out to the investors, that is, why there is a *positive wedge*

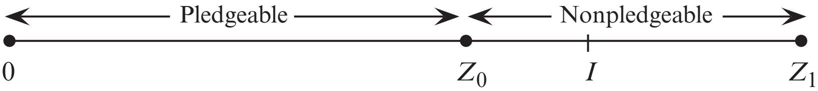

Figure 1.1
Limited pledgeability

(entrepreneurial rent) $Z_1 - Z_0 > 0$. We can put the explanations into two general categories: one based on *exogenous* constraints on payouts and another based on *endogenous* constraints. The prime example of exogenous constraints is a private benefit that only the entrepreneur can enjoy, such as the pleasure of working on a favorite project or the increased social status that comes with its success. A related intangible benefit arises from differences in beliefs. Entrepreneurs often have an inflated view of the chance that their project will succeed.[1] To the extent that the differences in beliefs are not based on better information, the extra utility the entrepreneur derives from overoptimism can, in a one-shot setting, be modeled as a private benefit that investors do not value. There are also tangible benefits that may be impossible to transfer fully, such as the increased value of human capital that comes with investment experience, or the future value that an entrepreneur may enjoy from the option to move after he has been revealed to be a good performer.[2]

In the second category, entrepreneurial rents are endogenous in the sense that while it is feasible to pay out all of the project's returns to the investors, attempts to reduce the entrepreneur's share below $Z_1 - Z_0 > 0$ will inevitably hurt the investors. Therefore it is optimal to let the entrepreneur enjoy a minimum rent. The simplest example is one where the entrepreneur can steal some of the output for private consumption or, equivalently, where the entrepreneur has to be given a share of the output in order to discourage him from diverting output to private consumption (Lacker and Weinberg 1989). Below we consider

1. Of course, the fact that entrepreneurs often fail, or that they express a high confidence in a project when asked, is as such no evidence of overconfidence and can be explained by either agency costs or confidence-maintenance strategies. However, Landier and Thesmar (2009) provide evidence of entrepreneurial overconfidence that is consistent with Van den Steen (2004). Simsek (2010) analyzes financing of projects sponsored by optimistic entrepreneurs. He shows that heterogeneity in beliefs has an asymmetric impact on financing, as financial market discipline operates only when entrepreneurial optimism concerns the likelihood of bad events. By contrast, entrepreneurs who are optimistic about good events can raise substantial amounts.

2. See, for example, Terviö (2009).

a standard moral hazard model with limited liability that leads to the same conclusion.

Because we assumed that the project is not self-financing, $I - Z_0 > 0$, investment will require a positive contribution from the entrepreneur. Let A be the maximum amount of capital that the entrepreneur can commit to the project either personally or through the firm. The project can go forward if and only if the pledgeable income exceeds the project's net financing need $I - A$, that is, when

$$A \geq \bar{A} \equiv I - Z_0 > 0. \tag{1.1}$$

Condition (1.1) puts a lower bound $\bar{A}$ on the amount of assets that the firm or the entrepreneur needs to have in order to be able to attract external funds. A firm with less capital than $\bar{A}$ will be *credit rationed*. It is, of course, possible that $A > I$, in which case no external funds are needed. This is an uninteresting case in the current model, so we will rule it out for the time being. But when we study liquidity shortages in chapter 3, $A > I$ is a legitimate and interesting case.

It bears repeating that a positive entrepreneurial rent $Z_1 - Z_0 > 0$ is necessary for credit rationing. If $Z_1 = Z_0$, then all projects with positive net present value ($Z_1 > I$) are also self-financing ($Z_0 > I$) and hence can move forward. Another necessary condition for credit rationing is that the firm is *capital poor* in the sense that

$$A < Z_1 - Z_0. \tag{1.2}$$

When (1.2) is violated, the firm has enough capital up front to pay for the ex post rents it earns, and therefore all projects with positive net present value can go forward. One can see this formally by rewriting (1.1) in the form

$$Z_1 - I \geq Z_1 - Z_0 - A. \tag{1.3}$$

The left-hand side is the net present value of the project. The right-hand side is the net rent enjoyed by the entrepreneur after investing all his net worth in the project. If the right-hand side is negative, all projects with a positive net present value can proceed. It is only when the firm is capital poor and (1.2) holds that valuable projects may be rejected. Stated more strongly, condition (1.2) has the important implication that for a capital poor firm there will always be projects with a positive net present value that have to be rejected because the firm does not have enough capital.

Let us finally note that the internal cost of capital is above the market rate (0) below the point where the firm is credit rationed as can be seen by considering the entrepreneur's utility payoff U.

$$
\begin{aligned}
U &= A + Z_1 - I, \quad \text{if} \quad A \geq \bar{A}, \\
U &= A, \qquad\qquad\quad \text{if} \quad A < \bar{A}.
\end{aligned} \tag{1.4}
$$

Because utility jumps up at $A = \bar{A}$, the value of funds inside the firm is strictly higher than outside the firm below $\bar{A}$.[3] When $A < \bar{A}$, total output can be increased by transferring funds from investors to capital poor entrepreneurs, but such transfers will not be Pareto improving. In models with non-transferable utility, Pareto optimality does not imply total surplus maximization.

1.2 A Simple Moral Hazard Model of the Wedge between Value and Pledgeable Income

1.2.1 The Wedge as an Incentive Payment

Our liquidity analysis proceeds largely without reference to the particular reasons behind the nonpledgeable income wedge $Z_1 - Z_0$. But to gain a better grasp of the economic significance of this analysis, it is worth going beyond the reduced-form model. In this section we analyze a specific model in which the wedge appears endogenously.[4] The analysis will highlight important determinants of the firm's debt capacity, illustrate the impact that credit rationing may have on the firm's choice of investments, and indicate the benefits and costs of using different kinds of collateral.

We employ a standard model of investment with moral hazard.[5] There is a single entrepreneur (firm) and a competitive set of outside investors. All parties are risk neutral. There is a single good used for consumption as well as investment. There are two periods. In the initial period, indexed $t = 0$, there is an opportunity to invest. The investment costs I. The gross payoff of the investment one period later ($t = 1$) is either R (a success) or 0 (a failure). The probability of success depends on an

3. Formally, the marginal internal cost of capital is equal to 0 up to $\bar{A}$ and jumps to infinity at $\bar{A}$. With a continuous investment choice this cost varies more smoothly and exceeds the market rate; see section 1.3.

4. Note that even if a number of explanations can be given for a positive wedge, we will take the wedge as a primitive in our analysis. This is why we provide an explicit model that justifies treating the wedge as exogenous in the analyses we will be considering.

5. The model is taken from Holmström and Tirole (1998), but it has many antecedents.

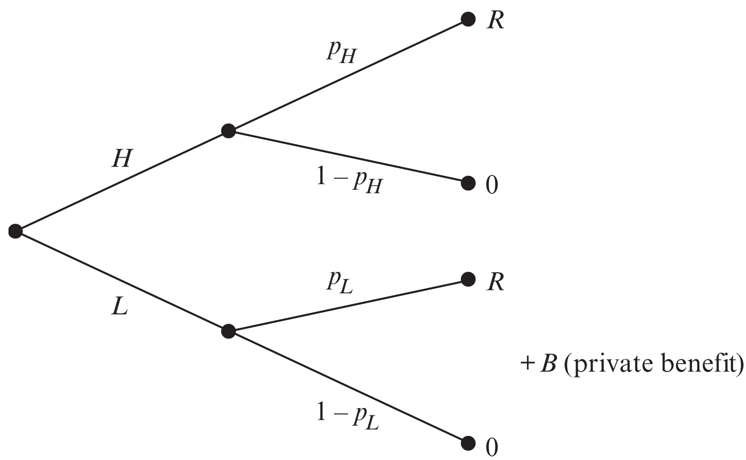

Figure 1.2
Description of moral hazard

unobserved action taken by the entrepreneur. The action represents the entrepreneur's choice of where to invest the funds, I. The intended purpose is to invest in an efficient technology H, which gives a high probability of success p_H. The entrepreneur also has the option to invest in an inefficient technology L, which gives a lower probability of success $p_L < p_H$, but provides the entrepreneur with a private benefit B. (For instance, the inefficient technology may only cost $I - B$, leaving B for the entrepreneur's private consumption.) The choice and payoff structure is described in figure 1.2.

We assume that there is no discounting between the periods and that the expected return of the investment is negative if the low action is taken and positive if the high action is taken:

$$p_H R - I > 0 > p_L R - I + B \tag{1.5}$$

Thus it is better not to invest at all than to invest and have the firm choose the inefficient technology L.

The entrepreneur has assets worth A. These assets are liquid in the sense that they have the same value in the hands of the entrepreneur as in the hands of investors. The firm is protected by limited liability. We assume again that $A < I$ so that the firm needs to raise $I - A > 0$ from outside investors in order for the project to go forward. Investors can access an unlimited pool of funds, and they demand an interest rate that we normalize to 0.

Investors can be paid contingent on the outcome of the project. Let X_s (X_f) be the entrepreneur's date-1 wealth in case the project succeeds (fails). Limited liability requires that $X_i \geq 0$, $i = s, f$. Investors receive $Y_s = R - X_s$ if the project succeeds and $Y_f = -X_f$ if it fails.

We are interested in the conditions under which the investment can go ahead. There are two constraints that must be satisfied. First, the investors need to break even,

$$p_H(R - X_s) + (1 - p_H)(-X_f) \geq I - A. \tag{1.6}$$

Second, the entrepreneur must be induced to be diligent,

$$p_H X_s + (1 - p_H)X_f \geq p_L X_s + (1 - p_L)X_f + B. \tag{1.7}$$

Simplified, this incentive compatibility constraint reads as

$$X_s - X_f \geq \frac{B}{\Delta p}, \tag{1.8}$$

where

$$\Delta p \equiv p_H - p_L > 0. \tag{1.9}$$

Incentive compatibility (1.8) paired with limited liability implies that the entrepreneur earns a positive rent. This rent is minimized by setting $X_f = 0$ and $X_s = B/\Delta p$. The rent cuts into the amount that can be paid out to investors. The firm's *pledgeable income* is defined as the maximum expected amount that investors can be promised when the entrepreneur is paid the minimum rent. The pledgeable income is

$$Z_0 = p_H\left(R - \frac{B}{\Delta p}\right). \tag{1.10}$$

To complete the link to the reduced form discussed earlier, denote the output $Z_1 = p_H R$. The positive wedge is then equal to the entrepreneur's minimum rent $Z_1 - Z_0 = p_H(B/\Delta p)$.

1.2.2 Factors Influencing Pledgeable Income

Bias toward Less Risky Projects The net worth of a firm may altogether prohibit it from investing, as discussed above. More generally, a firm's net worth will merely limit which projects it can invest in. Assume that there is a set of projects that the firm and the investors can jointly choose from. The firm can more easily satisfy (1.1) by choosing projects with a smaller investment scale I or a higher pledgeable income.

For example, in the incentive payment illustration, each project is characterized by a tuple (I, R, p_H, p_L, B). Pledgeable income increases in p_H and R and decreases in p_L and B, reflecting the fact that the entrepreneur's incentive problem is less severe when the efficient choice H becomes more attractive relative to the inefficient choice L. More interesting, consider variations in p_H and R that leave the expected payoff of the desired project Z_1 and the other parameters unaltered. Specifically, assume that p_H goes down while R goes up so that the project becomes more risky. Other things equal, the firm's pledgeable income decreases with such risk. A decrease in p_H increases the rent $p_H B/\Delta p$ that goes to the entrepreneur, since the entrepreneur's reward in the successful state (the only incentive instrument available) is less potent the lower is p_H. With a higher entrepreneurial rent, less can be promised to investors (Z_0 is lower), which raises the cutoff value $\bar{A}$. At the margin, capital-constrained firms will therefore accept safer projects at the expense of lower expected returns.

Diversification A variant on the theme above occurs when diversification helps reduce the need for own funds. Suppose that a single project can be replaced by two identical, half-sized projects of the sort we have discussed. Assume that the projects are stochastically independent and that the entrepreneur chooses separately, but simultaneously, whether to be diligent in executing each project. One can show that in this case the optimal incentive scheme pays the entrepreneur a positive amount only when both projects succeed. The entrepreneur in effect pledges the rewards that accrue from a successful project as collateral for the other project. This maximizes the pledgeable income.

For diversification to be of value, it is important that the projects be independent. If the projects are perfectly correlated (or the entrepreneur opportunistically choses them to be perfectly correlated), diversification does not raise the pledgeable income.[6]

Intermediation Another way of increasing the pledgeable income is to reduce the entrepreneur's opportunity cost of being diligent. Some projects are more conducive to misbehavior than others, for instance, projects that are exceptional, that do not have tangible investments, or that involve poor accounting. A capital poor firm can sometimes increase

6. For more on diversification in this type of model, see Conning (2004), Hellwig (2000), Laux (2001), and Tirole (2006, ch. 4).

its pledgeable income by turning to an intermediary that has monitoring expertise. A simple way to model monitoring is to assume that the intermediary can reduce B to a lower level b (and perhaps simultaneously reduce p_L) because it can place constraints on what the firm can do. Loan covenants serve this purpose: for instance, lending contracts frequently forbid the firm from paying dividends if certain financial conditions are violated. Covenants may also give the bank veto rights on the sale of strategic assets and spell out circumstances under which the bank can intervene even more aggressively by getting the right to nominate all or part of the board. Another potential interpretation of the monitoring activity is that the bank acquires information that is relevant for decision-making and uses it to convince the board not to rubberstamp (what turns out to be) the management's pet project. In the model, and apparently in reality, giving the firm less attractive outside options reduces entrepreneurial rents, increases pledgeable income and thus lowers $\bar{A}$. The carrot can be smaller if the stick is bigger.

Of course, intermediation is not free. To determine whether intermediaries can really increase pledgeable income, monitoring costs must also be taken into account. One can distinguish at least three kinds of monitoring costs from intermediation:

1. Direct costs are incurred by the intermediary as well as the firm due to the additional work involved in evaluating investments, processing loans, and monitoring compliance with covenants.

2. Constraints imposed on a firm as part of a loan covenant do not merely cut out illegitimate opportunities, they also cut out legitimate ones. A firm that cannot sell or acquire significant assets without the approval of a bank may have to forego valuable deals. Excluding profit opportunities of this kind lowers Z_1 and reduces the project's expected return.

3. Monitoring expertise is scarce and commands rents that depend on market conditions. In Holmström and Tirole (1997), we study a model where the monitor can itself act opportunistically and therefore has to be given a share in the firm's payoff. This increases $\bar{A}$ by an amount that gets determined by the demand for intermediation among credit-constrained firms. In equilibrium firms sort themselves into three groups as a function of their net worth: firms that have too little own capital to be able to invest, firms that have enough own capital to go directly to the market and so do not need intermediation, and firms that have

intermediate amounts of capital and invest with the help of intermediaries. In the last instance, funding comes both from the informed investors (intermediaries) and from the uninformed investors (the general market) that invest only because the intermediary's participation has reduced the risk of opportunism.

1.3 Variable Investment Scale

For the upcoming liquidity analysis we need a model where investment is variable so that we can study the important trade-off between the scale of the initial investment and the decision to save some funds to meet future liquidity shocks. A simple, tractable model is obtained by letting the investment vary in a constant-returns-to-scale fashion.

Let I be the scale of the investment (measured by cost), let ρ_1 be the expected total return, and ρ_0 the pledgeable income, both measured *per unit invested*. Thus I results in a total payoff $\rho_1 I$ of which $\rho_0 I$ can be pledged to outside investors. The residual $(\rho_1 - \rho_0)I$ is the minimum rent going to the entrepreneur.[7]

The moral-hazard model of section 1.2 fits this framework if we assume that a successful project returns RI and the private benefit to the entrepreneur from cheating is BI. In that case,

$$\begin{aligned} \rho_1 &= p_H R, \\ \rho_0 &= p_H \left(R - \frac{B}{\Delta p} \right). \end{aligned} \tag{1.11}$$

As before, we assume that projects are socially valuable but not self-financing:

$$0 < \rho_0 < 1 < \rho_1. \tag{1.12}$$

Consequently the entrepreneur needs own funds $A > 0$ to invest. For each unit of investment the firm can raise ρ_0 from outside investors, leaving the *minimum equity ratio* $1 - \rho_0 > 0$ to be covered by own funds. The repayment constraint is

$$A \geq (1 - \rho_0)I,$$

7. The parameters ρ_1 and ρ_0 correspond to the parameters Z_1 and Z_0 of the fixed investment model in section 1.1 at $I = 1$.

implying a maximum investment scale

$$I = kA = \frac{A}{1 - \rho_0}. \tag{1.13}$$

The *equity multiplier* $k \equiv 1/(1 - \rho_0) > 1$, the inverse of the (minimum) equity ratio, defines the firm's maximum leverage per unit of own capital. A firm with 10 units of own capital and a required minimum equity ratio of 20 percent can invest a maximum of 50 units.

If the firm chooses the maximum investment scale, the entrepreneur's gross payoff is

$$U^g = \frac{(\rho_1 - \rho_0)A}{1 - \rho_0} = \mu A,$$

where

$$\mu \equiv \frac{\rho_1 - \rho_0}{1 - \rho_0}. \tag{1.14}$$

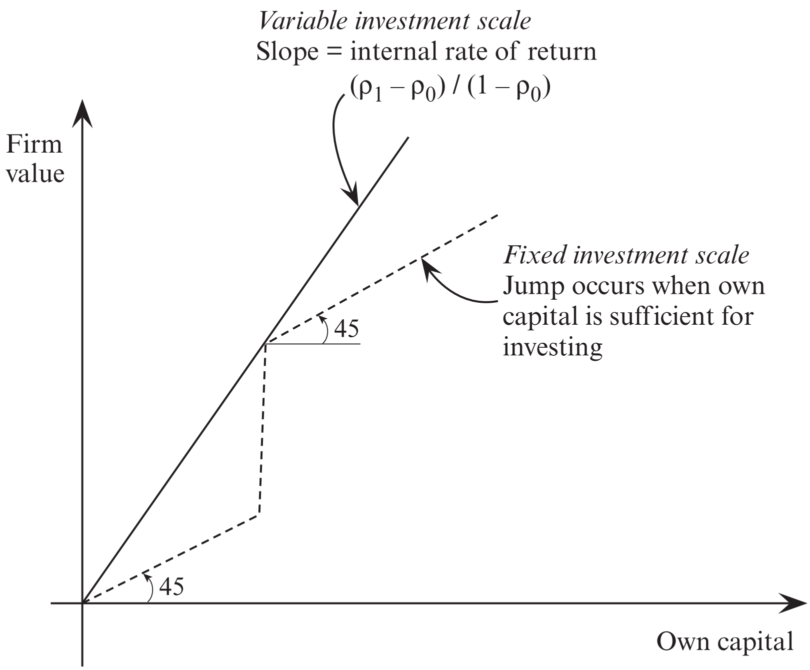

Figure 1.3
Internal rate of return

The entrepreneur's net utility is

$$U = (\mu - 1)A = \frac{\rho_1 - 1}{1 - \rho_0}A.$$

For each unit invested, the entrepreneur enjoys a rent $\rho_1 - \rho_0$. Thanks to the equity multiplier $k > 1$, the rent gets magnified, resulting in a gross rate of return on own capital $\mu > 1$; see figure 1.3. The rate is constant because of the constant-returns-to-scale technology. More important, the rate is greater than 1 because of (1.12), implying that the internal rate of return exceeds the market rate of interest. By transferring a unit of the good from investors to the entrepreneur, total social surplus $(\rho_1 I - I)$ could be increased by more than one unit. But such transfers are not Pareto improving, since the increase in total surplus cannot be arbitrarily split between the investors and entrepreneurs.

To see this, note that the entrepreneur maximizes his utility by choosing the maximum investment scale (1.14), since the rate of return on entrepreneurial capital exceeds the market rate. He puts all his wealth in the illiquid portion of the return (the nonpledgeable return $(\rho_1 - \rho_0)I$), leaving outsiders holding the firm's liquid assets. While total output could be raised by transferring wealth from passive investors to active entrepreneurs, investors cannot be compensated as they already hold all the firm's liquid claims. There is nothing that the government can do to improve on private contracting.

1.4 Comparative Statics and Investment Implications

Factors that increase ρ_0 or ρ_1 (or both) will increase the entrepreneur's utility and an increase in ρ_0 will also increase the investment scale I. Investors are simply paid their market rate of return, so they remain unaffected by these changes.

Recall that in the moral-hazard model, ρ_0 increases with R and p_H and decreases with B and p_L, while ρ_1 increases with R and p_H. If the firm could choose among investments that differed in their attributes ρ_0 and ρ_1, the firm would not simply choose the investment that maximizes the social net present value per unit, that is, the investment with the highest ρ_1. The pledgeable income is also critical as it determines the extent to which the firm can lever its capital. From (1.14) we see that the firm's willingness to substitute ρ_0 for ρ_1 is given by

$$\frac{d\rho_1}{d\rho_0} = 1 - \mu < 0. \tag{1.15}$$

The firm will choose projects with lower ρ_1 up to the point where the reduction in ρ_1 per unit of increase in ρ_0 equals the difference between the internal rate of return and the market rate of return. Each unit of pledgeable income ρ_0 is worth $\mu - 1$ units of ρ_1 because of scale expansion. This illustrates one of the central themes of credit constrained lending: the willingness to sacrifice net present value for an increase in pledgeable income.

1.5 Concluding Remark

This chapter introduced a simple agency model to create a link between a firm's net worth and its scale of investment. Even at this basic level it shows how moral hazard problems influence the firm's leverage and economic activity.[8] In all models of credit rationing, the leverage of net worth implies that the return on inside funds is higher than the market return on capital. Therefore the economy's total output can be increased by redistributing capital from investors to entrepreneurs. In reality the effectiveness of such redistributions is limited by the difficulty of identifying which entrepreneurs are able to use the capital constructively. The limited scope of venture capital funding makes this evident. In our model (and many others) there are no such informational limitations. However, as we argued, even if one were able to identify deserving entrepreneurs and redistribute to them funds from passive investors, this cannot be done in a manner that makes both entrepreneurs and investors better off. The next chapter shows that the situation changes when one adds to the model a period in which firms can anticipate future liquidity needs and make arrangements to fund them. What is redistribution in the one-period model becomes insurance in this two-period model. This simple observation about insurance lies at the heart of the analysis of liquidity provision both privately and publicly.

8. Bernanke and Gertler (1989) were the first to demonstrate the importance of net worth for investment and economic activity. They used a different model of moral hazard (the costly state verification model due to Townsend 1979).