Chapter 7 Supplement

Real and Imaginary Components

You may have heard people talk about the "sine and cosine components" of a Fourier transform. These are sometimes called the "real and imaginary components." These are just alternative ways of talking about amplitude and phase, and there is nothing "imaginary" about them.

This terminology comes from the fact that a sinusoidal waveform of a particular frequency can be represented by the sum of a sine wave and a cosine wave. As shown in figure S7.1A, a cosine wave is just a sine wave that has been shifted in time by one-quarter cycle (90°). Sine and cosine waves have an interesting property; namely, that a sinusoidal waveform of any amplitude and phase can be created by summing together a sine wave and a cosine wave of that same frequency. Figure S7.1B shows an example in which a cosine wave of amplitude 1.0 is added to a sine wave of amplitude 0.3, and the result is a sinusoid that has the same frequency as the sine and cosine waves but is slightly larger than, and shifted to the left of, the cosine wave. Instead of represent-

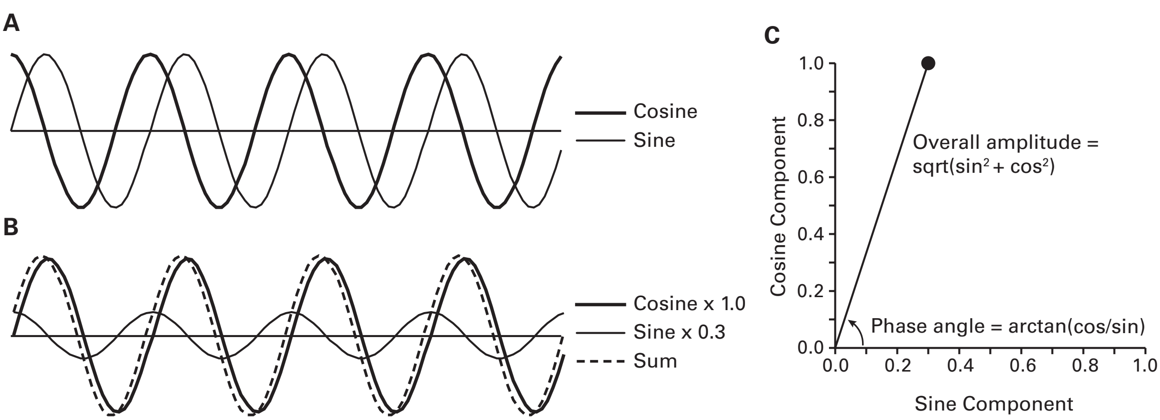

Figure S7.1
(A) Sine and cosine waves of the same amplitude and frequency. (B) Example of how adding a sine wave and a cosine wave yields a sinusoidal waveform of the same frequency but a different phase and amplitude. Any sinusoidal waveform can be reconstructed by adding together a sine wave and a cosine wave of the same frequency, but with different amplitudes. (C) Graphical depiction of the relationship between the sine and cosine components and the phase and amplitude of the overall sinusoidal waveform.

ing a given sinusoid in terms of its amplitude and phase, it is therefore possible to represent it in terms of the amplitudes of the sine and cosine waves that need to be added together to create this sinusoid. This is convenient mathematically, but it makes things much less intuitive.

Figure S7.1C shows how the phase and amplitude of the sinusoid are related to the strength of the sine and cosine components. If you plot the amplitude of the cosine component on the *X* axis and the amplitude of the sine component on the *Y* axis, the amplitude and phase of the overall waveform are represented by the point that represents the combination of the sine and cosine components. Specifically, the amplitude of the sinusoid is the length of the line from (0, 0) to the coordinate of the (cosine, sine) pair. The Pythagorean theorem tells us that the length of this line is the square root of the sum of the squared sine and cosine values. In our example, with a sine amplitude of 1.0 and a cosine amplitude of 0.3, the amplitude of the summed sinusoid is sqrt($1.0^2 + 0.3^2$) = 1.044. The phase is the angle of the line, which is equal to arctan(cosine/sine) = 1.279 radians = 73.3°.

This way of plotting the sine and cosine components uses *polar coordinates*, in which a given point is represented by an angle and a distance. This is the same system used to represent complex numbers, which have a *real* component and an *imaginary* component (where imaginary numbers involve taking the square root of a negative number). This is why people sometimes refer to the sine and cosine components as the real and imaginary components. However, this is just an analogy. There is nothing imaginary about the cosine component; it is every bit as real as the sine component. So don't worry that Fourier analysis involves something suspicious or spooky because it has an imaginary component. This is just an analogy.