Isaac Newton on Mathematical Certainty and Method

Niccolò Guicciardini

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1 Newton on Mathematical Method: A Survey

> For in those days I was in the prime of my age for invention & minded Mathematicks & Philosophy more then at any time since.
>
> — Isaac Newton, 1718

1.1 Early Influences

Mathematics played a prominent role in Newton's intellectual career. This was not, of course, his only concern. A polymath and polyhistor, Newton devoted years of intense research to the reading of the Books of Nature and Scripture, deploying the tools of the accomplished "chymist" (at the furnace and at the desk), instrument maker (he made his own instruments, among them the first reflecting telescope), experimentalist, astronomer, biblical interpreter, and chronologist. In all these fields mathematics entered as one of the most powerful and reliable tools for prediction and problem solving, and as the language that guaranteed accuracy and certainty of deduction. Newton would not have achieved most of his results without it.[1] It is no coincidence that the adjective mathematical enters into the title of his masterpiece.

When Newton matriculated at Cambridge in 1661, he possessed only a modicum of mathematical training. Two years later the first Lucasian Chair of Mathematics was conferred on Isaac Barrow, a scholar of broad culture who would play an important role in Newton's intellectual life. The existence of such chairs, which provided mathematical teaching at the universities, was something of a novelty in England.[2] Barrow therefore had to defend his discipline and lectured on the usefulness of mathematical learning. He did so in verbose and scholarly lectures, which Newton probably attended. Barrow patterned his peroration following the agenda set by Proclus, and he had in mind a late-sixteenth-century debate over the certainty of mathematics, which was sparked in 1547 by Alessandro Piccolomini's commentary

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Epigraph from MS Add. 3968.41, f. 85r. For a discussion of this memorandum see Westfall, *Never at Rest* (1980), p. 143, and "Newton's Marvelous Years of Discovery and Their Aftermath" (1980); Hall, *Philosophers at War* (1980), pp. 10–23. See also Whiteside, "Newton's Marvellous Year" (1966). The best guide to Newton's mathematical work is to be found in Whiteside's commentary to *Mathematical Papers*.

[1] Jed Buchwald and Mordechai Feingold are currently examining Newton's work on chronology. Their research reveals the importance of new mathematical techniques in treating astronomical and historical data.

[2] For the antecedents, see Feingold, *The Mathematicians' Apprenticeship* (1984).

Table 1.1 Mathematical Books Annotated by Newton in the 1660s

René Descartes	*Geometria, à Renato des Cartes*, Amsterdam, 1659–61
François Viète	*Opera Mathematica*, Leiden, 1646
Frans van Schooten	*Exercitationum Mathematicarum*, Leiden, 1657
William Oughtred	*Clavis Mathematicae*, 3d ed., Oxford, 1652
John Wallis	*Operum Mathematicorum Pars Altera*, Oxford, 1656
John Wallis	*Commercium Epistolicum*, Oxford, 1658

on pseudo-Aristotle's *Problemata Mechanica*.[3] The rising status of mathematics was opposed by some Aristotelian philosophers like Piccolomini, who maintained that mathematics did not possess the deductive purity of syllogistic logic and was not a science because it did not reveal causal relationships. Barrow's defense of geometry as a model of reasoning and his idea that since geometrical magnitudes are generated by motion, a causal relationship can be captured in such mechanically based geometry must have impressed the young scholar. These typically Barrovian ideas remained the backbone of Newton's views about mathematics.

Newton soon began to read advanced mathematical texts, possibly borrowing them from the Lucasian Professor. The mathematical books he had on his desk, which he annotated extensively, are listed in table 1.1. As is often repeated in later memoranda and hagiographic biographies, he devoted little attention to ancient geometry, which is at odds with his mature predilection for the ancients, which began to flourish in the 1670s. As far as we know, of the ancient corpus he studied only Euclid's *Elements* in Barrow's algebraized edition.[4] He learned algebraic notation from Oughtred's *Clavis Mathematicae* in the third 1652 edition, and from Viète's *Opera Mathematica* (1646). These last two works were based on the idea that algebra is not a deductive theory, like the *Elements*, but rather an analytical, heuristic tool that can extend the possibility of finding solutions to problems, especially geometrical problems. The annotations to Oughtred and Viète show how interested Newton was in this promising method of discovery.[5] Algebra was still a novel language in England. Oughtred had been a pioneer (his *Clavis* had first appeared in 1631), but in the 1660s there was still need for an updated text on algebra. In 1669, Newton became involved in the project of producing such a text-

[3] Piccolomini, *In Mechanicas Quaestiones Aristotelis* (1547). On Barrow's reading of Proclus, see Stewart, "Mathematics as Philosophy: Proclus and Barrow" (2000). On the debate initiated by Piccolomini, there is a vast literature; see Jardine, "Keeping Order in the School of Padua" (1997).
[4] *Euclidis Elementorum Libri XV Breviter Demonstrati* (1655).
[5] MP, 1, pp. 25–88.

book (see chapter 4). Algebra was interesting as a tool for practical applications (it answered the needs of cartographers, instrument makers, mechanics, accountants, land surveyors) but it was also promising for more theoretical purposes. The latter motivation was the stimulus for Newton.

The seminal text in Newton's mathematical formation is a highly abstract essay: Descartes' *Géométrie*. He borrowed and annotated the second Latin edition (1659–1661) by Frans van Schooten.[6] Here Descartes had proposed a novel method for the solution—he claimed in the opening sentence—of all the problems of geometry. It was on this text that Newton concentrated his attention. Descartes taught how geometrical problems could be expressed in terms of algebraic equations (this process was termed the resolution or analysis of the problem). He maintained that finding the equation and determining its roots, either by finite formulas or approximations, is not the solution of the problem (see chapter 3). It was not a surprise for the contemporaries of Descartes and Newton to read that in order to reach the solution, one had to geometrically construct the required geometrical object. A geometrical problem called for a geometrical construction (a composition or synthesis), not an algebraic result. Traditionally, such constructions were carried out by means of intersecting curves. Thus, Descartes provided prescriptions to construct segments that geometrically represent the roots and are therefore the solution of the problem.

By Newton's day the heuristic method proposed by Descartes was labeled common analysis. It was contrasted with a more powerful new analysis, which tackled problems about tangents and curvature of curves and about the determination of areas and volumes that cannot be reached by the finitist means envisaged by Descartes. Common analysis proceeds by "finite" equations (algebraic equations, we would say) in which the symbols are combined by a finite number of elementary operations. The new analysis instead goes beyond these limitations because it makes use of the infinite and infinitesimal.

Basically, Newton and his contemporaries understood both the common analysis and the new analysis, where respectively "finite" and "infinite equations" (infinite series and infinite products) were deployed, as heuristic tools useful in discovering a solution. Analysis, however, had to be followed by synthesis, which alone, in their opinion, could provide a certain demonstration. Barrow much concerned himself with synthesis and, in his lectures defending mathematical certainty, aimed to provide synthetic demonstrations of the results reached by the heuristic techniques characteristic of the new analysis. His young protégé was making inventive forays into the new analysis. Newton was aware, however, that a synthetic construction was needed, and he later turned to Barrow for inspiration.

[6] Newton worked on the second Latin edition, but he might also have encountered the smaller first Latin edition prepared by van Schooten, which appeared in 1649. A copy of the first edition (University Library (Cambridge) Adv.d.39.1) might have been in Newton's possession, but "its brief manuscript annotations are not in Newton's hand." MP, 1, p. 21.

1.2 First Steps

Newton's early notes on Descartes' *Géométrie* reveal how quick he was in mastering algebra applied to geometry. In 1665 he began to think about how the equation could reveal properties of the curve associated to it via a coordinate system. Actually, he began to experiment with alternative coordinate systems to orthogonal or oblique axes. He tried, for instance, what we call polar, bipolar, or pedal coordinates. He also began to work with transformation of coordinates. One line of research consisted in trying to extend algebraic treatment beyond the conic sections. In *De Sectionibus Conicis, Nova Methodo Expositis Tractatus*, which Newton read in the *Operum Mathematicorum Pars Altera* (1656), Wallis had developed an algebraic treatment of conics as graphs of second-degree equations in two unknowns. Newton began to extend the definitions of diameter, chord, axis, vertex, center, and asymptote to higher-order algebraic curves. In the late 1660s he made his first attempts to graph and classify cubic curves.[7]

Another line of research concerned the so-called organic description (or generation) of curves.[8] This was an important topic, since in order to determine the point of intersection of curves in the construction of geometrical solutions, it was natural to think of the curves as generated by a continuous motion driven by some instrument (an $o\rho\gamma\alpha\nu o\nu$). It is the continuity of the motion generating the curves that guarantees a point of intersection can be located exactly. Descartes had devised several mechanisms for generating curves. In *De Organica Conicarum Sectionum in Plano Descriptione Tractatus* (1646), which Newton read in *Exercitationum Mathematicarum* (1657), van Schooten had presented several mechanisms for generating conic sections. This research field was connected with practical applications, for instance, lens grinding and sundial design, but it was also sanctioned by classical tradition and motivated the highly abstract needs underlined by Descartes. Newton was able to devise a mechanism for generating conics and to extend it to higher-order curves (§5.4).

In 1665, Newton deployed organic descriptions in order to determine tangents to mechanical lines, that is, plane curves such as the spiral, the cycloid, and the quadratrix that Descartes had banned from his *Géométrie* (see chapter 3). The study of mechanical lines, curves that do not have an algebraic defining equation, was indeed a new, important research field. How to deal with them was unclear. Newton was able to determine the tangent to any curve generated by some tracing mechanism. He decomposed the motion of the tracing point P, which generates the curve, into two components and applied the parallelogram law to the instantaneous component velocities of P (see the parallelogram law on the top of the left margin

[7] MP, 1, pp. 155–244.
[8] MP, 2, pp. 134–42 and 152–5.

in figure 1.1). For instance, the point of intersection of two moving curves will generate a new curve whose tangent Newton was able to determine. Such a method for determining tangents without calculation pleased Newton as much as did his new techniques for the organic description of conics. This was an approach to the study of curves—alternative to the Cartesian algebraic—that Barrow had promoted and that in the 1670s Newton began to couple with ideas in projective geometry. Already in 1665 the master in the common and new algebraic analyses was experimenting with non algebraic approaches to geometrical problems.

In these early researches one encounters a characteristic of Newton's mathematical practice, a deep intertwining between algebra and geometry, that eventually led to unresolved tensions in his views on mathematical certainty and method. Indeed, it is often the case that in tackling a problem Newton made recourse to a baroque repertoire of methods: one encounters in the same folios algebraic equations, geometrical infinitesimals, infinite series, diagrams constructed according to Euclidean techniques, insights in projective geometry, quadratures techniques equivalent to sophisticated integrations, curves traced via mechanical instruments, numerical approximations. Newton's mathematical toolbox was rich and fragmentary; its owner mastered every instrument it contained with versatility. But he was also a natural philosopher who envisaged a role for mathematics that did not allow him to leave the toolbox messy, albeit efficient, and open for unauthorized inspection.

1.3 Plane Curves

How did the young Newton tackle a problem that was quite difficult in his day: the drawing of tangents to plane curves? Figure 1.1 shows the first folio of a manuscript dated by Newton (in retrospect?) November 8th, 1665, and entitled "How to Draw Tangents to Mechanicall Lines." In the left margin there are an Archimedean spiral, a trochoid, and a quadratrix.[9]

Tracing the tangent to the spiral was particularly handy. To a point b of a spiral with pole a (see figure 1.2) Newton associated a parallelogram having a vertex in b whose sides, the former bc directed along the radius vector ab and the latter bf orthogonal to it, are proportional to the radial speed and to the transverse speed of b. The diagonal bg determines the tangent at b. In other cases, the method was more difficult to implement, and Newton made a couple of blunders, which he soon corrected, in tracing the tangent to the quadratrix and to the ellipse.[10]

[9] In modern symbols these three curves have equations $r = c_0\theta$ (r, θ polar coordinates, c_0 constant), $x = c_1 t - c_2 \sin t$, $y = c_1 - c_2 \cos t$ (parametric equations, $c_2 < c_1$), and $x = y\cot(\pi y/2c_3)$ (x, y, Cartesian coordinates).
[10] MP, 1, pp. 379–80.

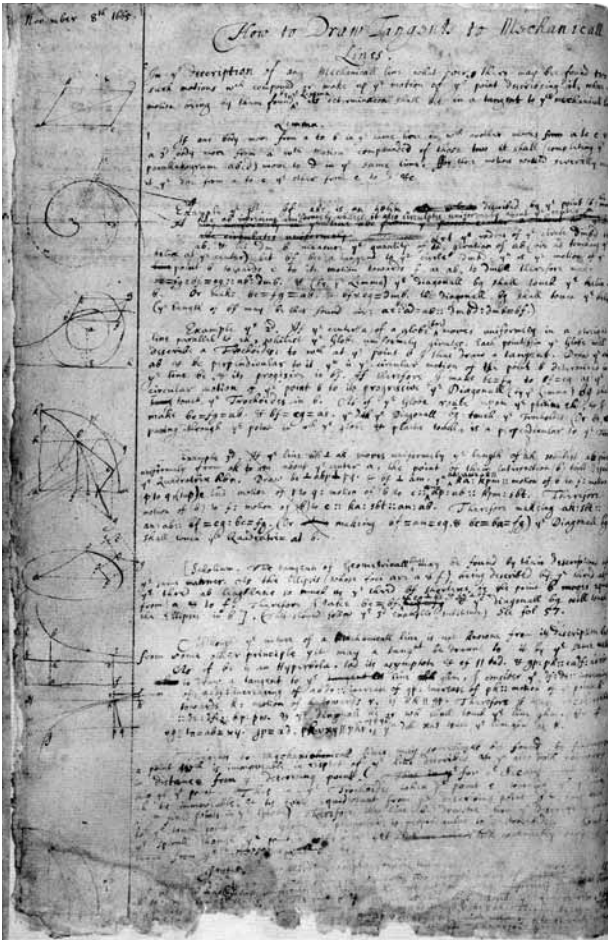

Figure 1.1

Newton's kinematic method for drawing tangents to mechanical curves. From top to bottom of the left margin, below the illustration of the parallelogram, are the following curves. (i) The Archimedean spiral is traced by a point that slides with constant speed along a straight line that rotates with constant angular speed. (ii) The trochoid (sometimes called curtate cycloid) is traced by a point on a disk that rolls without sliding along a straight line. (iii) The quadratrix is a curve traced by the intersection of a radius and a line segment moving at corresponding rates. A square and a circle are drawn so that one corner of the square is the center of the circle, and the side of the square is the radius of the circle. A radius rotates clockwise from the side of the square to the base at a constant angular speed. At the same time, a line segment falls from the top of the square at constant vertical speed and remains parallel to the base of the square. Both start moving at the same time, and both hit the bottom at the same time. Newton also considers two "Geometricall lines," namely (iv) the ellipse, and (v) the hyperbola. Source: Add. 4004, f. 50v. Reproduced by kind permission of the Syndics of Cambridge University Library.

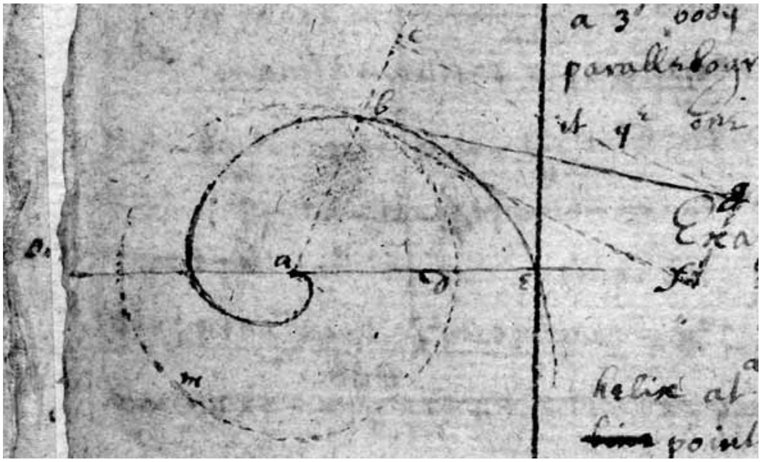

Figure 1.2
Tracing the tangent to the Archimedean spiral. Source: Add. 4004, f. 50v. Reproduced by kind permission of the Syndics of Cambridge University Library.

Newton applied his method for drawing tangents not only to mechanical but also to geometrical lines: the ellipse and the hyperbola.[11]

In his early papers, Newton intertwined the geometrical approach to tangents with the development of a new algorithm, which he called the method of series and fluxions. This method allowed the calculation of the tangent and curvature to all plane curves known in Newton's day. Later, I describe Newton's algorithm for the determination of tangents (§8.3.6) and its application to the conchoid (Cartesian equation $x^2y^2 = (c_1 + y)^2(c_2^2 - y^2)$). Undoubtedly, this algorithm, referred to in modern textbooks as the calculus, is the most celebrated discovery that Newton made in the years 1664–1666. This highly symbolic and algebraized tool of problem solving is discussed in part III. It should be stressed, however, that what appears, with the benefit of hindsight, to be Newton's greatest achievement was perceived as just one among many alternative approaches to problem solving by its inventor.

Infinite series allowed Newton to study the properties of mechanical curves, such as the cycloid (the curve traced by a point on the circumference of a circle that rolls along a straight line: the parametric equation of the cycloid generated by a circle with radius a is $x = a(t - \sin t)$, $y = a(1 - \cos t)$).[12] Most notably, they

[11] MP, 1, pp. 369–99. See also the beginning of the "October 1666 Tract on Fluxions." MP, 1, pp. 400–1. Kirsti Andersen studied this technique and presented her analysis at a meeting in Oberwolfach (Germany) in December 2005; see Andersen, "Newton's Inventive Use of Kinematics in Developing His Method of Fluxions" (2005).

[12] As Newton wrote in 1684, "To be sure, convergent equations can be found for the curved lines

allowed him to calculate curvilinear areas, curvilinear volumes, and arc lengths; these calculations were generally called "quadrature problems." So, squaring a curve meant calculating the area of the surface bounded by it. Nowadays we would use Leibnizian terminology and speak about problems in integration. Sections §7.4 and in §8.4.5 take up Newton's calculation of the area of the surface subtended by the cycloid and by the cissoid (Cartesian equation $y^2(a-x) = x^3$). The fact that Newton's method allowed him to tackle mechanical curves and quadratures is due to a mathematical fact of which he was well aware. Using Leibnizian jargon, we can say that while differentiation of algebraic functions (accepted by Descartes) leads to algebraic functions, integration can lead to new transcendental functions. Newton referred to what are now called transcendental functions as quantities "which cannot be determined and expressed by any geometrical technique, such as the areas and lengths of curves."[13] Infinite power series—in some cases fractional power series—were the tool that young Newton deployed in order to deal with these mechanical (transcendental) curves.

1.4 Fluxions

A Newtonian memorandum, written more than fifty years after the momentous intellectual revolution it describes, gives an account, basically confirmed by manuscript evidence, of his early mathematical discoveries:

> In the beginning of the year 1665 I found the Method of approximating series & the Rule for reducing any dignity of any Binomial into such a series. The same year in May I found the method of Tangents of Gregory & Slusius, & in November had the direct method of fluxions & the next year in January had the theory of Colours & in May following I had entrance into y^e inverse method of fluxions. And the same year I began to think of gravity extending to y^e orb of the Moon ... . All this was in the two plague years of 1665–1666. For in those days I was in the prime of my age for invention & minded Mathematicks & Philosophy more then at any time since.[14]

There would be much to say to decipher Newton's words and place them in context. For instance, the task of commenting on the meaning of the term philosophy would require space and learning not at my disposal.

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commonly dubbed 'mechanical,' and with their assistance problems on these curves are solved no differently than in simpler curves." MP, 4, p. 559. "Quinetiam ad curvas lineas vulgo dictas Mechanicas inveniri possunt aequationes convergetes et earum beneficio problemata in his curvis non aliter solvi quam in curvis simplicioribus." MP, 4, p. 558.

[13] MP, 3, p. 79. "quantitates ... quae nullâ ratione geometricâ determinari et exprimi possunt, quales sunt areae vel longitudines curvarum." MP, 3, p. 78.

[14] Add. 3968.41, f. 85r. This passage is contained in a draft (August 1718) of a letter that Newton intended for Pierre Des Maizeaux. It is discussed in Westfall, *Never at Rest* (1980), p. 143.

Note three things about this memorandum. (i) The "method of approximating series" is the method of series expansion via long division and root extraction. Newton achieved also other methods for expanding y as a fractional power series in x, when the two variables are related by an algebraic equation. These methods, later generalized by Victor-Alexandre Puiseux, allowed Newton to go beyond the limitations of the common analysis, where finite equations were deployed, and express certain curves locally in terms of infinite fractional power series, which Newton called infinite equations.[15] (ii) The "rule for reducing any dignity of any binomial" is now called the binomial theorem for fractional powers, which Newton attained in winter 1664 by interpolating results contained in Wallis's *Arithmetica Infinitorum* (included in *Operum Mathematicorum Pars Altera* (1656)).[16] Such methods of series expansion were crucial for attaining two goals: the calculation of areas of curvilinear surfaces and the rectification of curves. (iii) Newton does not talk about discovering theorems, but rather methods and a rule. This last fact is of utmost importance because it reveals that, in his view, his results belonged to the analytical, heuristic stage of the method of problem solving.

In October 1666, Newton gathered his early results in a tract whose *incipit* reads "To resolve Problems by Motion these following Propositions are sufficient."[17] He conceived this tract as devoted to a method of resolution (i.e., "analysis") of geometrical problems, which makes use of the concept of geometrical magnitudes as generated by motion. This method, referred to in Newton's memorandum as the "direct and inverse method of fluxions," is discussed in part III. Note that the inverse method was always conceived by Newton as deeply intertwined with the method of approximating series and with the binomial rule.[18]

1.5 In the Wake of the *Anni Mirabiles*

In 1669 the first challenge arrived for the young mathematician. A slim book entitled *Logarithmotechnia*, printed in 1668, the work of the German Nicolaus Mercator, came to his attention. What Newton saw was worrying. Mercator had used an infinite equation (in our terms, a power series expansion of $y = 1/(1 + x)$) in order to square the hyperbola (i.e., calculate the area of the surface subtended by the hyperbola). This result belongs both to pure mathematics and to practi-

[15] These techniques are discussed in many treatises on algebraic curves: e.g., Brieskorn and Knörrer, *Plane Algebraic Curves* (1986), pp. 370ff.
[16] MP, 1, pp. 89–142.
[17] The "October 1666 Tract on Fluxions" is Add. 3958.3, ff. 48v–63v, and is edited in MP, 1, pp. 400–48.
[18] For a recent evaluation of Newton's early work on series and fluxions in the period 1664–1666, see Panza, *Newton et les Origines de l'Analyse: 1664–1666* (2005).

cal applications. It is, in fact, useful in facilitating the calculation of logarithms, a need deeply felt by seventeenth-century practitioners in all fields from navigation to astronomy. This was one of the results that Newton had achieved via binomial expansion or long division. He was able to do much more than this and therefore summarized his results regarding infinite series applied to quadrature in a small tract entitled *De Analysi per Aequationes Numero Terminorum Infinitas* (1669).[19]

Barrow, who was well informed about Newton's discoveries, immediately sent *De Analysi* to a mathematical practitioner called John Collins. The choice could not have been happier. Collins was at the center of a network of British and Continental mathematicians whom he kept up to date with an intense and competent correspondence. After taking copies of *De Analysi*, Collins informed a number of his correspondents about Newton's discoveries. He also made Newton aware of the Scotsman James Gregory (or Gregorie), who was pursuing researches on series expansions at a level comparable to what could be found in *De Analysi*. Collins's correspondence was the vehicle that allowed Newton to establish his reputation as a mathematician. Collins's network overlapped with that of the Royal Society; its president, William Brouncker, the secretary, Henry Oldenburg, and Wallis were certainly interested in Newton's mathematical researches on infinite series.

In 1672, Newton was elected a Fellow of the Royal Society because of the construction of the reflecting telescope, not because of his mathematics. And it was because of his ideas concerning the role of mathematics in natural philosophy that he initially found himself in a difficult relationship with the Royal Society. When he presented his 1672 paper on the nature of light, Newton made it clear that the undisputable certainty of his "new theory about light and colors" was guaranteed by mathematical reasoning. This thesis displeased the secretary, Henry Oldenburg, and the curator of experiments, Robert Hooke, who refrained from subscribing to what they perceived as a dogmatic position (see chapter 2). Newton found himself embroiled in a dispute that led him, after some years of tiresome correspondence with critics, to be reluctant about printing his philosophical ideas. Famously, in a different context, he was to complain about philosophy as an "impertinently litigious Lady."[20] What is relevant here is that, in the mid-1670s, much to Collins's frustration, he withdrew from any project of printing his mathematical discoveries on series and fluxions.

As I argue in Part VI, Newton's policy of publication is consistent with his self-portraiture as a natural philosopher who—contrary to the skeptical probabilism endorsed by many virtuosi of the Royal Society—could attain certainty thanks to

[19] See chapter 7 for further information.
[20] Newton to Halley (June 20, 1686) in *Correspondence*, 2, p. 437.

mathematics. Printing the algebraic, heuristic method would have exposed him to further criticisms; what he aimed at was certainty, and this was guaranteed by geometry. The algebraic analysis—as he later said to David Gregory—was "entirely unfit to consign to writing and commit to posterity."[21] To appreciate Newton's views on mathematics, one should not underestimate how sharp a boundary he drew in contrast with his mathematical practice between algebra and geometry, and how strongly he believed that only geometry could provide a certain and therefore publishable demonstration.

Newton's perception of the two layers of algebraic analysis and geometrical synthesis is already evident in his *Tractatus de Methodis Serierum et Fluxionum*, composed in 1670–1671.[22] The beginning of this long treatise is occupied by a revision and expansion of *De Analysi*. In the remaining twelve sections (labeled as problems) Newton "methodized" his researches into fluxions that he had first laid down in the October 1666 tract.[23] Here he developed the *analytical* method of fluxions, which was divided into two parts: (i) the direct method (mainly calculations of tangents and curvatures) and the inverse method (mainly calculations of areas and rectifications of curves). *De Methodis* ends with extensive tabulations of areas of surfaces subtended to curves. Newton soon developed (in an "Addendum" written in 1671) the idea that a *synthetic* form of the method of fluxions was required (see chapter 9). This more rigorous version, where no infinitesimals occur, was based on limit concepts and geometrical-kinematical conceptions and was systematized in a tract entitled "Geometria Curvilinea," written about 1680. The synthetic method of fluxions—the method of first and ultimate ratios—informs most of the *Principia* (1687).

Thus, in 1671—just after the completion of *De Methodis*, a summa of his analytical researches on series and fluxions—Newton began to rethink the status of his early researches, which are based on heuristic analogies and the use of infinitesimals, namely, on techniques that are far from the standards of exactness that he aimed at as a natural philosopher. In the 1670s he spent great effort in systematizing them, in rethinking their foundation, and in attempting alternative approaches. Several factors contributed to the more mature phase of Newton's mathematical production that followed the creative burst of the *anni mirabiles*. I note a few of these factors in the next section and elaborate on them in subsequent chapters.

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[21] "Algebram nostram speciosam esse ad inveniendum aptam satis at literis posterisque consignandum prorsus ineptam." University Library Edinburgh MS Gregory C42, translated by D. T. Whiteside in MP, 7, p. 196. See also *Correspondence*, 3, p. 385.
[22] See chapter 8 for further information.
[23] "partly upon Dr Barrows instigation I began to new methodiz ye discourse of infinite series." Newton to Collins (July 20, 1670) in *Correspondence*, 1, p. 68.

1.6 Maturity

Young men should prove theorems, old men should write books.[24]

In 1669, Newton was elected Lucasian Professor, in succession to and thanks to the patronage of Barrow. He began preparing a first set of lectures on optics in which he claimed that certainty in natural philosophy can be guaranteed by the use of geometry (see chapter 2). A concern with certainty in mathematical method thus emerged in the context of Newton's early optical researches and remained anchored to them until maturity when, in the last Query 23/31 (1706/1717) of the *Opticks*, he wrote a famous peroration in favor of the use of the method of analysis and synthesis in natural philosophy. The investigation of difficult things, he claimed, could be pursued in natural philosophy only by following the steps of the mathematicians' method of enquiry. Newton wished to validate his natural philosophy mathematically, outstripping the skeptical probabilism that was rampant in his day, as he complained. Synthesis, not analysis, was the method that could guarantee the level of accuracy and certainty required for such an ambitious task. Further, as a successor of Barrow in the Lucasian Chair, Newton probably felt that his new status implied delivering mathematics in rigorous and systematic form. He began writing mathematical treatises characterized by length, maturity, and apparent uselessness (they seldom went to the press).[25]

Newton's involvement in preparing his next set of lectures on algebra led him to conceive the idea that analysis could also be approached differently from the way promoted by the moderns: in short, there could be a geometrical analysis, a geometrical rather than an algebraic method of discovery. Up to this point in this chapter, I have somewhat incorrectly equated analysis with algebra, and synthesis with geometry. But it is necessary to avoid such equivalences because they were not accepted by Newton and by many of his contemporaries. Not only synthesis but also analysis could be geometrical.

Lucasian Lectures on Algebra stemmed from a project on which Newton had embarked since the fall of 1669, thanks to the enthusiasm of John Collins: the revision of Mercator's Latin translation of Gerard Kinckhuysen's Dutch textbook on algebra. Newton's involvement in this enterprise was an occasion to rethink the status of common analysis. He began experimenting with what he understood as ancient analysis, a geometrical method of analysis or resolution that, in his opinion, the ancients had kept hidden. In his *Lucasian Lectures on Algebra*, which he deposited in the University Library of Cambridge in 1684 and from which William

[24] Godfrey H. Hardy, quoted by Freeman Dyson, in Albers, "Freeman Dyson: Mathematician, Physicist, and Writer" (1994), p. 2.
[25] But on Newton's attitude toward print publication versus manuscript circulation, see chapter 16.

Whiston edited the *Arithmetica Universalis* (1707), Newton extended Cartesian common analysis and arrived at new results in this field. But even in this eminently Cartesian text one can find traces of his fascination with the method of discovery of the ancients. The ancients, rather than using algebraic tools, were supposed to have a geometrical analysis that Newton wished to restore. This was a program shared by many in the seventeenth century. He also made it clear that synthesis, or composition, of geometrical problems had to be carried on—*contra* Descartes—in terms wholly independent of algebraic considerations (see chapter 4). The fascination with ancient analysis and synthesis, a better substitute, he strongly opined, for Cartesian common analysis (algebra) and synthesis (the techniques on the construction of equations prescribed by Descartes), prompted Newton to read the seventh book of Pappus's *Collectio* (composed in the fourth Century A.D. and printed alongside a Latin translation in 1588). He became convinced that the lost books of Euclid's *Porisms*, described incompletely in Pappus's synopsis, were the heart of the concealed ancient, analytical but entirely geometrical method of discovery (see chapter 5).[26]

Newton intertwined this myth of the ancient geometers with his growing anti-Cartesianism. In the 1670s he elaborated a profoundly anti-Cartesian position, motivated also by theological reasons. He began looking to the ancient past in search for a philosophy that would have been closer to divine revelation. The moderns, he was convinced, were defending a corrupt philosophy, especially those who were under Descartes' spell. Newton's opposition to Cartesian mathematics was strengthened by his dislike for Cartesian philosophy. Descartes in the *Géométrie* had proposed algebra as a tool that could supersede the means at the disposal of Euclid and Apollonius. Newton worked on Pappus's *Collectio* in order to prove that Descartes was wrong. He claimed that the geometrical analysis of the ancients was superior to the algebraic of the moderns in terms of elegance and simplicity. In this context, Newton developed many results in projective geometry and concerning the organic description of curves. His great success, achieved in a treatise entitled "Solutio Problematis Veterum de Loco Solido" (late 1670s) on the "restoration of the solid loci of the ancients," was the solution by purely geometrical means of the Pappus four-lines locus. This result, much more than the new analysis of infinite series and fluxions, pleased Newton because it was in line with his philosophical agenda (see chapter 5).

The importance of projective geometry emerged also in the study of cubics, when Newton found that these algebraic curves can be subdivided into five projective classes. His interest in the classification of cubic curves dates to the 1660s, but it was only in the late 1670s that, by deploying advanced algebraic tools, he achieved

[26] It seems that Newton did not know that Descartes expressed similar views in the "Responsio ad Secundas Obiectiones" in *Meditationes de Prima Philosophia* (1641) (AT, 7, pp. 155–6).

the array of results that later, in the mid-1690s, were systematized in *Enumeratio Linearum Tertii Ordinis*, a work that first appeared in print as an appendix to the *Opticks* (1704) (see chapter 6).

One should not forget another factor that determined Newton's option for geometry in the 1670s: the encounter with Huygens's *Horologium Oscillatorium*. In his masterpiece, printed in 1673, Huygens had employed proportion theory and *ad absurdum* limit arguments (method of exhaustion) and had spurned as far as possible the use of equations and infinitesimals (in his private papers he did employ symbolic infinitesimalist tools, but he avoided them in print).[27] Huygens offered an example to Newton of how modern cutting-edge mathematization of natural philosophy could be presented in a form consonant with ancient exemplars. The Lucasian Professor immediately acknowledged the importance of Huygens's work, and one might surmise that his methodological turn of the 1670s—which in part led him to cool his relationship with Collins and avoid print publication of his youthful algebraic researches—was related not only to a reaction against Cartesianism, but also to an attraction toward Huygens's mathematical style.

When Newton composed the *Principia*, in 1684–1686, he had a panoply of mathematical methods in his toolbox, methods that he could deploy in the study of force and motion. He gave pride of place to the synthetic method of fluxions (first elaborated in a treatise composed about 1680 and entitled "Geometria Curvilinea"), claiming in Section 1, Book 1, that this was the foundation on which the *magnum opus* was based. But in several instances, as a close reading of the text of the *Principia* makes clear, he appealed to quadrature techniques that belong to his algebraized new analysis. These quadratures were not, however, made explicit to the reader. Newton chose instead to insert in the body of the text a treatment of ancient analysis and its application to the solution of the so-called Pappus problem. In Part IV I discuss the policy of publication that led Newton to structure the text and the subtext of the *Principia* in ways consonant with his views on mathematical method.[28]

After the publication of the *Principia*, Newton ceased to be an isolated Cambridge professor. He had to defend and establish his rising position in the political and cultural world of the capital, where he moved in 1696 as Warden of the Mint. A first challenge, in 1691, from David Gregory (§8.5.1) on quadrature techniques induced him to work hard in the early 1690s on the composition of a treatise, *Tractatus de Quadratura Curvarum*, which appeared in 1704 as an appendix to the *Opticks*. *De Quadratura* opens with an introduction in which Newton claims that the method of fluxions is based on a conception of magnitudes generated by motion

[27] Yoder, *Unrolling Time* (1988).
[28] See also Guicciardini, *Reading the Principia* (1999).

and on limiting procedures that are consonant with the methods of the ancients (see chapter 9).[29]

Newton's growing fascination with the myth about the *prisca sapientia*, a pristine superior wisdom of the ancients, that characterizes his thought after the publication of the *Principia* resonates with his extensive researches on the ancient analysis that he carried on in the 1690s and early 1700s. His aim was to show that the youthful analytical method of fluxions could be reformulated in terms acceptable by ancient standards. He even explored a totally new method of discovery and proof. Newton left hundreds of manuscript pages, which culminated in an unfinished "Geometriae Libri Duo," devoted to his attempts to write a treatise on projective geometry written in a style reconstructed following the authority of Pappus (see part V). These aborted attempts are the more philosophy-laden texts belonging to Newton's mathematical *Nachlass*, since he made a deep effort to clarify the relations among the various sectors of his mathematical method: analysis, synthesis, algebra, geometry, mechanics, and natural philosophy. These terms have been used in this first chapter in an improperly ambiguous way. But commenting on Newton's works on method in subsequent chapters will allow me to clarify this terminology and decode Newton's somewhat arcane mode of expression.

Newton encouraged his acolytes to pursue researches in ancient analysis and never missed the opportunity for praising those, such as Huygens, who resisted the prevailing taste for the symbolism of the moderns, the "bunglers in mathematics."[30] When the polemic with Leibniz exploded, he could deploy his classicizing and anti-Cartesian theses against the German (see part VI). Thus, Newton's last mathematical productions, publications, and (often anonymous) polemical pieces were driven by a philosophical agenda difficult to reconcile with his mathematical practice.

[29] The other appendix, *Enumeratio Linearum Tertii Ordinis*, was also written in the 1690s, deploying notes on cubics dating from the 1670s. See chapter 6.
[30] Hiscock, *David Gregory, Isaac Newton and Their Circle* (1937), p. 42.