Isaac Newton on Mathematical Certainty and Method

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Preface

There is no doubt that Isaac Newton is one of the most researched giants of the scientific revolution. It is triviality to say that he was a towering mathematician, comparable to Archimedes and Carl Friedrich Gauss. Historians of mathematics have devoted great attention to his work on algebra, series, fluxions, quadratures, and geometry. Most notably, after the publication of the eight volumes of *Mathematical Papers* (1967–1981), edited by D. T. Whiteside, any interested reader can have access to Newton's multifaceted contributions to mathematics. This book has not been written with the purpose of challenging such a treasure of scholarship and information.[1] Rather, I focus on one aspect of Newton's mathematical work that has so far been overlooked, namely, what one could call Newton's philosophy of mathematics.[2] The basic questions that motivate this book are What was mathematics for Newton? and What did being a mathematician mean to him?

It is well known that Newton aimed at injecting certainty into natural philosophy by deploying mathematical reasoning. It seems probable that he entitled his main work *The Mathematical Principles of Natural Philosophy* (1687) in order to state concisely and openly what constituted the superiority of his approach over the Cartesian *Principles of Philosophy* (1644). The role of this program in Newton's philosophical agenda cannot be overestimated. Little research has been devoted, however, to Newton's views on mathematical certainty and method, views that are obviously relevant to his program, for if mathematics is to endow philosophy with certainty, it must be practiced according to criteria that guarantee the certainty of its methods. But the new algebraic methods that Newton mastered so well, and that are the salient characteristic of seventeenth-century mathematical development, appeared to many far from rigorous. Newton participated in the debate on the certainty of mathematical method and elaborated his own answer. However, his views on mathematical method have attracted scant attention from historians,

[1] So writes Whiteside, the doyen of Newtonian studies, about his edition of Newton's mathematical papers: "Let it be enough that the autograph manuscripts now reproduced are no mere resurrected historical curiosities fit only once more to gather dust in some forgotten corner, but will require the rewriting of more than one page in the historical textbook." MP, 7, pp. vii–viii. I dare to hope that my book will in part answer Whiteside's *desideratum*; it would have been impossible to write without a many-years-long full immersion in his eight green volumes.

[2] This is a term that Newton would not have understood.

whereas we know a great deal about, say, René Descartes' or Gottfried Wilhelm Leibniz's philosophies of mathematics.[3]

In order to implement and divulge his innovative approach to a mathematized natural philosophy, Newton tackled a series of questions that have been overlooked or misunderstood by historians, such as When are mathematical methods endowed with certainty? How can one relate the common and new algebraic analyses of the moderns to the venerated methods of the ancients?[4] When is a geometrical construction exact and elegant? What guarantees the applicability of geometry to mechanics? In tackling these issues Newton mobilized deeply felt convictions concerning his role as a philosopher and as a mathematician. He positioned himself against the probabilism endorsed as a moral value by most of the virtuosi of the Royal Society, such as Robert Hooke and Robert Boyle, but he also held a polemical position against two great giants in the common and new analyses, Descartes and Leibniz. On the other hand, Newton found affinities with the thought of Isaac Barrow, Christiaan Huygens, and more obliquely Thomas Hobbes. He reconstructed the development of mathematics from ancient times so as to depict himself as a legitimate heir of the Greek tradition while distancing himself from the moderns. Newton's antimodernism had important consequences for his policy of publication, for the general outlook of his foundational thought, for the mathematical structure of his *Principia*, and for the negotiations he set afoot in order to trade his mathematics in the milieu of British mathematicians and to affirm the superiority of his method over Leibniz's.

In this book I depart from a tendency that has prevailed in the literature devoted to the history of seventeenth-century mathematical methodology. Generally speaking, historians of mathematics have tended to concentrate on questions related to the definitions of basic terms such as *fluxion*, *infinitesimal*, *limit*, and *moment*, that is, on questions concerning *rigor*. These questions, which relate to the "labyrinth of the continuum," certainly had a great importance for a deep thinker like Leibniz. Not so for Newton, who was more concerned with questions about the legitimacy, elegance, and exactness of procedures for solving geometrical problems, that is, with questions about *method*. Leibniz's concerns appear much closer to modern foundational issues in the philosophy of mathematics. Newton's discourse on method is much more opaque to modern readers trained in contemporary foundational literature. This is a natural consequence of scientific training in an era of mathematical

[3] Just to take a telling example, in Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (1996), an informed and thorough study of the philosophy of mathematics in the seventeenth century, Newton's name occurs only three times and in passing references.

[4] The terms common analysis and new analysis referred to Cartesian algebra carried on via polynomial equations and to a more general algorithm where infinitesimals, infinite series, and infinite products could be deployed.

practice shaped by the exigencies that emerged after the early-nineteenth-century rigorization of the calculus, axiomatization of geometry, and development of abstract algebra. Since then, definitions of basic terms have become a prominent feature of the foundations of mathematics. Newton did devote attention to, say, the nature of infinitesimals, but he was much more concerned with methodological issues related to the analysis (or resolution) and synthesis (or composition) of geometrical problems. In this he showed himself to be influenced by Descartes, who was deeply involved in the program of redefining his algebraic method of analysis as an alternative to the analysis of the ancients ("Analysis Veterum").[5] Descartes also traced the boundaries between legitimate (exact) and illegitimate (not exact) means of geometrical construction and classed them on the basis of criteria of simplicity that broke with the ancient tradition conveyed by Pappus's *Collectio*. Newton fiercely rejected Descartes' canon of problem resolution and composition, and proposed an alternative that looked to ancient exemplars as models superior to the mathematics practiced by the "men of recent times." But Newton's admiration for the ancients—arguably intertwined with many facets of his philosophy of nature and religion—opened a gap between his views on mathematical method and his mathematical practice.

Several hitherto unexplained aspects of Newton's mathematical work are related to this condition of stress and strain characterizing his thoughts on mathematical method. Why did Newton fail to print his method of series and fluxions before the inception of the priority dispute with Leibniz? Did he use his new analysis in the *Principia*, as he claimed in retrospect? And if so, why did he hide his competence in his new methods of quadrature when writing the *Principia*? Why does his *Arithmetica Universalis*, a work devoted to algebra, end with pointed criticisms of the use of algebraic criteria in the construction of geometrical problems? Why did Newton, a master of symbolic manipulation, express deep disparagement of the algebraists (whom he, according to hearsay, labeled the "bunglers of mathematics")?[6] Which strategies did he adopt in order to maintain the superiority of his method compared to Leibniz's extraordinarily efficient algorithm? Why did he engineer *Commercium Epistolicum* (1713), which was meant to prove his priority in the discovery of the calculus, using documents that proved everything he had achieved on series and quadrature but revealing nothing about the rules of the calculus? Why did he—*contra* Leibniz—attribute these rules to Isaac Barrow rather than to himself? Was not therefore his policy during the priority dispute lacunose and contradictory?

Historians have found these questions embarrassing. It is not unusual to encounter attempts to formulate answers in terms of psychological motivations, a

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[5] Bos, *Redefining Geometrical Exactness* (2001).
[6] Hiscock, *David Gregory, Isaac Newton and Their Circle* (1937), p. 42.

sign—in my view—of the difficulty of making sense of Newton's policy and convictions. So, we often read that it was Newton's neurotic shyness, or a lack of interest in mathematics that suddenly crept into his mind, that prevented him from printing his method. Further, we are told that in the *Principia* he studiously tried to be obscure in order to avoid dispute and criticisms because of his obsessive feeling of being persecuted. We even encounter the thesis according to which, in the polemic against algebra and calculus, he was trying to hide his indebtedness toward an intellectual father (Descartes) whose image he found oppressive. When we move to consider the polemic with Leibniz, we find studies that highlight Newton's obsessive approach to the priority dispute, his lack of fairness, his egotism, and even his political motivations related to the Hanoverian succession. Little effort is made to try to discern in such a muddied context (and muddied and political it certainly was!) coherent methodological positions held by Newton and by his opponents.

I believe we should be able to find better answers by studying more carefully Newton's writings on the nature of mathematics, which are so abundant in his manuscripts. True, the search for Newton the philosopher of mathematics is at times frustrating. What we find is a disconnected, sometimes contradictory, constellation of pronouncements scattered in the margins of mathematical manuscripts, in aborted prefaces and appendices, in letters and personal notes. They serve their purpose in a dialogical context, providing defensive grounds for mathematical practices, orienting aims, and establishing hierarchies. However, in these writings Newton reveals himself as a mathematician who—even when shattered by psychological disturbances, stymied by academic rivalries, and motivated by political interests—is able to endorse a fairly clear view of method and certainty. It is this view that prompted him to articulate his mathematical work according to codes of communication that were understood by his contemporaries, especially by his close acolytes, but that turn out to be so puzzling for modern readers. In short, even in the heated context of the priority dispute Newton has something to tell us about mathematical method. We should consider his theses, even though his stature as a philosopher of mathematics is inferior to a Leibniz or a Descartes.

In this book I study Newton's methodology of mathematics by analyzing his main works, from the early treatises on series and fluxions to the writings addressed against Leibniz. Even though my book is not devoted to Newton's mathematical results, it is important to try to understand his mathematics, because his views on mathematical method interacted with his mathematical practice in a complex way. Therefore, the reader will find some pages in which I analyze Newton's mathematics at work. I have been extremely selective, and the examples I have chosen are not meant to offer an exhaustive view of Newton's mathematics; he was simply too prolific to allow condensing the wealth of his results in a single book. I have tried, however, to choose examples that are simple enough to be followed by a reader equipped with a modicum of mathematical expertise and that are representative of a method or an approach that Newton developed. The expert mathematician will find most of the examples far too simple. Newton was indeed a great problem solver, and his best mathematics can be admired by reading *Mathematical Papers* and its editor's profound commentary.

In matters of notation I have avoided translating Newton's mathematics into modern terms.[7] The mathematician not trained in history will find Newton's mathematical language and practice somewhat puzzling. For instance, Newton and his contemporaries did not talk about functions, but about continuously varying magnitudes. Newton did not use an equivalent of Leibniz's integral sign consistently; most often he used connected prose and referred to "the area under the curve" or "the fluent of." He was also somewhat confused about the distinction between definite and indefinite integral, and never rendered constants of integration explicit. Therefore, he often used the singular ("the fluent of"), but he was of course aware that the indefinite integral identifies a class of functions. Further, he dealt with the convergence of infinite series in very intuitive terms; their convergence was tested in cavalier terms. We have to wait for Augustin Louis Cauchy at the beginning of the nineteenth century for a modern theory of convergence. The list of oddities could continue. I have made no effort (with the exception of a few explanatory footnotes) to avoid the distinctive character of Newton's mathematics compared to modern usage.

The book is divided into six parts. Part I provides some preliminaries: a survey of Newton's mathematical work and of the development of his ideas on mathematical method that began to mature just after the creative burst of the *anni mirabiles* (chapter 1); a comparison between his youthful program in natural philosophy with the one endorsed by influential contemporaries like Descartes, Hooke, and Boyle (chapter 2); a presentation of Descartes' ideas on analysis and synthesis as Newton found them in the *Géométrie* (chapter 3). In fact, Newton, who was in his mathematical practice so much a Cartesian, stood in opposition to Descartes with respect to method.

Part II considers the first period of Newton's methodological thought. He began distancing himself from Cartesian method in writings that date from the 1670s in which he compared common analysis (i.e., Cartesian algebra) to ancient analysis. The occasion for these reflections was Newton's involvement in the project of revising a textbook on algebra by the Dutchman Gerard Kinckhuysen and his commitment to prepare lectures on algebra (chapter 4). It is in this context that Newton began reading Pappus with the purpose of recovering the ancient analysis (chapter 5). In this period Newton also worked on cubic curves (chapter 6); and it is in this research that the tensions between his mathematical practice and his views on method surfaced.

[7] So, for instance, I avoid the integral sign and prefer to talk about the "quadrature of a curve" rather than the "integral of a function."

Part III is devoted to Newton's attempts to provide a synthetic version of what he labeled "method of series and fluxions." Newton's early researches on series (chapter 7) and fluxions (chapter 8) were carried on in terms unacceptable to his more mature standards of validation. Consequently, he aimed at developing a synthetic method of fluxions—a "more natural approach," as he says—consonant with the practice of the ancient geometers. These researches culminated in the early 1680s in a treatise entitled "Geometria Curvilinea," in which Newton elaborated his method of first and ultimate ratios, a method that informs the most mature presentation of his method of series and fluxions offered in the *Principia* and *De Quadratura* (chapter 9).

Part IV considers the mathematical methods employed by Newton in the *Principia* (chapter 10). I devote particular attention to the strategies he chose in order to accommodate his analytical methods in common analysis (chapter 11) and new analysis (chapter 12) in the body of a text that he presented as written according to the "ancient and good geometry."

Part V concerns perhaps the most philosophically freighted texts that Newton wrote in the 1690s and early 1710s. In these decades his belief in the myth about a *prisca sapientia* of the ancients prospered and determined his self-portraiture as an heir to the mathematics of Euclid and Apollonius. Consequently, he wrote at length on the relations among analysis, synthesis, algebra, natural philosophy, and mechanics (chapters 13 and 14).

Newton's views concerning mathematical method emerge again in the polemic with Leibniz, which occupied him especially from 1710 (chapter 15). In order to trace the rationale for Newton's polemical strategy, Part VI devotes attention to his policy of publication adopted in circulating manuscripts, in correspondence (chapter 16), and later in printing some of his tracts on the new analysis that he had composed years before (chapter 17).

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