1 The Battle of Numbers in the Middle Ages

Formations of the Battle of Numbers

According to Adam Ries, it is necessary to distinguish between "calculation on the lines and with the quill": numbers can be positioned as counters on the lines of an abacus, the antique calculating board, or they can flow in the form of Hindu-Arabic digits from the quill.[1] But when Ries extolled the virtues of writable digits in the early modern period, he did so in a medium that did not stand in a neutral relation to the represented numerical concepts. Gutenberg's book printing preserved and reproduced writing operations better than it did anything else. When the Occidental and Oriental forms of calculation first encountered each other in Italy and Spain in the Middle Ages, it was not merely different modes of representing calculative operations that came to the fore. Rather, it turned out that the numerical conceptions differed at all levels of their material incarnation. The most dramatic difference emerged in the comparison of their place-value systems: Whereas on the abacus—the *tabula abachi*—the place that does not count is simply not incarnated by a stone, the Hindu-Arabic numeral system indiscriminately indicates a value and the lack of the same through signs. The news of zero is therefore placed by some authors, with a certain degree of justification, at the beginning of the history of the modern period.[2]

The history of the Battle of Numbers, however, first created a platform on which various mathematicians were able to enter into competition.[3] What began in substance in the eleventh century received its name in the twelfth: *Rhythmos* and *machia* were combined by clerics into *Rithmomachia*[4]—a coinage in which the first lexical component not only means *arithmos*, or "number," but is also read as a musical quality. Yet the Roman Boethius had uncoupled mathematics from music in the sixth

century when he established the very numerical proportions from which the Battle of Numbers now derived its configurations. Cassiodorus took still further the separation of numerical conceptions from all "material supplementation"[5]—the division of the quadrivium, which was based on different applications, seemed invalid to him, and he promptly summarized it as *mathematica*. The Battle of Numbers, however, again sets in motion an operational approach to arithmetic. By bringing the confrontation of even and odd numbers onto the game board, the Battle of Numbers aligns with the basic concept of Pythagorean mathematics.

Initially, the term Battle of Numbers was not associated with the attribute of play. Only relatively late is there mention of *ludus*[6] in connection with the *conflictus numerorum*.[7] In light of the conflicts at the level of numerical practices, which were fought out with the Battle of Numbers, one cannot be certain that its limits are those of a game. The contrast with and distance from the pure game becomes conspicuous, at the latest, through its reception in the baroque period. In its collecting mania, that age takes up the Battle of Numbers as nothing more than a scarcely understood game with mute signs.[8]

Yet the *Rithmomachia* is probably the first instrument that is not only described in writings, but also emerges from writing itself (figures 1.1 and 1.2). One searches in vain for diagrammatic designs of this complexity in previous epochs. The Battle of Numbers disseminated its forms of inscription with a comprehensiveness that erases the difference between writing and calculation, at a moment when the written calculation of Arabic mathematicians found its way into Western Europe.

One of the most prominent figures among the scholars of the twelfth century, Hermann the Lame, assigns the Battle of Numbers to the arsenal of medieval instruments—including the astrolabe, the abacus, and the monochord—and stresses its instrumental character.[9] It thereby serves primarily as a means of practice in figuratively understood numbers. The goal is to arrange one's own pieces on the opponent's side of the game board in accordance with the proportion doctrine of arithmetical, geometric, or musical harmonies. The calculation and game principles coincide with the mathematical founding acts of the Pythagoreans and to this day pose riddles to archeologists and philologists in their attempts at reconstruction.[10] Nonetheless, the Battle of Numbers is distinguished from the astrolabe and monochord by the fact that it does not refer to external realities such as stars or

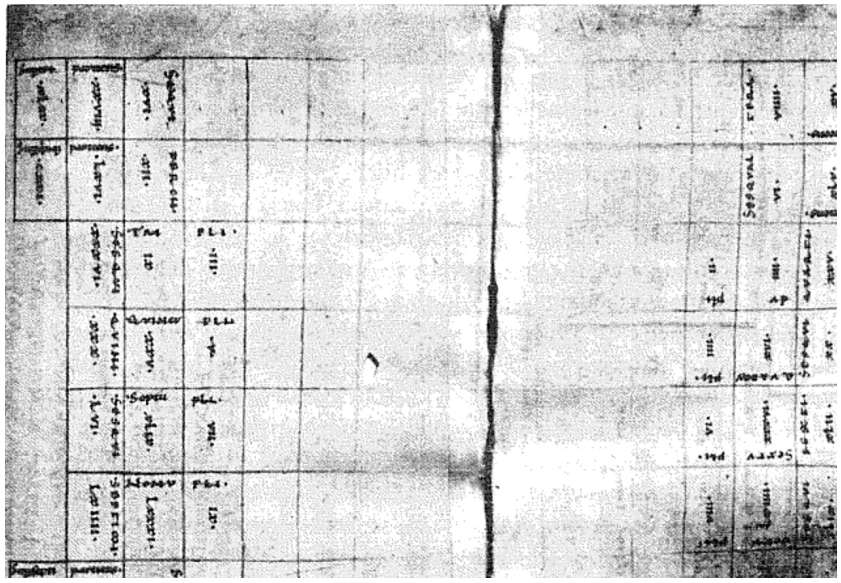

Figure 1.1
The oldest known depiction of the game board for the Battle of Numbers, prepared for the cathedral school in Hildesheim around 1100, accompanying instructions by Odo von Tournai.
Source: Bistumsarchiv Trier, BATr Abt. 95, no. 6, fol 79r. Reprinted with permission.

sound images. And as for the abacus, it is employed for a whole variety of practices: it serves merchants as much as geometricians.[11] The Battle of Numbers, on the other hand, turns the translational achievement of the abacus on its head. As opposed to the abacus, which has as its only object calculation itself, the Battle of Numbers incorporates more and more symbolic and objective contexts in the course of its development: musical intervals, battle formations, and thus whole world orders are enacted in the Battle of Numbers, without particular figurative and iconic efforts being undertaken in the process. In the manuscripts of the Battle of Numbers, which were produced for over six centuries at least, the game pieces are rarely described through colors and geometric shapes. The Battle of Numbers is surprisingly symbol-laden for an epoch in which the imaginary reigns above all. Unlike in the case of chess, for example, to this day no game board has been found for the Battle of Numbers. This proves *ex negativo* that the Battle of Numbers was bound only to the possibilities of the medium of parchment.

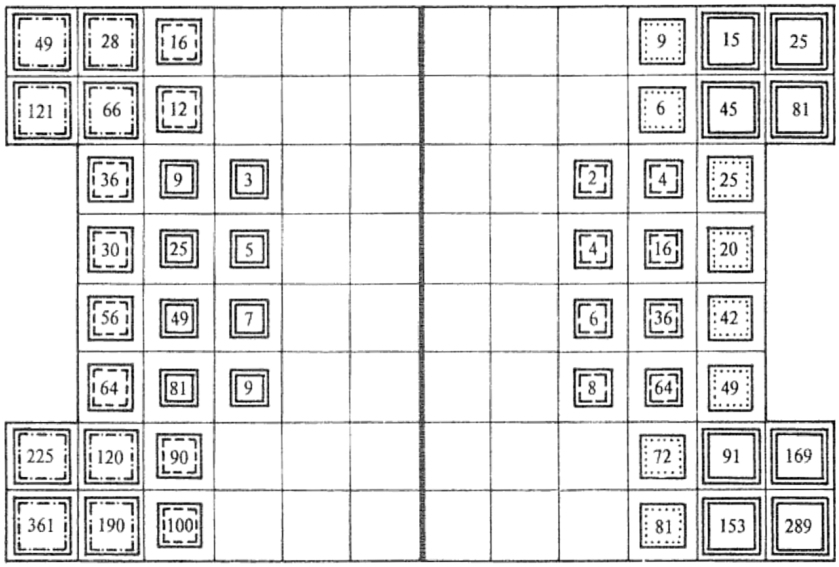

Figure 1.2
Game board for the Battle of Numbers reconstructed by Arno Borst according to the Liège table with even and odd game pieces derived according to various classes of proportions.
Source: Borst 1990, 278. © 1990 WBG. Reprinted with permission.

The Implantation of Mathematics

Arno Borst has reconstructed the discursive milieu of South German cathedrals around the year 1000, within which the Battle of Numbers arises. The catalyst was the so-called Worms school quarrel. The two cathedral schools of Würzburg and Worms struggled for the favor of pupils and ultimately for that of the Salian Emperor Conrad II himself. By itself, the quarrel would not necessarily have led to a retreat from the principle of orally competitive rhetoric. But apparently the Emperor's chancellor and cousin explicitly decreed that it should be fought out in writing, and a monk named Asilo came up with the idea of composing a Battle of Numbers.[12] The cause of the quarrel itself—the efficient calculation of sums of arbitrarily long series and setting up of ratios—favors the writing surface and evokes forms of inscription. Early commentators already characterize the Battle of Numbers as a *novellae plantantiones*.[13] It makes possible a tentative

writing,[14] which gains traction through an arrangement that repeatedly evokes new orders: Begun in the form of a circular letter and continued in composite manuscripts, the scattered writings on the Battle of Numbers nonetheless escape all luxury volumes and canonical writings.[15] The "disposable literature"[16] in which the number conflict is fought out does not flow into the dogmatic stock of knowledge. One exception, however, seems significant: in a single case, comments on the Battle of Numbers are taken up in a luxury manuscript alongside venerable texts on the *regula* and *ordo* of the monastic discipline. Whether this exception rests solely on a mistake—provoked by the frequent use of the signifier *regula*[17]—or whether a space for play is in fact being granted in the enumeration of monastic rules is an open question.

What unites and divides the three introductory and four additional liberal arts of the Middle Ages is their use of letter-based or numerical sign systems. Only the focus on the use of writing characterizes all the subjects of the *artes liberales*. If a secure logic of counting is first inherent in Roman numerals, it is still possible for Greek letter-numbers to make what is counted nameable through the alphabet. The simplicity of that which can be straightforwardly announced and said could always be elevated to the last explanatory resort[18] alongside that which can be geometrically shown in Pythagorean mathematics—especially as mathematics and music theory are linked down to their technical terminology.[19] However, the Greek sources became more and more linguistically inaccessible to the Western empires of the Middle Ages.[20] Increasingly, therefore, it was possible to perform operations with Greek signs only as such. In Greek letters, Carolingian monks discover the link that translates orders of writing into numerical orders: in the cryptograms of the papal couriers, names can be encoded through numbers, and sums that yield names written in Greek open up—beyond all calendrical calculations—a glimpse of looming apocalyptic events.[21] Tangibly practiced arithmetic nonetheless differs fundamentally from its inscription up to the first millennium: whereas monochords, sand tables, wooden abaci, their *psephoi* and *apices*, and even finger positions took on the most diverse spatial and temporal configurations, the act of setting them down in writing leads to orders of inscription that are bound to the direction of reading and writing and are ultimately immovable.[22] Until the appearance of the Battle of Numbers in the eleventh century, there are—as far as can be ascertained—no instances of

movable and discrete elements that exhibit numerals and do not arrest their arrangement. Rather, established numerical designations refer on their writing substrata directly to movable elements—for example, the signless counters of the abacus or the strings of the monochord—in a continuous, sequential fashion. That writing in the mode of continuity does not constitute a triviality first becomes clear with the onset of Arabic algorithmic notation: The backward movements of reading, the space-seeking directions of writing, the cross-outs—undertaken by reading and writing operations in rapid alternation on discrete signs—are all basic in themselves. But no one had previously been compelled to take them up. Conversely, a prominent passage by Herodotus demonstrates that the use of the abacus follows the movement of writing: "In writing letters and in calculating with stones the Greeks move the hand from left to right, the Egyptians from right to left."[23] The Battle of Numbers will first systematically open up further dimensions of the field for semiotic operations through horizontal, vertical, and diagonal ways of moving the game pieces and calculating stones. It will stack signs into pyramids and raise them from the surface into the spatial realm. In short, due to the loosening of the grip that prescribes the direction of writing, multidimensional spaces open up, in which sign systems are subjected to an elementalization. Doctrines of the abacus limit the movement of the counters to specific axes, lest the logic of the place-value system be thwarted. In the Battle of Numbers, on the other hand, there are three interconnected levels that can emerge as numerical representations: What counts equally and simultaneously are the fields of the chess-like game board, the number of the game pieces and the numerals on the game pieces. The Battle of Numbers ceases to function as an instrument for calculating numerical relations. It is not as much about numerics as it is about numerology—the maximization of numerical relations and referents, not the calculation of quantities. The Battle of Numbers skillfully limits the calculation of numerical relations: only pieces with low numbers can be combined into a large number of products and sums that correspond to the pieces with the highest numbers and can thereby win. Conversely, for pieces with the highest numbers, only division can be used to eliminate pieces with lesser numbers from the field through one of their divisors.

The high density of arithmetical relations that the Battle of Numbers produces must be managed with mental calculations. Increasingly, tables

of ratios are available to struggling players, and the Battle of Numbers degenerates—to its inventors' chagrin—into a war of tables.

The numerals of diverse cultures and epochs find a playing field in the Battle of Numbers. A battle for supremacy of the various numerical concepts is literally fought out here: Roman, Arabic, Greek.[24]

One disadvantage of Roman numerals clearly exposed by the Battle of Numbers is that with higher numbers, they tend to require a great deal of writing surface, which is just as hard to apply to game pieces that are all the same size. But Greek letter-numbers and gobar digits—to an equal extent—might have first demonstrated that scalarity could also be applied to numerals and that—in the case of gobar digits—the directions of writing or reading could shift. The Battle of Numbers stands at the intersection of a decoding of the sunken numerals of the Greek and Roman epochs and of the future ones of the Orient.[25]

"Caracteres"—a new term that emerges from this juncture—implies the dissolution of the strict separation between written numerals on the one hand and the operationality—in itself devoid of characters—of the instrumentariums on the other hand. From that point on, numerals achieve autonomy in the course of abiding traditions of writing. Meanwhile, their instrumental implementations in the form of the abacus and other calculative apparatuses have long since disappeared. Their reconstruction becomes a speculative question. And so scholars of the Middle Ages train themselves for the first time in mathematical descriptions, for the understanding of which the materiality of parchment suffices.[26] Even before the turn of the millennium, Gerbert of Aurillac did not simply presuppose the abacus in his *Regulae de numerorum rationibus*. Rather, he completely redesigned it, in order to practice the numerical relations that appear in the sentences of his source.[27] One reason that the calculating stones can no longer be presupposed is that they become a hybrid construct on which the stamp of writing is imprinted for the first time; in order to provide them with gobar digits, Gerbert ordered that they be fashioned out of horn.[28] Caracteres thus designate very precise numerals, which for the first time appear on the side of mobile elements like game pieces and calculating stones. The crossings of the place-value systems that thereby occur might have initially produced incalculabilities above all. But beyond that, a combinatorial matrix with movable letters emerges, on which—not least of all—the Gutenberg Galaxy will be based.

Scholars are divided as to whether the Battle of Numbers does not already arise in Walther von Speyer's *Libellus Scolasticus* of 984.[29] A personified geometry begins here as "a playful battle"[30] with the above-mentioned caracteres. However, columns of the abacus numbers one and ten dominate the event, and not—as with the monk Asilo half a century later—Boethius's classes of proportions. Nonetheless, Walther condenses—in the form of dactylic verse—numerical proportions, calculative operations on the lines, numerical figures, and musical interval formations into the program of *mathesis*. In the development of the Battle of Numbers, everything that still sounds metaphorical here will take on a calculable and playable form on the same written basis.

Semiotic Turn

What is the status now of the fragility of things, the persistence of the grapheme and material and semiotic transferences? Regarding the partition of the pieces on the game board, the first writings on the Battle of Numbers reveal nothing; nor do they provide any game diagrams. Nonetheless, the first extant tabular arrangements of the pieces show at a glance a highly differentiated grouping. Their schema follows exactly Greek military formations.[31] The pieces are permitted to move in different increments. With each move they travel one, two, or three fields.[32] It is as if heavily armed hoplites, more mobile foot soldiers, and riders were waging their attack on the wings of the game board. To think strategies and numerical figurations together is a Greek achievement.[33]

With the Battle of Numbers—despite or precisely because of its abstraction—religious scholars brought in a military reality. Roman war chronicles already spoke of their armies as of signs: Thus, phrases such as "signa provere" and "signa constituere"[34] stand for the advance and halt of whole troop units, which are themselves no longer addressed. The Roman military counted among the "signa" not only flags, but also acoustic signals. Specific chords of individual horns had only a single addressee—the sergeant and standard bearer, the *signifer*. He translated the acoustic signal sequences into optical ones.

The eleventh century, in which the Battle of Numbers arose, appears to have drawn from such sources of the use of signs. According to Carl Erdmann's investigation into the emergence of the "idea of the crusade," it is

reflected less in a Christian iconology than in semiotic practices that are typical of medieval battlefields. Thus, a theosophy became possible that no longer ethically condemned or justified wars, but itself created reasons for war. Erdmann's attention is therefore directed primarily at the holy flags that arose at the turn of the millennium.[35] With the beginnings of the Christian sense of mission, the *ordinatio*—the power of consecration—established the hierarchy of the Church, separating bishops from priests, priests from the laity, and sacred objects from profane things. But only in the eleventh and twelfth century was a boundary crossed in the semiotic orders: the consecration of flags and swords assigned insignia of a military order to the churchly order. Strictly speaking, flags had hitherto exhibited a trinity that profoundly opposed the Christian one. Flags were not only incriminated as lance weapons and—still more devastatingly—through images of idols. On top of that, they counted among the signa—the standards. As such, they made the battle and combat legible; they regulated beginning, middle, and end. They were no longer separable from the war that they waged. Chiastically, the Church designed its own flags, provided them to the armies, and—conversely—led crusades under the flags of kings. The battle was no longer waged merely with signs but over signs. Depictions and miniatures of the crusades differentiate the often completely similar Franks and Saracens on foreign and unknown ground solely by the fact that the former displayed insignia and the latter did not. The victorious end of the battle was sealed with the reconquest of holy flags by the king who captured them.

From that point on, signs gained an autonomy of previously unknown magnitude. Probably unsurpassed in this regard was the *carroccio*, a wagon bearing the standards of those Lombard cities that preserved their independence in 1176 in the victory over Barbarossa. Before each battle, the *carroccio* was fetched from the cathedral by a city contingent made up not only of soldiers, brought to the marketplace, equipped with all sorts of insignia and finally taken to the battlefield. During the battle itself, a group of guards protected the wagon, while on its platform trumpeters sounded tactical signals; notaries wrote orders, recorded losses, and prepared commendations, punishments, and compensations; and priests cared for the wounded and administered sacraments to the dying. "Thus, the classic *carroccio* served several purposes at once for the northern and central Italian city communities: as a sort of mobile generals' hill, command

center, optical point of reference," as a dressing station and refuge for weary soldiers.[36] Above all, however, the wagon bearing the standard ensures a self-contained war, because to capture it means to take possession of the *signum civile*—without extending the battle to the city itself.

In the Battle of Numbers, one game piece—the pyramid—is now elevated above all the others. It embodies several square numbers at once. The taking of all the other game pieces is executed through expressions of arithmetic. But the taking of this piece is articulated only through a military terminology.[37] If the pyramid—which is vulnerable in comparison to other game pieces—is taken, then all the other pieces that count among the square numbers of the pyramid are rendered invalid.[38] No other piece contains such purely referential dependencies. The rules of the emerging chivalric orders will provide the same semiotic logic for the battle: if the standard bearer falls, then the troops assigned to him admit defeat as well.[39] Thus, the Battle of Numbers overlaps with the rules of the chivalric orders and has, so the theory goes, created a codex for their peculiar position midway between military and clerical status.[40]