

# **Equilibrium, Trade, and Growth**

**Selected papers of Lionel W. McKenzie**

**edited by Tapan Mitra and Kazuo Nishimura**

**The MIT Press  
Cambridge, Massachusetts  
London, England**

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This book was set in Palatino on 3B2 by Asco Typesetters, Hong Kong and was printed and bound in the United States of America.

Library of Congress Cataloging-in-Publication Data

McKenzie, Lionel W.

Equilibrium, trade, and growth : selected papers by Lionel W. McKenzie / edited by Tapan Mitra and Kazuo Nishimura.

p. cm.

Includes bibliographical references and index.

ISBN 978-0-262-13501-6 (hardcover : alk. paper)

1. Economics, Mathematical. 2. Equilibrium (Economics). 3. Economic development.

I. Mitra, Tapan, 1948-. II. Nishimura, Kazuo, 1946-. III. Title.

HB135.M413 2009

330—dc22

2008029323

10 9 8 7 6 5 4 3 2 1

# Index

- Accumulation paths, 350, 352, 378, 393, 403, 404, 411, 425–426. *See also* Capital accumulation  
efficient, 333, 334, 340  
infinite, 352, 394  
optimal, 12, 411, 426 (*see also* Growth paths, optimal; Optimal Accumulation path)  
and turnpike theory, 353–354, 397, 403–404, 417–419, 441–445 (*see also* Turnpike theorem)  
regular, 413
- Activity analysis  
and analysis of competition, 216  
and efficiency theorem, 219  
and efficient facets, 225  
in equalization problem, 237  
facets of, 235n23  
and factor price equalization theorem of Samuelson, 6  
fundamental theorem of, 269  
McKenzie's interest in, 5, 6, 7  
and selection of productive processes, 215  
and world trade, 233
- Adequacy Assumption, of Bewley, 195
- Admissible price vectors, set of, 196
- Aggregate consumer set, 105
- Alaoglu's theorem, 207, 210–211
- Algorithms for finding equilibria, 144–145  
in Houthakkar model, 145  
in MacKinnon model, 150  
Scarf algorithms, 149, 150
- Allais, W., 37
- Allocation, of net trades, 199
- Almon, Clopper, 145–146, 154, 155
- Amano, Akihiro, 9
- Antisymmetric relation, 192
- Araujo, A., 411, 412–413, 414, 415, 418, 419
- Arrow, Kenneth  
and always-desired or always-productive goods, 105  
and applications of Liapounov's approach, 135n2  
and argument for existence of equilibrium for competitive market, 107, 114  
on compensated equilibrium, 163, 164  
and competitive economy, 180  
and competitive equilibrium, 107, 159  
and consumer preferences, 182  
and consumption set, 189–190  
at Econometric Society meeting, 36  
on equilibrium, 119n2  
and general production model, 91n1  
and irreducibility, 27, 28  
and linear production models, 66  
vs. McKenzie approach, 3  
and matrices with dominant diagonals, 276–277  
and optimal growth conference, 12  
and price stability, 342n2  
and resource relatedness, 15, 28, 31  
and stability of equilibrium (lecture), 135n1  
and survival assumption, 15, 165  
and *tâtonnement* stability, 34, 36  
and world trade model, 78n2
- Arrow-Debreu model, 24  
and competitive equilibrium, 115, 181  
constant returns model of, 25, 26  
and consumer sets, 103, 104  
and divisibility, 25  
and external economies between firms, 183  
and irreducibility, 28  
and production model, 114

- Arrow-Debreu paper on existence of competitive equilibrium, 6–7, 8, 15, 38  
 Asymptotic convergence for optimal paths of capital accumulation. *See* Turnpike theorem(s)  
 Asymptotic stability, 431  
   of optimal stationary point, 459–462  
 Asymptotic theorems on optimal T-period paths, 381  
 Asymptotic turnpike theorem, 437–439  
 Atsumi, Hiroshi, 9, 12, 346, 346–347, 350–351, 353, 394  
 Atsumi-Weizsacker criterion for optimality over an infinite horizon, 47, 346, 347, 394  
 Atsumi-Weizsacker definition of optimality, 370  
 Autonomous control system, 145  
  
 Back, Kerry, 31, 189, 191  
 Balanced family, 199–200  
 Balanced game, 181  
 Balanced growth paths  
   and DOSSO turnpike theorem, 9  
   existence of, 286–287  
   and Samuelson-Solow theorem, 339  
 Banach limit, 210  
 Banach space, 207, 411, 412, 419  
 Banhabib, J., 53  
 Barriers to trade, and Graham analogue, 262  
 Beckman, Martin, 6  
 Benveniste, L., 416, 437  
 Berge, C., 206, 432  
 Bewley, Truman, 54, 189, 191, 423, 439, 463n  
 Bewley's Adequacy Assumption, 195  
 Block, H. D., 342  
 Boldrin, M., 53  
 Boyd, John H. III, 28, 31, 32  
 Brauer, A. T., 277  
 Brock, W. A., 12, 14, 52, 417, 419, 423  
 Brouwer fixed-point theorem, 38, 102, 111–114, 159  
 Brunner, Karl, 6  
 Burke, J., 195  
 Burmeister, E., 425  
 Business cycle equilibrium, 37, 54–55  
  
 Cantor process, 444, 458  
 Capital accumulation  
   and consumer utility, 46–47  
   endogenous technical change as, 425  
   optimal, 43–46, 47–50, 345, 349, 350, 353  
     (*see also* Accumulation paths; Growth paths, optimal; Optimal accumulation path)  
   in Ramsey's program, 345–346  
 Capital accumulation, theory of, 37  
 Capital accumulation models  
   and competitive equilibrium, 194, 196, 197–199  
   continuous time, 209  
   inappropriate assumption for, 189  
   and turnpike theorems, 9, 10 (*see also* Turnpike theorem)  
 Capital accumulation as optimal in final state, 377  
   comparison paths for, 378–379  
   and convergence on facet, 385–390  
   and convergence to facet, 381–385  
   production model for, 377, 378  
   valuation ceilings for, 379–381  
 Capital goods, change in price and rental of, 423  
 Capital stocks, 393  
   expandable, 433, 443  
   growth of, 11–12  
   sufficient, 433, 443  
   in turnpike theory with von Neumann facet, 446  
   value for, 443  
 Cass, David, 14, 51, 54–55, 419, 423  
 Cassel, Gustav, 63, 215  
 Cauchy-Lipshitz theorem, 129, 133  
 Charge, pure, 210  
 Chichilnisky, G., 189  
 Chipman, John, 6, 42, 276, 286  
 Choquet's theorem, 204  
*Classical General Equilibrium Theory* (McKenzie), 4  
 Classical theorem on existence of competitive equilibrium. *See* Existence of competitive equilibrium, classical theorem on  
 "Climate," and international trade, 216–217  
 Cobb-Douglas assumption for intermediate product inputs, 147  
 Cobb-Douglas cost function, 311  
 Cobb-Douglas production functions, 146  
   and factor price equalization, 253–254  
 Cobb-Douglas utility function, 38, 142, 150  
 Coles, J. L., 54

- Collective demand function, 85
- Comparative advantage, 24, 38–41
  - and specialization, 215 (*see also* Specialization)
- Comparison paths, for capital
  - accumulation as optimal in final state, 378–379
- Compensated equilibrium, 163, 164
- Competitive economy, 23–24, 180
- Competitive equilibrium(ia), 6–7, 17n2, 86.
  - See also* Existence of competitive equilibrium
  - of abstract market, 106–107
  - activities model of, 24
  - Arrow-Debreu paper on, 7
  - and Debreu-Scarf theorem, 32
  - definition of, 163
  - and diagonal dominance, 281–282
  - efficiency of, 23
  - and efficient specializations, 269
  - and factor price equalization, 239, 282–284, 295, 12
  - and firms, 25–26
  - and Graham's model, 68–70, 71, 72, 73, 75, 76, 77, 79n18
  - in infinite-horizon model, 196–197 (*see also* Existence of competitive equilibrium with infinite horizon)
  - in McKenzie's Presidential Lecture, 15–16, 28
  - and McKenzie as student, 6
  - and maximum world output, 215
  - profit conditions of, 215–216
  - and quasi-equilibrium, 29–31
  - and specialization, 267
  - turnpike theory extended to, 54
  - uniqueness of, 10
  - and Wald, 91n1
- Competitive markets, theory of (basic propositions deduced from), 275
- Cone of diversification or specialization, 42–43
- Consumer preferences, 7–8, 12
  - dependent, 8, 83, 182
- Consumer sets, in argument for existence of general equilibrium for competitive market, 103–104, 105–106
- Consumer utility, and capital accumulation, 46–47
- Consumption, postponing of, 394–395
- Consumption sets, 189, 190, 194, 205
- Convexity, assumption of, 160
- Convex sets
  - and existence theorem, 73
  - and McKenzie as student, 6
- Core, 32
  - and competitive equilibrium, 32, 181, 182, 202, 205
  - of economy, 199
  - and firms, 25
  - and irreducibility, 193
  - nonemptiness of, 181, 190–191
- Cost function, and factor price equalization, 296
- Cost function inversion, 10–11, 296–297
  - Pearce conjecture on and counter-example, 11, 297–303, 305–311
- Cowles Commission, 5, 6, 7, 38
- Cowles Foundation at Yale University, 8, 119n1
- Darwin, Charles, 24
- DasGupta, Swapn, 461n
- Debreu, Gerard. *See also* Arrow-Debreu model
  - and always-desired or always-productive goods, 105
  - and argument for existence of equilibrium for competitive market, 107, 114
  - and competitive economy, 180
  - and competitive equilibrium, 119n1, 107, 159
  - in Cowles Commission group, 6
  - on equilibrium, 119n2
  - and external economies, 182
  - and general production model, 91n1
  - and irreducibility assumption, 27, 28, 32
  - on irreducible markets definition, 137
  - vs. McKenzie approach, 3
  - and matrices with dominant diagonals, 276
  - and preference relation, 174
  - on “quasi-equilibrium,” 163
  - and social equilibrium, 7
  - and stability of equilibrium (advice), 135n1
  - and survival assumption, 15, 164, 165, 166, 169
  - and uncertainty, 190, 209
  - and uniqueness debate, 11
  - and world trade model, 78n2
- Decentralization, Mirrlees on, 154–155

- Demand  
 as consistent preference, 90–91  
 positive excess, 34–37
- Demand functions  
 collective, 85  
 in competitive equilibrium, 86  
 in consumer-preferences model, 84–86  
 continuity of, 88–90, 91  
 excess, 284  
 in existence proof, 63  
 in Graham's model, 65–66, 69, 73–74, 77  
 and individual's survival without trading, 15  
 proof of existence of, 25, 26  
 single value, 25, 90  
 and survival assumption, 165
- Demand theory, 7–8, 32–34  
 without utility index, 8, 95–100
- "Demand Theory without a Utility Index"  
 (McKenzie), 8
- Diewiert, W. E., 148
- Discounted utility, 441
- Discounting, 13–15  
 and neighborhood turnpike theorem, 51, 52, 53, 54  
 in optimal growth path mode, 351  
 and Ramsey, 12, 13, 50  
 and turnpike theorem primal proof, 423, 424, 427, 428, 431, 432, 433  
 in turnpike theory with von Neumann facet, 441, 442, 443, 452, 453, 462
- Disposal  
 and argument on general equilibrium for competitive market, 101, 117  
 and capital accumulation as optimal in final state, 389  
 and Graham model, 76  
 and factor price equalization, 250, 252–253  
 and Pearce's theorem, 305–306, 311  
 and turnpike theorem of Morishima, 336
- Disposal, free, 178–179  
 and argument for general equilibrium for competitive market, 101, 105, 107, 119n2  
 and competitive equilibrium, 160, 165  
 and efficiency prices, 369  
 and expansible stock, 408  
 and factor price equalization, 241, 249, 257n9, 258n36  
 and general equilibrium theory, 27  
 and general production model (Arrow-Debreu), 91n1  
 and Graham's model, 76  
 in Leontief model, 248  
 in optimal growth path model, 348, 350, 352  
 and Pearce's theorem, 310, 311  
 and role of firm, 181  
 and single valued demand function, 90  
 and turnpike theorems, 335, 350, 352, 399, 401, 428, 433, 436, 443
- Dobell, R., 425
- Domestic goods, and factor price equalization, 246–247
- Dominant diagonal blocks, in proof of turnpike theorem, 414
- Dominant diagonal condition, 311–312, 418, 419
- Dominant diagonal matrices. *See* Matrices with dominant diagonals
- Dorfman, R. P., 9, 43. *See also* Turnpike theorem of Dorfman, Samuelson and Solow
- Duality considerations, in turnpike proofs, 423, 431
- Duke University  
 McKenzie as professor at, 5, 6, 7–8  
 McKenzie as student at, 4  
 recommendation to, 24
- Econometrica*, 8, 178, 305
- Econometric Society  
 Houthakkar paper at European meeting of, 145  
 McKenzie corrects false conjecture at, 36  
 McKenzie Fisher-Schultz lecture to, 49  
 McKenzie papers presented to, 8, 34, 38, 44, 45, 159, 342n1, 419n1  
 McKenzie presidential address to, 15, 28, 185n1  
 papers on competitive equilibrium presented to, 159
- Economic growth. *See* Accumulation paths; Capital accumulation; Growth paths, optimal
- Economic Journal*, 5, 43
- Economics, McKenzie's choice of, 24
- Economy, definition of, 163
- Edgeworth equilibria, 201–203, 209
- Efficiency  
 of competitive equilibrium, 23  
 and free trade, 215, 216, 232–233  
 and fundamental theorem of activity analysis, 269  
 in world production, 262, 272n7

- Efficiency prices, 368–370
- Efficiency theorem, in Graham model, 219–225
- Efficient accumulation paths, 333, 340
- Efficient facets, in Graham model, 225–228
- Efficient growth paths, 319–321, 333
- Efficient specialization, 263, 269, 270–272, 272n12
- Elasticities, method of, 144
- Entrepreneurial factors
  - in classical theorem on existence of competitive equilibrium, 180, 182
  - in competitive equilibrium model with infinite horizon, 195
  - and perfect competition, 163
  - in production model, 102, 104, 114, 115, 138–139
- Equality of factor prices. *See* Factor price equalization
- Equal treatment core, 201
- Equilibrium(a) for economic models
  - (computation of), 153–154
  - algorithms for, 144–145
  - Almon/Leontieff model, 145–146, 147, 154, 155
  - Brookings model, 147–148
  - for estimating effects of taxing capital income (Shoven/Whalley vs. Harberger), 149, 154
  - Klein/Tinbergen model, 147, 154
  - and linear systems, 148
  - MacKinnon model, 150–151, 153
  - Saito model, 146–147, 154
  - of Tsukui (optimal growth), 151–152, 153, 154
- Equilibrium position, of Graham model, 228–229
- Equilibrium stability, proofs of, 123–135
- Euler equation, 463
- Euler-Lagrange condition or equations, 412, 413, 414–415, 459
- Euler relation, 423
- Excess demand functions, 284
- Existence of balanced growth paths, 286–287
- Existence of competitive equilibrium, 6–7, 26–27, 31, 86–88, 281
- Existence of competitive equilibrium, classical theorem on, 159–164
  - and external economies, 182–183
  - finiteness and convexity assumptions in, 160
  - and free disposal, 178–179
  - preference relation weakened in, 171–177
  - and survival assumption, 164–171, 177, 184–185
- Existence of competitive equilibrium with infinite horizon, 189–191
  - core of as nonempty, 199–201
  - and Edgeworth equilibria, 201–203, 209
  - model of, 192–199
  - proof of, 191, 203–209
  - and similar results, 209
- Existence of equilibrium point, in Graham's model, 71–75, 77–78
- Existence of equilibrium in world trade model, 143
- Existence of general equilibrium for competitive market (argument for), 101–102
  - and assumption on irreducible markets (Assumption 6), 118–119
  - competitive equilibrium in, 106–107
  - consumer sets in, 103–104
  - and general existence theorem, 111–113
  - Hicksian model in concordance with, 113–115
  - postulates behind, 101
  - production set in, 104–105
  - and properties of  $C_i(p)$  (preferred combinations of goods), 115–117
  - and relations between consumer and production sets, 105–106
  - and special existence theorem, 107–111, 112
  - and Wald's counter-example, 117–118
- Existence of optimal paths, 370–374. *See also* Growth paths, optimal; Optimal accumulation path
- Existence and uniqueness of equilibrium in economic systems, theorems on, 63
- External economies, in classical theorem on existence of competitive equilibrium, 182–183
- Facet(s), 47. *See also* von Neumann facet of activity analysis, 235n23
  - in Graham model, 225–228, 229, 235n23
- Factor price equalization, 7, 38, 41–43, 237, 282–284
  - activity analysis in approach to, 237
  - assumptions underlying, 295
  - basic theorems on, 240–244

- Factor price equalization (cont.)
  - and Cobb-Douglas production function, 253–254
  - and competitive equilibrium, 239, 312
  - and equilibria for economic models, 145
  - and factor saturation, 247–248
  - and intermediate products, 238–239, 246, 253, 258n30
  - and inversion of cost function, 296–303, 305, 311
  - and Koopman, 6, 7
  - and Leontief model, 248–251, 258n30
  - model of, 237–238
  - and Samuelson, 10, 254–256, 283, 295, 296
  - and substitution or intermediate products, 251–253
  - and variable factor supplies, 245–246
- Famines, 16
- Favard's theorem, 306, 310
- Feasibility effects, 182–183
- Feasible allocation, 199
- Fenchel, W., 448
- Final state turnpike theory, 9–10
- Finitely additive measure, 210
- Finiteness, assumption of, 160
- Firms, 25–26
  - in classical theorem on existence of competitive equilibrium, 179–182
  - efficient distribution of production between, 261
- Fisher-Schultz lecture, 49
- Fixed-point method of finding
  - equilibrium, 149, 151, 155, 155n.3
  - and Graham model, 149
  - and "sandwich method," 150
  - of Scarf, 152, 153
  - and Shoven and Whalley, 154
  - von Neumann's use of, 159
- Fixed-point theorem of Brouwer, 38, 102, 111, 159
- Fixed point theorem of Kakutani, 6, 38
  - and competitive equilibrium, 7, 26, 86–88
  - and existence proof (Graham model), 75, 144
  - and neighborhood turnpike theorem, 51
  - and preference relation, 174
  - and quasi-stationary model (turnpike theorem), 429
  - and survival assumption, 167
- Flynn, J., 431
- Forecasting, models for, 145, 153–154
  - of Klein, 147
- Forecasting of prices, 146
- Free disposal. *See* Disposal, free
- Free trade
  - and complexity of interdependence, 232
  - efficiency of, 215, 216, 232–233
- Frobenius matrix, 371
- Fromm, Gary, 148
- Futures economy, of Hicks, 190
- Gale, David
  - and closed linear production model, 333
  - and competitive equilibrium, 119n1, 159, 160, 177
  - and cost function inversion, 296, 297
  - on expansible stock, 433
  - and irreducibility, 8, 27
  - and Liapounov functions, 423
  - on optimal accumulation path, 426
  - and optimal growth programs, 12, 152, 346, 347, 373
  - and preference relation, 171
  - and production model, 377
  - and turnpike theory, 46–47, 48, 399, 401
  - and uniqueness debate, 10–11
- Gantmacher, F. R., 10, 355–356
- General equilibrium, claim to support for, 155
- General equilibrium model
  - of Saito, 146–147, 154
  - of world trade (Graham), 141–143, 144 (*see also* Graham model of international trade)
- General equilibrium theory, 25, 26
  - and irreducibility, 27, 28
- General existence theorem, 111–113
  - modification of, 138
- Golden-rule equilibrium, 12
- Golden-rule stock, 14–15
- Goodwin, R. M., 276
- Graham, Frank D., 5, 23, 24
- Graham analogue, 262–263, 267
  - limits of, 263–267
  - and migratory factors, 270
- Graham model of international trade, 37–41, 63, 64–66, 78n8, 141–143, 144, 153, 215, 216–219, 261, 270
  - and competitive economy, 180
  - and competitive equilibrium, 68–70, 71, 72, 73, 75, 76, 77, 79n18
  - efficient facets in, 225–228
  - equilibrium position of, 228–229



- existence of equilibrium point in, 71–75, 77–78
  - and fixed point method, 149
  - generalizations of, 75–77
  - implications of, 66–68
  - and intermediate products, 229–232, 233
  - properties of (existence proof), 71, 76
  - specialization in, 219, 261, 270–272
  - theorem on efficiency in, 219–225
  - uniqueness of equilibrium output in, 70–71
  - Graham table of comparative costs, 39, 217, 231, 262, 264, 265
  - Great Depression, and McKenzie's interest in economics, 5
  - Gross substitutes, 35–36
    - weak, 35, 284–285
  - Growth model, 197–199
    - "reduced form" version of, 12–13
  - Growth paths, efficient, 319–321, 333
  - Growth paths, optimal, 347. *See also*
    - Optimal accumulation paths
  - asymptotic behavior of, 347, 351, 353, 361
  - and business cycle, 54
  - in convergence on von Neumann facet, 354–368
  - and efficiency prices, 368–370
  - existence of, 370–374
  - model for, 347–349
  - Ramsey on, 345–346
  - recent work on, 346–347
  - and turnpike theorem, 44, 54, 349–354
- Hadamard's theorem, 275
- Hahn, Frank
  - and compensated equilibrium, 163, 164
  - and consumer preferences, 182
  - and consumption set, 189–190
  - and irreducibility, 27
  - vs. McKenzie approach, 3
  - and resource relatedness, 15, 28, 31
  - and stability problem, 276–277
  - and survival assumption, 165
- Hahn-Banach theorem, 210
- Halmos, Paul, 6
- Hamiltonian function, 423
- Hammond, P. J., 28, 30
- Hansen, T., 152, 153
- Harberger, A. C., 149
- Harrod, Roy, 5
- Harvard Classics, 4, 24
- Harvard University, McKenzie as
  - professor at, 14, 439n, 463n
- Hawkins, D., 276
- Hawkins-Simon conditions, 287
- Heal, G. M., 189
- Henderson, Hibert, 5
- Herstein, I. N., 276
- Hicks, John, 9
  - and competitive economy, 180–181
  - festschrift* for, 12, 18n17
  - fruitful lecture of, 44
  - on international trade, 261
  - and matrices with dominant diagonals, 276
  - and method of finding equilibrium, 144
  - and Morishima turnpike theorem, 342n1
  - and paths on von Neumann facet, 374n3
  - and stability of equilibrium, 123
  - Value and Capital* written by, 5
- Hicksian model, 24
  - in concordance with production model, 113–115
- Hildenbrand, W., 209
- Hirsch-Pugh stable manifold theorem, 462
- Houthakker, Hendrik, 144–145
- Hurwicz, Leonid, 6, 33, 34, 36, 135n2, 276–277, 342n2
- "Ideal Output and Interdependence of Firms" (McKenzie), 5
- Implicit function theorem, 411, 412, 414
- Inada, Ken-ichi, 377
- Income effects, and factor price equalization, 245
- Income multiplier model, stability of, 286
- Infinite optimal accumulation paths, 12, 347, 368, 371, 372, 373, 374, 394–396, 397–398, 402, 405, 407, 408
- Infinite optimal programs, 345, 346
- Infinite production paths, price sequences supporting, 370, 395
- Infinity of goods, 190. *See also* Existence of competitive equilibrium with infinite horizon, 189–191
- Integrability problem, 34
- Interiority, for existence of equilibrium, 166, 177, 178–179
- Intermediate products
  - and activity analysis, 269
  - and classical treatment of specialization, 261

- Intermediate products (cont.)
  - and factor price equalization, 238–239, 246, 253, 258n30
  - in Graham analogue, 266–267
  - and Graham model, 229–232, 233, 261
  - and Leontief model, 249–250, 258n30
  - and linear activities analogue, 267
  - prohibition of trade in, 233
- International Economic Association, 46
- International trade (world trade)
  - and efficient distribution between firms, 261
  - Graham's model of, 37–41, 64–66, 141–143, 144, 180 (*see also* Graham model of international trade)
  - and profit conditions of competitive equilibrium, 215–216
  - specialization in, 215
- Intertemporal equilibrium, 209. *See also* Existence of competitive equilibrium with infinite horizon
- Inversion of cost function, counter-example to Pearce's conjecture on, 11, 297–303
- Irreducibility (irreducible economy or markets), 8, 24, 26, 27, 28–32, 101
  - and argument for general equilibrium for competitive market, 106, 119
  - and classical theorem on existence of competitive equilibrium, 177
  - and competitive equilibrium, 101
  - definition of, 105–106, 137–138
  - and famines, 16
  - strong, 193
- Irreflexive relation, 192
- James, S. F., 11, 295
- Japanese Economic Review*, McKenzie paper in, 4
- Jeanjean, Patrick, 49, 423, 427
- Johnson, Harry, 41
- Jones, Ronald, 8, 41
- Journal of Economic Theory*, 14
- Kakutani's fixed point theorem. *See* Fixed point theorem of Kakutani
- Kaplansky, Irving, 6
- Kemeny, J. G., 46, 375
- Kemperman, Johannes, 303n1
- Keynesian multiplier theory, 286
- Khan, Ali, 28, 31
- Klein, Lawrence, 146, 148, 154, 155
- Koopmans, Tjalling
  - and activity analysis, 5, 6, 7, 38, 216, 219
  - and efficiency prices, 69
  - and efficient point set, 219
  - and Graham model, 78n2
  - and optimal growth programs, 152, 153, 346, 347, 350–351
  - and overtaking criterion, 394
  - on survival, 15
  - transportation model of, 220
- Kuhn, Harold, 159, 297
- Kurz, M., 13
- Kydland, F., 54
- Labor supplies, in Graham's model, 64, 76
- Leontief model, 23, 248–251, 276
  - and Almon model, 145
  - with capital coefficients, 50
  - and balanced growth paths, 286
  - and competitive economy, 24
  - and competitive equilibria, 281–282
  - and convergence of optimal paths to saturation paths (Atsumi), 347
  - and convergence on von Neumann facet, 365, 377
  - and intermediate products, 249–250, 258n30
  - and labor, 50
  - and linear methods, 154
  - and matrices with dominant diagonals, 287
  - and productive processes, 215
  - and quasi-dominant diagonal, 35
  - and Radner proof, 10
  - and Saito model, 146
  - and turnpike theorem, 44, 315
- Leontief technology, 289
- Levhari, D., 460, 461
- Liapounov, M. A., 123
- Liapounov function
  - and stability of equilibrium, 124, 126, 135n2
  - and tâtonnement stability, 36
  - and turnpike theorems, 52–53, 411, 423, 441, 448, 449
- Liapounov (neighborhood) stability
  - of optimal stationary point, 456–458
  - of stationary optimal stock, 15
  - and turnpike theorems, 412, 424, 431, 437
  - of von Neumann facet, 441, 448–456
- Linear activities analogue, 263, 267–269

- Linear Programming and Economic Analysis* (Dorfman, Samuelson and Solow), 7, 9, 333
- Liviatan, N., 460, 461
- Lucas, R. E., 54
- McFadden, D., 12, 46–47
- McKenzie, Lionel, 3–4, 16  
early life of, 4–5  
Ph.D. of, 8  
as professor at Duke, 5, 6, 7–8  
as professor at Harvard, 14, 439n, 463n  
as professor at University of Rochester, 3, 8–9, 43  
student period in life of, 5–6  
and students of, 3–4, 16
- Mackey topology, 191
- MacKinnon, James, 150–151, 153
- MacLane, Saunders, 6
- Majumder, M., 53
- Malinvaud, Edmond, 6, 370
- Malinvaud prices, 395
- Marimon, R., 54
- Market failure, famine as, 16
- Markowitz, Harry, 6
- Marschak, Jacob, 5, 6
- Marshall, Alfred, 25, 34, 144, 161, 163, 180
- Mas-Colell, Andreu, 160, 171, 172, 177, 189
- Mathematical Methods in the Social Sciences (conference), 8
- Mathematical Social Sciences Board  
Squam Lake conference of, 14  
summer workshop sponsored by, 47
- Mathematics, and inversion of cost function, 11
- Matrices with dominant diagonals, 8, 34–35, 36, 275–277  
basic mathematical theorems relating to, 277–281  
basic mathematical theorems relating to (further developments), 287–291  
and existence of balanced growth paths, 286–287  
and factor price equalization, 282–284  
positive solutions resulting from, 281–282  
and stability of income multiplier model, 286  
and stability of trading system, 284–286
- Maximal balanced growth, 24–25  
for capital accumulation as optimal in final state, 379  
and optimal growth theory, 151  
and Samuelson-Solow assumption, 45  
in turnpike theorem (DOSSO), 317–318, 320, 321, 322  
in turnpike theorem (Morishima), 340  
in turnpike theorem (Radner), 44  
in turnpike theory with von Neumann facet, 45  
and von Neumann model, 46, 159  
as von Neumann ray, 333 (*see also* von Neuman ray)
- Maximization, general theory of, 351
- Measure  
finitely additive, 210  
purely finitely additive, 210
- Menger, Karl, 159
- Method of elasticities, 144
- Metzler, L., 276, 277
- Middle Georgia College, McKenzie attends, 4, 24
- Migratory factors, and production possibility locus, 269–270
- Mill, John Stuart, 142, 144
- Minimum income functions, 24, 33–34
- Minkowski separation theorem, 369
- Mirrlees, James, 154–155, 346
- MIT, McKenzie as instructor at, 5
- Mitra, T., 53
- Model of production. *See* Production model
- Montruchio, L., 53
- Moore, James (Jim), 15–16, 160, 163, 165, 166, 171, 177
- Moore, John, 27
- Morgenstern, Oskar, 5, 8, 46, 377  
bibliography of work of, 420–421
- Morishima, Michio, 9, 43–44, 315. *See also* Turnpike theorem of Morishima  
and polyhedral model, 390  
and von Neumann facet, 374n3
- Morse, Marston, 6, 7
- Mosak, J. L., 144
- Movshovich, S. M., 379
- Mukerji, Anjan, 37
- Nagasaki Prefectural University Review*, McKenzie paper in, 4
- Nash, John, 7, 78n2
- Negishi, T., 27, 37, 54, 276–277
- Neighborhood turnpike theorem, 14–15, 25, 50–54, 431–437

- Neoclassical economics
  - and McKenzie, 3
  - and turnpike theorems, 23, 53
- Neoclassical growth model, and turnpike theory with von Neumann facet, 442
- Neoclassical model, and concave utility function, 425
- Neoclassical production model, 44
- Neumann, John von. *See* *at* von Neumann
- Nikaido, Hukukane, 3, 10–11, 159, 296, 297
- Nishimura, K. G., 53, 437
- Non-neutral cycles, 24
- “On Equilibrium in Graham’s Model of World Trade and Other Competitive Systems” (McKenzie), 6
- Optimal accumulation path, 12, 411, 441.
  - See also* Growth paths, optimal
  - and capital accumulation in final state, 377, 381, 382, 384, 385, 386, 390
  - and convergence on von Neumann facet, 355, 362, 367
  - finite, 394, 402, 404, 405 (*see also* Optimal T-period path)
  - infinite, 12, 347, 368, 371, 372, 373, 374, 394–396, 397–398, 402, 405, 407, 408
  - and Liapounov stability, 424
  - nontrivial, 448
  - and support prices, 405, 407
  - and turnpike theorems, 353–354, 397, 402–404, 417–419, 441 (*see also* Turnpike theorem)
  - in turnpike theory with von Neumann facet, 45, 442, 443, 445, 446, 448, 450, 454, 455, 456, 456–457, 462
  - unique, 462
- Optimal capital accumulation, 43–46
  - and turnpike theorem, 47–50
- Optimal growth paths. *See* Growth paths, optimal
- Optimal growth theory
  - and Hansen/Koopmans model, 152–153
  - and Tsukui model, 151–152, 153, 154
- Optimality, Atsumi-Weizsacker definition of, 370
- Optimality theory, and monopolistic competition, 43
- Optimal savings, Ramsey on, 12
- Optimal stationary path, 424, 428, 431, 434–435, 436, 438
  - non-trivial, 431, 445, 448, 450, 451, 452, 454, 457
  - for turnpike theory with von Neumann facet, 441, 442, 445–446, 448, 450–451, 452, 453, 454, 455, 457, 459, 463
- Optimal stationary point
  - asymptotic stability of, 459–462
  - Liapounov stability of, 456–458
- Optimal stationary program, 400
- Optimal stationary state, 412
- Optimal stationary stock, 14, 445–446, 449, 452, 453, 457, 459
- Optimal T-period path, 353, 379, 394–395, 397, 402–403
  - asymptotic theorems on, 381–385
- Optimization problems, dynamic (Ramsey type), 13
- Origin of Species* (Darwin), 24
- Osumi, Keisuke, 463n
- Overtaking criterion, 394
- Pareto, and stability of equilibrium, 123
- Path of accumulation. *See* Accumulation paths
- Paths, optimal. *See* Growth paths, optimal
- Pearce, Ivor, 11, 295, 297
- Pearce’s theorem (conjecture), 11, 297–299, 305–308
  - counter-example to, 11, 298–303, 309–311
  - as generalized, 309
- Peleg, B., 131, 189, 190, 191, 205, 426
- Phelps, E. S., 347
- Pollution, 395
  - and free disposal, 401
  - as state variable, 425
- Positive excess demand, 34–37, 123
- Prediction
  - through finding equilibria, 153–154
  - of prices, 154–155
- Preference orderings, 8, 86, 90, 91, 96, 102
- Preference relation, in classical theorem on existence of competitive equilibrium, 171–177
- Prescott, E. C., 54
- “Price Mechanism in a Planned Economy” (Mirrlees), 154
- Price sequences, and infinite paths, 370, 395, 396, 400, 403, 406, 408
- Price vectors, admissible (set of), 196
- Princeton Graduate College, McKenzie attends, 5, 37

- Production model, 83–84
  - for capital accumulation optimal in final state, 377, 378
  - and competitive equilibrium, 91n1
  - and maximal balanced growth, 333
- Production paths, infinite, 370, 395
- Production possibility locus
  - and Graham analogue, 261, 262, 264, 265–266
  - and linear activities analogue, 267–268
  - and migratory factors, 269–270
- Production set, in argument for existence of general equilibrium for competitive market, 104–106
- Productive processes, 215, 238
- Profit, in competitive equilibrium, 86
- Profit conditions
  - of competitive equilibrium, 215–216, 237, 240, 247
  - and von Neumann facet, 45
- Pseudo-utility functions of Shafer, 28
- Purely finitely additive measure, 210
- Quasi-dominant diagonals (q.d.d.), 24, 34–36, 276, 277–279, 280, 289, 290
- Quasi-equilibrium, 29–31, 163, 164, 165, 203
- Quasi-stability, 135nn3,5
- Quasi-stationary models
  - for turnpike theorem primal proof, 423, 427–431
  - for turnpike theory with von Neumann facet, 441, 445
- Radner, Roy
  - and “strict convexity,” 339–340, 347
  - and turnpike theorem, 9, 12, 43–44, 45, 315, 316, 333, 377 (*see also* Turnpike theorem of Radner)
  - and utility as stock, 374n2
  - and value loss lemma, 12, 45
- Radner theory for closed models of growth, 353
- Ramsey, Frank, 345–346
  - and discounting, 13
  - on optimal savings, 12
  - and overtaking criterion, 394
- Ramsey model, 43, 46–47, 49, 152
  - multi-sector, 50, 461
- Ramsey problem, 152–153, 377
- Ramsey theory of optimal taxation, 8
- Rationality, strongly individual, 193
- Reciprocal roots, and turnpike theory, 315, 323–326
- Resource relatedness, 15, 28, 31
- Revealed preference, weak axiom of, 37, 144, 159
- Review of Economic Studies*
  - McKenzie papers in, 6, 8
  - symposium on optimal growth presented by, 12
- Rhodes scholarship, won by McKenzie, 5
- Ricardo, David, 144, 333
- Risk aversion, uniform absolute, 416
- Robinson, Joan, 347
- Rockafellar, R. T., 14
- Role of the firm, in classical theorem on existence of competitive equilibrium, 179–182
- Rosett, Richard, 8–9, 40
- Saito, Mitsuo, 146–147, 154
- Samuelson, Paul A.
  - and capital goods price or rental, 423
  - on factor price equalization, 6, 10, 254–256, 283, 295, 296
  - and intermediate products, 232
  - on labor as factor, 217
  - and linear production models, 66
  - and McKenzie at MIT, 5
  - and Ramsey program, 346
  - revealed preference approach of, 99
  - and stability of equilibrium, 123, 276
  - and turnpike theory, 9, 13–14, 43, 45, 315, 462, 463n (*see also* Turnpike theorem of Dorfman, Samuelson, and Solow)
- Samuelson-Solow theorem, 44–45, 339
- “Sandwich method,” 150
- Sard’s theorem, 11
- Saturation, 345–347
- Savage, Jimmy, 6
- Saving rate, and Ramsey on economic growth, 345
- Scarf, Herbert, and Scarf algorithm, 145, 149, 150, 152, 153, 155n3, 181
- Scarf’s theorem, 32, 190–191, 201, 209
- Scheinkman, Jose
  - and continuous time models, 423
  - and discounting, 14
  - and first derivative for value function, 437
  - and Liapounov functions, 423
  - on unique optimal stationary stock, 457

- Scheinkman, Jose (cont.)  
 and proof of turnpike theorem, 51–52,  
 411, 412–413, 414, 415, 416, 418, 419  
 and turnpike theory with von Neumann  
 facet, 460, 461, 462, 463n  
 “Scholar’s Progress, A” (McKenzie), 4  
 Sen, A., 16  
 Set of admissible price vectors, 196  
 Shafer, Wayne J., 171, 173, 177, 182  
 Shell, K., 14, 51, 419, 423  
 Shoven, John, 149, 154  
 Simon, H. A., 276  
 Slater, Morton, 6, 38  
 Slutsky equation, 33, 95, 96, 99  
 Smith, Adam, 4, 24  
 Social budget planes, 72  
 Social production set  
 and general equilibrium for competitive  
 economy, 25  
 and theory of existence of equilibrium, 179  
 Social utility function, in turnpike proof,  
 425  
 Solow, R., 9, 43, 45, 276, 315, 346. *See also*  
 Turnpike theorem of Dorfman,  
 Samuelson, and Solow  
 Sonnenschein, Hugo, 171, 173, 177, 182  
 Sources, bibliographic, 465–466  
 Specialization, 39–41, 267  
 and bilateral comparison, 233  
 classical treatment of, 261, 266  
 complete, 270  
 efficient, 263, 269, 270–272, 272n12  
 and efficient facets, 225–226, 227  
 and factor price equalization, 244, 251  
 and free trade, 233  
 general theory of, 269–270  
 in Graham analogue, 262, 265, 267  
 in Graham model, 219, 261, 270–272  
 incomplete, 10  
 and intermediate products, 230, 231, 232  
 in international trade, 215  
 and linear activities analogue, 267  
 in optimal growth path model, 348  
 Special existence theorem  
 for abstract market, 107–111, 112  
 “Specialization and Efficiency in World  
 Production” (McKenzie), 6  
 Squam Lake, Mathematical Social Sciences  
 Board Conference in, 14  
 Stability  
 asymptotic, 431  
 of optimal stationary point, 459–462  
 of competitive equilibrium, and matrices  
 with dominant diagonals, 281  
 of equilibrium, proofs of, 123–135  
 of income multiplier model, 286  
 of trading system, 284–286  
 Stanford University  
 Arrow lecture at, 135n1  
 conference on optimal growth at, 12  
 Mathematical Methods in the Social  
 Sciences conference at (1959), 8, 35  
 summer workshop at (1965)  
 Stationary optimal path. *See* Optimal  
 stationary path  
 Stationary optimal point. *See* Optimal  
 stationary point  
 Stationary optimal program, 400  
 Stationary optimal state, 412  
 Stationary optimal stock, 14, 445–446, 449,  
 452, 453, 457, 459  
 Stigum, B. P., 189  
 Stokey, N., 54  
 “Straight-down-the-turnpike” theorems,  
 402, 404, 405  
 Strict preference relation, 192  
 Strong individual rationality, 193  
 Strong irreducibility, 193  
 Substitution and substitution circuits,  
 272n11  
 and efficient specialization, 271  
 and factor price equalization, 245, 252–  
 253  
 and Graham analogue, 265  
 in Graham model, 220, 222, 223, 224, 225,  
 226–227, 229, 233, 235n23  
 Support prices  
 for nontrivial stationary optimal paths,  
 445  
 and turnpike theorem primal proof, 427  
 and turnpike theorems with technology  
 and welfare function variable, 405–409  
 and turnpike theory with von Neumann  
 facet, 446, 452  
 Survival assumption, 15–16, 27, 29, 164–  
 171, 177  
 Appendix on, 184–185  
 Survival issue, 15–16  
 Sutherland, W., 13  
 Swan, R. G., 303n1, 309  
 Takayama, Akira, 8  
*Tâtonnement* stability, 23, 24, 34–37  
 Taxation, and Ramsey theory, 8

- Technological feasibility, and competitive equilibria, 183
- Theory of International Values, The* (Graham), 141, 144
- Thompson, G. L., 46, 377
- Tinbergen, J., 147
- T-period optimal path. *See* Optimal T-period path
- Trade, international. *See* International trade
- Trade (exchange), need for. *See* Resource relatedness; Survival assumption
- Trading system, stability of, 284–286
- Transformation function, 315–316
- Transitive relation, 192
- Tsukui, Jinkichi, 151–152, 153, 154, 155
- Turnpike problem, 377
- Turnpike property, 46
- Turnpike theorem(s) (asymptotic convergence for optimal paths of capital accumulation), 9, 11–12, 45–46, 333, 349, 354
- Aaujo-Scheinkman proof of extended, 411–419
- asymptotic, 437–439
- with discounting, 14–15, 51–52
- extended to competitive equilibria, 54
- extended to quasi-stationary models, 52
- and Leontief model, 23, 315
- myopic optimizations in, 424
- “neighborhood turnpike theorem,” 14–15, 25, 50–54, 431–437
- and optimal growth, 54, 151–152
- proofs of, 423
- and Ramsey paper, 12
- value loss approach in, 12, 45, 441 (*see also* Value loss approach in turnpike theory)
- Turnpike theorem of Dorfman, Samuelson, and Solow (DOSSO), 9–10, 43, 44, 45, 315–317, 330, 333, 375
- and efficient growth paths, 319–321, 333
- and linear systems, 321–323
- and McKenzie version of Morishima theorem, 334
- and maximal balanced growth, 317–318
- and reciprocal roots, 315, 323–326
- turnpike path in, 326–330
- Turnpike theorem of Morishima, 9, 43–44, 315, 333–342
- Turnpike theorem primal proof, 423–424
- and neighborhood turnpike theorem, 431–437
- quasi-stationary model for, 427–431
- reduced model for, 424–426
- uniform strict concavity in, 426–427, 432
- Turnpike theorem of Radner, 9, 12, 43–44, 45, 315, 316, 333
- Turnpike theorem and Ramsey model, 46–47, 49, 50
- Turnpike theorems with technology and welfare function variable, 402–404
- and existence of optimal paths, 394–398
- model of, 393–394
- and other initial stocks, 397–398
- in stationary case, 399–402
- support prices for, 405–409
- Turnpike theory, final state, 9–10
- Turnpike theory with von Neumann facet, 45–46, 377, 441–442
- and asymptotic stability of optimal stationary point, 459–462
- background results for, 445–448
- and example of Weitzman-Samuelson model, 14, 15, 462–463, 464
- and Liapounov stability of facet, 448–456
- and Liapounov stability of optimal stationary point, 456–458
- model for, 442–445
- Turnpike theory and von Neumann model, 9, 44, 46, 152, 315, 377
- Uekawa, Yasuo, 4, 9
- Uncertainty, about tastes and technology, 346
- Uniform strict concavity, in turnpike theorem primal proof, 426–427, 432
- Uniqueness of competitive equilibria and factor price equalization, 282
- and matrices with dominant diagonals, 281
- Uniqueness debate, 10–11
- Uniqueness of equilibrium, and Graham’s model, 70–71, 143
- University of Chicago, McKenzie as student at, 5–6
- University of Rochester
- conference on optimal growth at, 12
- McKenzie as professor at, 3, 8–9, 43
- summer workshop at, 47
- Utility, derived at each point in time, 12–13

- Utility function. *See also* Social utility function  
 bounded, 345  
 and existence of competitive equilibrium with infinite horizon, 209  
 reduced-form, 13  
 Utility gain, in turnpike proof, 426  
 Utility index, 95. *See also under* Demand theory  
 Utility saturation  
 and Atsumi program, 347  
 Ramsey on, 345  
 Uzawa, Hirofumi, 135n2, 159
- Valuation ceiling, 354  
 for capital accumulation as optimal in final state, 379–381  
*Value and Capital* (Hicks), 5, 180, 190  
 Value loss approach in turnpike theory, 12  
 an Araujo-Scheinkman proof, 415, 416, 417–419  
 and Liapounov function, 423  
 and optimal paths, 396, 397, 398  
 and stationary case, 400, 401  
 and support prices, 405  
 and uniform strict concavity, 427  
 and von Neumann facet, 45, 441, 449, 454, 455–456, 461, 462  
 “Visit lemma,” 14–15  
 von Neumann, John, 6, 9, 10  
 and competitive equilibrium, 159  
 and existence proof, 64  
 on general equilibrium models of trade, 38  
 and Graham on algorithm for equilibrium discovery, 143  
 production model of, 377  
 and von Neumann ray, 333  
 von Neumann equilibria, 25, 46  
 von Neumann facet, 25, 45–46, 47, 48  
 convergence on (capital accumulation as optimal in final state), 377, 385–390  
 convergence on (optimal growth paths), 354–368  
 convergence to (capital accumulation as optimal in final state), 377, 381–385  
 Liapounov stability of, 448–456  
 in multi-sector Ramsey model, 50  
 nontrivial, 461, 462  
 stable, 458  
 and turnpike theory, 51–52, 53, 54, 350, 441–442, 448–456, 459, 461, 462, 463
- Von Neumann growth rate, 351  
 von Neumann model, 9, 10, 44, 46, 152, 315, 347, 461, 462  
 von Neumann point, 47, 350, 352, 371  
 von Neumann ray, 10, 45, 333–334, 340  
 von Weizsäcker, Christian, 346, 395–396.  
*See also* Atsumi-Weizsacker
- Wald, Abraham  
 and competitive equilibrium, 6, 7, 91n1, 159–160  
 and general equilibrium, 63, 117–118  
 and productive system, 257n12  
 and role of firm, 180  
 and world trade model, 78n2
- Walras, Leon, 23  
 and competitive equilibrium, 161, 163  
 and general equilibrium, 153  
 and productive processes, 215  
 and Adam Smith, 24  
 and social production set, 25, 179–180  
 and stability of equilibrium, 123  
 and *tâtonnement* stability, 34, 37
- Walras’s Law, 125, 275  
 and gross substitute matrix, 35, 36  
 and stability of equilibrium, 125, 126, 132, 134  
 and stability of trading system, 284, 285  
 and theory of competitive markets, 275
- Walras’s production model, 83
- Weak Axiom of Revealed Preference, 37, 144, 159
- Weak gross substitutes, 35, 284–285
- Wealth of Nations* (Smith), 4, 24
- Weintraub, Roy, 4
- Weitzman, Martin, 13, 14, 15, 405, 462, 463
- Weitzman prices, 427
- Weizsäcker, Christian von, 346, 395–396.  
*See also* Atsumi-Weizsacker
- Whalley, John, 149, 154
- Woodbury, M. A., 276, 277
- World trade. *See* International trade
- World War II, and McKenzie, 5
- Yaari, M. E., 31, 189, 190, 191, 205
- Yano, Makoto, 54, 439n, 463n
- Yosida-Hewitt theorem, 203, 207, 210
- Zabel, Edward, 8–9
- Zero profit condition, 106
- Zilcha, I., 426