

The Equilibrium Manifold

**Postmodern Developments in the Theory of General
Economic Equilibrium**

Yves Balasko

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