the teacher of little Jane is *the same teacher* as the teacher Tim had last year. If you refer to the person in question as Leslie, something must individuate and handle the identity of what bears the name "Leslie." None of the categories mentioned will do, because Leslie was Leslie before she was a woman, a wife, a mother, a teacher, or a friend. What does the job for the bearer of the name is the kind PERSON.

The moral to draw is that the work of individuation and identity tracing for all the cases we have considered is done by kinds. This suggests the following Fundamental Postulate:

Fundamental Postulate No reference without a kind to provide for individuation and identity of what is referred to. This means that to have an individual as the object of thought or desire, one must appeal, at least implicitly, to a suitable kind to individuate the object and handle its identity. Now kinds are abstract objects, in the sense that they do not enter the story of causal interactions taking place in space and time. What makes reference so special is the fact that it presupposes appeal to such abstract objects. Abstract objects cannot be perceived, for they give rise to no sights or sounds or any other perceptual signals, but they can be talked about. The ubiquitous appeal to abstract objects is what is special about reference, and hence cognition. [It is interesting to note that the entry in *Webster's Dictionary* for "individuate," namely "to distinguish from others of the same species," agrees with the fundamental postulate, "species" in the dictionary entry corresponding with "kinds" in the fundamental postulate. (See the postscript to this chapter. For an explanation, see the preface.)]

The Fundamental Postulate intimates an answer to the puzzle of universals and individuals that became particularly acute in the chapter on Duns Scotus and Ockham. The postulate says that what handles the problem of individuation is a kind. But then what individuates the kinds, distinguishing one from another, are their members. Kinds and their members mutually determine each other, just as sentences determine the grammatical category and interpretation of their constituent words, while sentences themselves are determined by their constituent words in particular syntactic categories with particular senses.

Following this line of thought, we discover that the question Duns Scotus asked, which he inherited from antiquity, was ill posed. Duns Scotus

asked how, starting from a universal, one can have an individual that falls under the universal. This is to take universals (best conceptualized as descriptions of essential structure) as basic, and individuals as derivative. It is a mistake to take universals as primitive in this sense for a reason that can be presented by means of a familiar example. To express the universal for dogs, one would have to appeal to another kind: dogs are *animals* of a particular sort, that is, animals that satisfy certain conditions. This in turn leads to the notion that animals are *living creatures* of a certain sort, that is, that satisfy certain conditions. And so on up the scale until one runs out of kinds altogether and individuation and identity are not provided for or, what is virtually the same, one appeals to a universal kind, say THING. Now we have no grasp of the supposed kind THING. If asked to count the things in a room, should I count as one thing a book, or each word in the book on the grounds that a word is a thing, or the letters, or the corners of the pages, etc.? Since there is no answer, it follows that the word "thing" in the supposed sense fails to individuate. The upshot is that one has run out of manageable kinds, and this at the highest and most comprehensive level. It follows from the Fundamental Postulate that universals cannot be constitutive of kinds in the way in which Duns Scotus thought. Instead, they are logically secondary to kinds: they are descriptions of the individuals in a kind, descriptions that presuppose the individuals they purport to describe.

But what is the relation between all the kinds that may be associated with a particular person: GIRL, WOMAN, WIFE, MOTHER, TEACHER, FRIEND, and so on? If there are guests coming to supper, there may be among them members of all these kinds. The number of places to be laid at table, however, corresponds to the number of persons: one place for one person. This means that for certain purposes we identify a particular wife, woman, mother, teacher, and friend with a particular person. For other purposes we may need to distinguish [them in different ways]. For example, only teachers may be allowed to teach and use the teachers' room; it is not enough to be a person. These operations of identifying and distinguishing are basic operations of the human mind.

The psychological literature almost universally treats the relevant relations among kinds as set-theoretic inclusions. It says, for example, that GIRL and WOMAN are both included in PERSON. This has to be wrong.

Take *g* in GIRL, and let *w* in WOMAN be the woman she grows up to be. If the relations in question really were inclusions, *g* would have to be equal to a certain *p* in PERSON, and *w* would have to be equal to the same *p*. That is, $g = p$ and $w = p$, which inescapably yields that $g = w$. But this cannot be, since a girl is not a woman, nor a woman a girl. The problem disappears if we replace inclusions with maps: *u:* GIRL → PERSON and *u′*: WOMAN → PERSON. These maps associate a person with each girl and a person with each woman. Naturally, we require that any situation of which *g* is a constituent should also have $u(g)$ as a constituent. In other words, the domain of existence of the underlying person should be at least as extensive as that of the associated girl. There is no problem in having $u(g) = u(w)$, because it does not imply that $g = w$.

The same move enables us to handle the somewhat different relation between kinds like PASSENGER and PERSON. If a person travels three times with Air Canada, for purposes of the annual report the company will count three passengers in association with the one person. Each of these passengers is a person, but while there are three passengers, there is only one person. The relation between the two kinds is therefore not an inclusion. With maps in place of inclusions, there is no problem, because several passengers can be mapped onto a single person. We can make a related point dramatically by calling the person Leslie. Each passenger is Leslie [i.e., each passenger has the property of being Leslie], yet while there are three passengers, there is only one Leslie. It follows that we need to distinguish in cognition between "Leslie" in subject position in a sentence and "to be Leslie" in predicate position. The transformations of subjects into predicates and predicates into subjects are fundamental operations of the human mind that Aristotle was fully aware of. They become apparent when, in the manner of Aristotle, one is attentive to intuitions about the rules for interpreting terms in varying grammatical positions. (Those interested in reading more about these ideas will find a nontechnical account in La Palme Reyes, Macnamara, and Reyes, 1994.)

There are other transformations that are equally fundamental. Take the kind CHICKEN. A chicken is an atomic object in the sense that cutting it in two does not yield two chickens, whereas dividing a quantity of water yields two quantities of water. One transformation of CHICKEN yields the plural CHICKENS, where the objects, being groups of chickens,

are no longer atoms. A group of chickens can usually be divided into two groups of chickens. This transformation is performed by an operator similar to the power-set operator of Sets, with the difference that it must make provision for domains of existence. Another transformation yields the portion of meat associated with a chicken; yet another yields the portion of food, which may be larger than the portion of meat, since besides the meat there is the chicken content in the soup that can be obtained; and yet another transformation yields the portion of biological matter, which is larger than the food, since it includes the beak and feathers. This time the transformation has the added complication that the size of the portions of meat, food, and matter depend on when the chicken is killed. Chickens usually put on weight as they grow older. These facts must be handled appropriately in the theory, but I will not go into detail beyond saying that it seems unlikely that they can be handled without recourse to category theory.

Everywhere we find an interdependence of the physical objects of cognition on the abstract, and that gives us a clue as to how to handle one of our problems, Plato's problem of truth. Plato wondered how perceptual contact with a fleeting and changeable world should give rise to permanent and unchanging truths. The main part of the answer seems to be that when perceptions are interpreted, the immutable abstract objects enter to freeze things.

We were less interested in the metaphysical question of what makes truth permanent than in the psychological one of how we come to appreciate the permanence of truths that we grasp in connection with impermanent presentations. It seems that all our thinking presupposes that time does not change facts. We are as alive to the unchanging as to the changing. This, in turn, indicates that we somehow appreciate that we interpret the changing against an unchanging background. The unchanging background is supplied by the abstract objects.

The problem of truth is intimately connected with that of learning. Children who are learning the name "Freddie" for a particular dog must specify a kind, the kind DOG, to individuate the bearer of the name "Freddie" and handle his identity. When they learn the word "dog," they must specify the same kind DOG, but now as the reference of "dog." In

addition, children must also specify the kind WORD to individuate the words ''Freddie'' and ''dog'' as linguistic entities and handle their identity, noting that the one is a proper name, the other a count noun. Otherwise, they would be unable to use them with appropriate syntax. This is not to say that they must know the relevant English words (''word,'' ''proper name,'' ''count noun''), but they should have available to them some symbols that can perform the same functions. Bringing together language and cognition in this way draws attention to the relation between the two. Words are combined into structures that we call sentences, and sentences describe structured relations among structured objects. The mutually inverse operations of interpreting sentences and encoding information about the world must respect the structure of the objects on which they operate. An operator that can do this is precisely what in category theory is known as a functor. Once again, we see the need for category theory if our understanding of cognition is to advance.

Since abstract objects play a crucial role in learning and since they are not perceptible, we see once again that learners must go beyond the purely perceptual. This in turn indicates that learning is not controlled by the environment in anything like the manner supposed either by the British empiricists or by the behaviorists.

What is the explanation of this reaching beyond perception and of the logical resources that are involved in doing so? The reader who has followed attentively this far will have concluded that the explanation has to include unlearned logical resources. The argument for them is perfectly straightforward. Allow with Brentano that all cognitive states and events involve reference, and allow that reference can occur only in an appropriate context. A proper name, for example, refers only in collaboration with a count noun to specify the kind to which the bearer belongs, and proper name and collaborating count noun must be in subject position in a sentence [e.g., ''The dog Freddie died in 1991'']. Do not be misled here: a word that features many times as a proper name (e.g., ''Nixon'' in ''Nixon was President of the United States'') may also feature as a common noun (e.g., ''Don't try to do a Nixon on me'') or as an adjective (e.g., ''That's a Nixon idea if I ever heard one''). What determines things is the sentence in which the word occurs. Now consider the first word a child learns, and consider the sentence that constitutes its context. The other constituents in the sentence cannot have been learned, since by hy-

pothesis this is the first word learned. It follows that the remaining constituents cannot have been learned and therefore that there are unlearned logical resources.

The argument is readily extended to other representational systems. If in the system there are learned logical resources, consider the first one to have been learned. From here the argument we have just seen can be repeated mutatis mutandis.

The problem of learning, to which Plato points, can only be handled by compensating for the logical indeterminacy of learned responses with psychological preparedness. That is, learners must come to their task well prepared for an appropriate response. Such preparedness may well be guided in part by prior experience and training, but ultimately one must depend on unlearned operations, hypotheses, constraints, especially at the initial stages of learning.

A good theory of learning specifies the unlearned resources that are necessary for a particular domain. We have seen that there have to be some. When one has specified all that is necessary for learning a particular item, one can choose the unlearned elements by means of two criteria. (a) The item to be learned is not associated with a distinctive perceptual characteristic. For example, there is something perceptually distinctive about dogs—difficult though it may be to specify what that is—so that one can learn that there is the kind DOG. There being nothing perceptually distinctive about proper names, one cannot learn the existence of the kind PROPER NAME on the basis of perceptually distinctive features. (b) The item to be learned is primitive in our conceptual lives, or one has reason to believe that it is. This means that the item cannot be defined, and so the second avenue to learning is closed. The two criteria, jointly applied, indicate those elements that are unlearned. For example, if you agree that morally good actions are not perceptually distinct from actions that are not morally good and that moral goodness cannot be defined, you will probably agree that there is important unlearned content in the concept of moral goodness. This is not to deny, of course, that there are also environmentally inspired elements in this concept (see Macnamara, 1991).

What about access to ideals, which I specified as Augustine's problem? Since an ideal is a limit that some series tends to but never in our experi-

ence reaches, it follows that a grasp of ideals cannot be attributed to experience. This means that the ability to posit ideals must be a capacity that the human mind brings to experience rather than one that it derives from experience. The ideals in question include ideals of measurement, of the straightness of lines and the smoothness of surfaces, of personal charm and personal beauty, of justice, of knowledge, of goodness of action, and so forth. Augustine himself attributed ability to grasp such ideals to divine illumination operating within the mind. Aquinas took it to be part of natural endowment, and of course he must be right, but it is not easy to situate the ability in a general theory of cognition.

We saw that Kant attempted to make a radical division of labor, assigning ideals to some discipline other than psychology and assigning to psychology the "facts" about the operation of the human mind. For example, he assigned to logicians/philosophers the ideals of errorless interpretation and reasoning; to psychologists he assigned the "facts" of how people interpret and reason. We saw, however, that the distinction is unsustainable. Either it is a fact that we have access to logical ideals or it is not. If it is a fact, the ideals are also the concern of psychologists; if it is not a fact, we should forget all about them, for they are an illusion. On the assumption that we do have access to ideals, it follows that psychologists should be on the track of a single theory to handle both logical ideals and sometimes logically erroneous performance.

The division is responsible for much mischief in psychology. Its detrimental effects are to be seen not only in the psychology of reasoning but also in the psychology of decision theory and the psychology of mental health, which, as we noted in the chapter on Freud, usually ignores ethical ideals.

It seems to me that it was Hobbes's decision to do psychology in the model of something that is not psychology, namely kinematics, that started the trend of neglecting the mind's access to ideals. Hobbes was followed in this by those who modeled psychology on mechanics, chemistry, thermodynamics, telephone exchanges, biology, and computers. The result is the discounting of an essential property or set of properties of the human mind, with a consequent distortion of psychological theory. An influential voice raised against this whole way of doing things is that of Noam Chomsky, who in the area of linguistics insists that linguists and

psychologists alike should combine in the effort to account for linguistic competence, that is, errorless ideal knowledge of grammar. In this work I follow his lead. Indeed, he advocated that his basic ideas should be applied in areas other than psycholinguistics. Evidence of an attempt to deal with the relevant ideals is a test of the seriousness of any psychological theory proposed for our consideration.

Colin McLarty's definitions of category and functor will be helpful (McLarty, 1992):

Category [A category has *objects A, B, C,* . . . and *arrows f, g, h,* . . . . Each arrow goes from an object to an object. To say that *g* goes from *A* to *B,* we write *g: A* $\rightarrow$ *B,* or say that *A* is the *domain* of *g,* and *B* the *codomain.* We may write Dom(*g*) = *A* and Cod(*g*) = *B.* Two arrows *f* and *g* with Dom(*f*) = Cod(*g*) are called *composable.* If *f* and *g* are composable, then they must have a *composite,* an arrow called $f \circ g$. Every object *A* has an *identity arrow,* 1_A. The axions read as follows. For every composable pair *f* and *g,* the composite $f \circ g$ goes from the domain of *g* to the codomain of *f.* For each object *A,* the identify arrow 1_A goes from *A* to *A.* Composing any arrow with an identity arrow (if the two are composable) gives the original arrow. And composition is associative.]

Functor [A *functor* **F** from a category **A** to a category **B,** written **F:** **A** $\rightarrow$ **B,** assigns to each object *A* of **A** an object **F** *A* of **B,** and to each arrow *f* of **A** an arrow **F***f* of **B,** meeting the following conditions. (1) It preserves domains and codomains: given *f*: *A* $\rightarrow$ *B* of **A,** we have **F***f*:**F***A* $\rightarrow$ **F***B*. (2) It preserves identities: for any *A* of **A,** $\mathbf{F}(1_A) = 1_{\mathbf{F}A}$. (3) It preserves composition: if *f* and *g* are composable in **A,** then $\mathbf{F}(g \circ f) = \mathbf{F}g \circ \mathbf{F}f$, where the second composite is formed in **B.**]

Postscript

Is there, then, no reference in perception? In this connection, recall Berkeley's idea that visual perception should be conceptualized as a language. If we go along with this, as I am tempted to, the question of reference becomes pressing. But why go along with it? While this is not the place for a full treatment of the idea, I will indicate one line of motivation, connected with intentionality. We saw that perceptual experiences can

be as intentional as cognitive ones. We have as little chance of discerning the forms of perceptual representations by introspection as of discerning those of cognitive representations. The reason is that in perception what are presented are perceptual objects and their perceptual properties; just as in cognition what are presented are objects and their properties. Brentano explains the intentionality of cognitive states and events by appealing to reference, reference being the property that establishes the "aboutness" of such states and events. Since this is the only explanation of intentionality that seems remotely plausible, it is tempting to employ it also in the explanation of perceptual intentionality. This is to conceptualize visual representations as sentences in a language of vision, but now as interpreted sentences, sentences whose terms are interpreted into perceptual objects and their perceptual properties and relations. For the record, this seems to have been Brentano's own move, for he came to treat both perception and cognition as mental, as involving reference to something as an object. This does not mean, however, running the two together in the manner of the radical empiricists, because, as we have seen, perceptual objects and their perceptual properties are not the objects and properties of cognition. Yet it does mean that the Fundamental Postulate applies also in the domain of perception, and that abstract objects (different ones) play a role in perception similar to the role they play in cognition. That will have to suffice for the purposes of this chapter.

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Notes

Chapter 3

1. [“Contradiction” is here used in the Hegelian sense, not in the logical sense, of the word. To understand the difference, consider arithmetic. When we say that there is no contradiction in arithmetic, we mean that it cannot be that both a sentence and its contradiction are theorems. This is the logical sense. On the other hand, according to Lawvere, the main contradiction of arithmetic is the law of distributivity, $a(b + c) = ab + ac$, which transforms a product into a sum and is the motor of further and deeper developments, for instance the introduction and development of categories of spaces. This is the Hegelian sense.]

Chapter 6

1. [Chapters 6 and 7 are not exegesis in the strict sense. The Bible has been read in the Christian tradition, and this is what Macnamara does. He is studying Genesis and John’s Gospel in the light of the whole tradition. Technically, this is what scholars call *Wirkungsgeschichte.* The ideas about the image of God in Genesis have had a long history. Augustine developed them. For him, the principal image of the Trinity was the human mind: memory, intellect, and will. He also developed, however, a social analogy: lover, beloved, and love (book VIII of *De Trinitate* in Augustine, 1872). The social image was especially important for Richard of St. Victor in his *De Trinitate,* book III (1979). One can read about this in William Hill, *The Three-Personed God* (1982). I wrote an article entitled “The Trinity as divine community” in 1988.

The part about naming the animals, below, is stretching it a bit. The main point exegetically seems to be that God gives Adam stewardship over the creation. Still, the type of reflection found in the present book has some basis if one takes Genesis in the light of the whole tradition. It does seem that the Bible’s belief in the creator grounds faith in the orderedness of the world.

One of the critical remarks in reviews of the manuscript was that Macnamara was trying to harmonize his faith in the Bible with his beliefs about human nature.

This seems to me a valid point. It is also quite classical: faith seeking understanding. So in summary, chapters 6 and 7 present for the most part a valid point of view, but they are reading the Bible spiritually, *lectio divina,* in the light of the whole tradition, not just as a scientific exegetical study.—John M. O'Donnell, S.J.]

Chapter 10

1. [We think that what Macnamara wanted to say was that a theory can describe (i.e., define) its own syntax if it contains arithmetic (technically, primitive recursive arithmetic suffices). Of course, the syntax is defined in the language of the theory, whence his rather clumsy statement. Furthermore, the author somewhat obscures the issue by bringing in semantics and talking of "a system that the language describes," when Godel's theorem is purely "syntactical." Another factual error of the author is to confound, in one statement, two theorems. The negative statement about the nondefinability of semantical notions (in particular, "truth") is a famous theorem of Alfred Tarski and should be credited to him. In the light of this clarification, the second statement of Macnamara's could be formulated as follows: "The semantics of the theory of the brain cannot be defined in the language of that theory." This formulation requires that the theory of the brain, whatever that is, should be axiomatizable.]