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A MACRODYNAMIC THEORY OF BUSINESS CYCLES[1]

BY M. KALECKI

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I

IN the following all our considerations concern an economic system *isolated and free of secular trend.* Moreover, we make with respect to that system the following assumptions.

1. We call real gross profit B the total real income of capitalists (business men and private capitalists), amortization included, per unit of time. That income consists of two parts, that consumed and that accumulated:

$$B = C + A. \tag{1}$$

Thus, C is the total volume of consumers' goods consumed by capitalists, while A—*if we disregard savings of workpeople, and their "capitalistic" incomes*—covers goods of all kind serving the purpose of reproduction and expansion of fixed capital, as well as increment of stocks. We shall call A "gross accumulation."

The personal consumption of capitalists, C, is not very elastic. *We assume that C is composed of a constant part, C_1, and a variable part proportionate to the real gross profit λB:*

$$C = C_1 + \lambda B \tag{2}$$

where λ is a small constant fraction.

From equations (1) and (2) we get:

$$B = C_1 + \lambda B + A$$

and

$$B = \frac{C_1 + A}{1 - \lambda}, \tag{3}$$

i. e., the real gross profit B is proportionate to the sum C_1+A of the constant part of the consumption of capitalists C_1 and of the gross accumulation A.

[1] The term "macrodynamic" was first applied by Professor Frisch in his work "Propagation problems and impulse problems in dynamics" (*Economic Essays in Honour of Gustav Cassel*, London, 1933), to determine processes connected with the functioning of the economic system as a whole, disregarding the details of disproportionate development of special parts of that system.

The gross accumulation A is equal to the sum of the production of capital goods and of the increment of stocks of all kinds.[2] *We assume that the total volume of stocks remains constant all through the cycle.* This is justified in so far as in existing economic systems totally or approximately isolated (the world, U.S.A.) the total volume of stocks does not show any distinct cyclical variations. Indeed, while business is falling off, stocks of finished goods decrease, but those of raw materials and semi-manufactures rise; during recovery there is a reversal of tendencies. From the above we may conclude that *in our economic system the gross accumulation A is equal to the production of capital goods.*

2. We assume further that *the "gestation period" of any investment is θ.* Of course, this by no means corresponds to the reality; θ is merely the average of various actual durations of "gestation periods," and our system in which θ is a constant value is to be considered as a simplified model of reality.

Whenever an investment is made, three stages can be discerned: (1) investment orders, i.e., all the orders for capital goods to serve the purpose of reproduction or expansion of industrial equipment; the total volume of such orders allocated per unit of time will be called I; (2) production of capital goods; the volume of that production per unit of time, equal, as said above, to the gross accumulation, is called A; (3) deliveries of finished industrial equipment; the volume of such deliveries per unit of time will be called L.[3]

The relation of L and I is simple. Deliveries L at the time t are equal to investment orders I at the time $t-\theta$:

$$L(t) = I(t - \theta). \tag{4}$$

($I(t)$ and $L(t)$ are investment orders and deliveries of industrial equipment at the time t.)

The interrelationship of A and I is more complicated.

Let us call W the total volume of unfilled investment orders at the moment t. As each investment needs the time θ to be filled, $1/\theta$ of its volume must be executed in a unit of time. Thus, the production of capital goods must be equal to $1/\theta \cdot W$:

$$A = \frac{W}{\theta}. \tag{5}$$

[2] Industrial equipment in course of construction is not included in "stocks of all kinds"; thus, change in the volume of the industrial equipment in course of construction is involved in the "production of capital goods."

[3] While A is the production of *all* capital goods, L is only that of *finished* capital goods. Thus, the difference $A-L$ represents the volume of industrial equipment in course of construction, per unit of time.

As regards W, it is equal to the total of orders allocated during the period $(t-\theta, t)$. Indeed, since the "gestation period" of any investment is θ, no order allocated during the period $(t-\theta, t)$ is yet finished at the time t, while all the orders allocated before that period are filled. We thus obtain the equation:

$$W(t) = \int_{t-\theta}^{t} I(\tau)d\tau. \tag{6}$$

According to equations (4) and (5) we get:

$$A(t) = \frac{1}{\theta}\int_{t-\theta}^{t} I(\tau)d\tau. \tag{7}$$

($A(t)$ is the production of capital goods at the time t.)

Thus A at the time t is equal to the average of investment orders $I(t)$ allocated during the period $(t-\theta, t)$.

3. Let us call K the volume of the existing industrial equipment. The increment of that volume within the given period is equal to the difference between the volume of deliveries of finished equipment and that of equipment coming out of use. If we denote by $K'(t)$ the derivative of K with respect to time, by $L(t)$ the volume of deliveries of industrial equipment per unit of time (as above), and by U the demand for restoration of equipment used up per unit of time, we get:

$$K'(t) = L(t) - U. \tag{8}$$

We can assume that the demand for restoration of the industrial equipment—U—remains constant all through the cycle. The volume of the existing industrial equipment K shows, it is true, certain fluctuations,

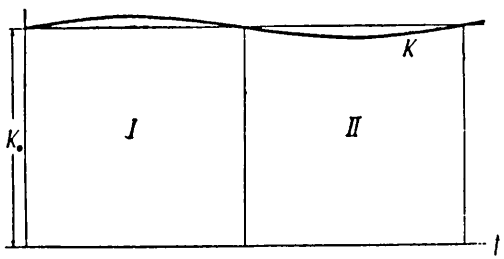

FIGURE 1

e.g., in the first part of the cycle K is above the average, and one might think that then the demand for restoration of equipment ought to be above the average too. Yet, it should be borne in mind that the new equipment is "young" and that its "rate of mortality" is very low, as the average "lifetime" of industrial equipment is much longer than the duration of a cycle (15–30 years as against 8–12 years). Thus, the fluctuations of the demand for restoration of equipment are of no importance, and may safely be disregarded.

4. The proportions of the investment activity at any time depend on the expected net yield. When the business man will invest a capital k in the construction of industrial equipment, he will first evaluate the probable gross profit b, while deducting (1) the amortization of the capital k, i.e., βk (β—the rate of amortization); (2) the interest on the capital k, i.e., pk (p—the interest rate); (3) the interest on the future working capital, the ratio of which to the invested capital k will be denoted by $\gamma - p\gamma k$. The probable yield of the investment will thus be:

$$\frac{b - \beta k - pk - p\gamma k}{k} = \frac{b}{k} - \beta - p(1 + \gamma).$$

The coefficients β and γ may be considered constant all through the cycle. p is the money rate at the given moment, b/k is the probable future yield evaluated after that of the existing enterprises. The volume of the existing industrial equipment is K, the total real gross profit is B. Thus, the average real gross profit per unit of the existing fixed capital is B/K (that quotient will be called further gross yield B/K).

We may conclude that $\frac{b}{k}$ is evaluated after B/K, and that investment activity is controlled by the gross yield B/K and the money rate p. As a matter of fact, the function of B/K and p is not the very volume of investment orders I, but the ratio of that volume to that of industrial equipment K, i.e., I/K. In fact, when B and K rise in the same proportion, B/K remains unchanged, while I rises (probably) as did B and K. Thus, we arrive at the equation:

$$\frac{I}{K} = f\left(\frac{B}{K}, p\right) \tag{9}$$

where f is an increasing function of B/K and a decreasing function of p.

It is commonly known that, except for *financial panic* (the so-called crises of confidence), the market money rate rises and falls according to general business conditions. We make on that basis the following simplified assumption: *The money rate p is an increasing function of the gross yield B/K.*

From the assumption concerning the dependence of the money rate p on the gross yield B/K, and from (8), it follows *that I/K is a function of B/K.* As B is proportionate to C_1+A, where C_1, is the constant part of the consumption of capitalists, and A the gross accumulation equal to the production of capital goods, we thus obtain:

(10) $$\frac{I}{K} = \phi\left(\frac{C_1 + A}{K}\right)$$

ϕ being, of course, an increasing function. We further assume *that ϕ is a linear function,* i.e., that:

$$\frac{I}{K} = m\frac{C_1 + A}{K} - n$$

where the *constant m is positive,* ϕ being an increasing function. Multiplying both sides of the equation by K we get:

(11) $$I = m(C_1 + A) - nK.$$

* * *

We have seen that between I (investment orders), A (gross accumulation equal to the production of capital goods), L (deliveries of industrial equipment), K (volume of the existing industrial equipment), and the time t, there are interrelationships:

(4) $$L(t) = I(t - \theta)$$

(7) $$A(t) = \frac{1}{\theta}\int_{t-\theta}^{t} I(\tau)d\tau$$

(8) $$K'(t) = L(t) - U$$

resulting from technics of the capitalistic production, and the relation:

(11) $$I = m(C_1 + A) - nK$$

resulting from the interdependence between investments and yield of existing enterprises. From these equations the relation of I and t may be easily determined.

Let us differentiate (11) with respect to t:

(12) $$I'(t) = mA'(t) - nK'(t).$$

Differentiating the equation (7) with respect to t, we get:

(13) $$A'(t) = \frac{I(t) - I(t - \theta)}{\theta}$$

and from (4) and (8):

(14) $$K'(t) = I(t - \theta) - U.$$

Putting into (12) values of $A'(t)$ and $K'(t)$ from (13) and (14), we have:

$$(15) \qquad I'(t) = \frac{m}{\theta}\left[I(t) - I(t-\theta)\right] - n\left[I(t-\theta) - U\right].$$

Denoting the deviation of $I(t)$ from the constant demand for restoration of the industrial equipment U by $J(t)$:

$$(16) \qquad J(t) = I(t) - U,$$

we can transform (15) as follows:

$$J'(t) = \frac{m}{\theta}\left[J(t) - J(t-\theta)\right] - nJ(t-\theta)$$

or

$$(17) \qquad (m + \theta n)J(t-\theta) = mJ(t) - \theta J'(t).$$

The solution of that equation will enable us to express $J(t)$ as a function of t and to find out which, if any, are the endogenous cyclical fluctuations in our economic system.

II

It may be easily seen that the equation (17) is satisfied by the function $De^{\alpha t}$ where D is an arbitrary constant value and α a definite value which has to be determined. Replacing $J(t)$ by $De^{\alpha t}$, we get:

$$D(m + \theta n)e^{\alpha(t-\theta)} = Dme^{\alpha t} - D\alpha\theta e^{\alpha t}$$

and, dividing by $De^{\alpha t}$, we obtain an equation from which α can be determined:

$$(18) \qquad (m + \theta n)e^{-\alpha\theta} = m - \alpha\theta.$$

By simple transformations we get further:

$$e^{-m}(m + \theta n)e^{m-\alpha\theta} = m - \alpha\theta$$

and setting

$$(19) \qquad m - \alpha\theta = z$$

$$(20) \qquad e^{-m}(m + \theta n) = l$$

we have

$$(21) \qquad le^{z} = z$$

where z is to be considered as a complex number:

$$(22) \qquad z = x + iy.$$

Thus, (19) can be given the following form:

(23) $$\alpha = \frac{m - x}{\theta} - i\frac{y}{\theta}$$

and (21) be transformed into:

(24) $$x + iy = le^x(\cos y + i \sin y).$$

Adopting the method of Tinbergen,[4] we discern two cases: Case I—when $l > 1/e$, and Case II—when $l \leqq 1/e$.

Case I. As Tinbergen has shown, in that case all the solutions will be complex numbers, and they will be infinite in number. Let us arrange them by increasing y_k:

$$\cdots x_k - iy_k, \cdots x_2 - iy_2,\ x_1 - iy_1,\ x_1 + iy_1,\ x_2 + iy_2 \cdots x_k + iy_k \cdots.$$

(It is easy to see that when $x_k + iy_k$ is a root of (24), that equation is satisfied as well by $x_k - iy_k$).

From the equation (23) we get values of α:

$$\alpha_k = \frac{m - x_k}{\theta} - i\frac{y_k}{\theta}$$

and

$$\alpha_{-k} = \frac{m - x_k}{\theta} + i\frac{y_k}{\theta}.$$

Functions:

$$D_k e^{\alpha_k t} = D_k e^{(m - x_k)t/\theta}\left(\cos y_k \frac{t}{\theta} - i \sin y_k \frac{t}{\theta}\right)$$

and

$$D_{-k} e^{\alpha_{-k} t} = D_{-k} e^{(m - x_{-k})t/\theta}\left(\cos y_k \frac{t}{\theta} + i \sin y_k \frac{t}{\theta}\right)$$

satisfy (17).

The general solution of (17), which is at the same time a differential and a functional equation, depends upon the form of the function $J(t)$ in the initial interval $(0, \theta)$; that form is quite arbitrary. Yet, we can develop (with sufficient approximation) the function $J(t)$ in the initial interval into the series $\sum D_k e^{\alpha_k t}$ where the constants D_k depend upon the form of the function $J(t)$ in the initial interval.[5] As functions $D_k e^{\alpha_k t}$ satisfy (17), the function $\sum D_k e^{\alpha_k t}$, which represents with sufficient approximation $J(t)$ in the initial interval, will be a general solu-

[4] "Ein Schiffbauzyklus?" *Weltwirtschaftliches Archiv*, B. 34, H.1.

[5] *Loc. cit.*, p. 158.

tion of (17).[6] That solution is, of course, a real one, thus D_k and D_{-k} must be complex conjugate numbers, and $J(t)$ can be represented as follows:

$$
(25)\qquad \begin{aligned} J(t) &= e^{(m-x_1)t/\theta}\left(F_1 \sin y_1 \frac{t}{\theta} + G_1 \cos y_1 \frac{t}{\theta}\right) \\ &\quad + e^{(m-x_2)t/\theta}\left(F_2 \sin y_2 \frac{t}{\theta} + G_2 \cos y_2 \frac{t}{\theta}\right) \cdot \cdot \end{aligned}
$$

On the basis of that solution we cannot yet say anything definite about the character of fluctuations of $J(t)$, as the constants F_k and G_k depend upon the form—unknown to us—of the function $J(t)$ in the initial interval. But here we can take advantage of the following circumstance. It may be inferred from Tinbergen's argument when he solves the equation (24) that

$$
(26)\qquad x_1 < x_2,\ x_1 < x_3 \cdot \cdot \cdot .
$$

Let us divide $J(t)$ by:

$$
e^{(m-x_1)t/\theta}\left(F_1 \sin y_1 \frac{t}{\theta} + G_1 \cos y_1 \frac{t}{\theta}\right).
$$

According to the inequality (26), for a sufficiently great t the sum of all the expressions other than the first one will be equal to an arbitrarily small value ω:

$$
\frac{J(t)}{e^{(m-x_1)t/\theta}\left(F_1 \sin y_1 \frac{t}{\theta} + G_1 \cos y_1 \frac{t}{\theta}\right)} = 1 + \omega.
$$

At a time sufficiently distant from the initial interval, the following equation will be true with an arbitrarily small relative error:

$$
(27)\qquad J(t) = e^{(m-x_1)t/\theta}\left(F_1 \sin y_1 \frac{t}{\theta} + G_1 \cos y_1 \frac{t}{\theta}\right).
$$

That equation represents harmonic vibrations with an amplitude decreasing, constant, or increasing, according as $x_1 \gtreqless m$. Their period, and the degree of progression or degression they show, do not depend on the form of the function $J(t)$ in the initial interval. (It is worth mentioning that, as follows from Tinbergen's analysis, vibrations represented by (27) have a period longer than 2θ, while vibrations represented by the expressions on the right side of the equation [25] which we dropped, have a period shorter than θ).

[6] *Loc. cit.*, p. 157.

If now we fix the origin of the time axis so as to equate $J(t)$ from (27) to zero for $t=0$, that equation will assume the form:

$$J(t) = F_1 e^{(m-x_1)t/\theta} \sin y_1 \frac{t}{\theta}$$

or, taking into consideration (16):

$$(28) \qquad I(t) - U = F_1 e^{(m-x_1)t/\theta} \sin y_1 \frac{t}{\theta}.$$

Case II. In that case (24) has two real roots z_1' and z_1'', among complex roots like $x_1 \pm iy$. As in the first case, we get here, for a time sufficiently distant from the initial interval:

$$J(t) = D_1' e^{(m-z_1')t/\theta} + D_1'' e^{(m-z_1'')t/\theta}.$$

It follows from that equation that there are no cyclical vibrations.

The results of the above analysis can be summarized as follows: *Cyclical variations occur in our economic system only when the following inequality is satisfied:*

$$l > \frac{1}{e},$$

transformed, by putting the value of l from (20) *into*:

$$(29) \qquad m + \theta n > e^{m-1}.$$

As we know, m is positive (*see p.* 331). *We can easily prove that a necessary, though insufficient, condition, at which* (29) *is satisfied, i.e., there are cyclical variations, is that n be positive too.*

Fluctuations of I at a time sufficiently distant from the initial interval (0, θ) *will be represented by the equation*:

$$(28) \qquad I(t) - U = F_1 e^{(m-x_1)t/\theta} \sin y_1 \frac{t}{\theta}.$$

The amplitude of fluctuations is decreasing, remains constant, or rises, according as $x_1 \gtreqless m$.

The period is equal to

$$(30) \qquad T = \frac{2\pi}{y_1}\theta.$$

On the basis of equations

$$(7) \qquad A(t) = \frac{1}{\theta}\int_{t-\theta}^{t} I(\tau)d\tau$$

and

$$(4) \qquad L(t) = I(t - \theta)$$

we can show L and A as functions of t, and see that these values are fluctuating, like I, around the value U. K is obtained by integration of:

$$(8) \qquad K'(t) = L(t) - U.$$

It also fluctuates around a certain constant value, which we denote by K_0. The whole calculation will be given in the next chapter with respect to a particular case when the amplitude of fluctuations is constant.

III

If, while $x_1 = m$, the amplitude of fluctuations remains constant, (28) assumes the form:

$$(31) \qquad I(t) - U = a \sin y_1 \frac{t}{\theta}$$

where a is the constant amplitude.

That case is of a particular importance as it appears to be nearest to actual conditions. Indeed, in reality we do not observe any *regular* progression or degression in the intensity of cyclical fluctuations.

Putting the value of I from (31) into (7) and (4) we get

$$(32) \qquad \begin{aligned} A - U &= \frac{1}{\theta}\int_{t-\theta}^{t}\left(a \sin y_1 \frac{\tau}{\theta} + U\right)d\tau - U \\ &= a \frac{\sin \frac{y_1}{2}}{\frac{y_1}{2}} \sin y_1 \frac{t - \frac{\theta}{2}}{\theta} \end{aligned}$$

and .

$$(33) \qquad L - U = a \sin y_1 \frac{t - \theta}{\theta}.$$

From (8) and (33)

$$K'(t) = a \sin y_1 \frac{t - \theta}{\theta}.$$

Integrating:

(34) $$K - K_0 = - a \frac{\theta}{y_1} \cos y_1 \frac{t - \theta}{\theta}$$

where K_0 is the constant around of which K is fluctuating, equal here to the average volume of the industrial equipment K during a cycle.

In a similar way, the average values of I, A, and L, during a cycle will be equal in our case of constant amplitude to the constant U around which I, A, and L, are fluctuating.

Taking into consideration the condition of a constant amplitude $x_1 = m$ we shall get now from (20) and (24):

(35) $$\cos y_1 = \frac{m}{m + \theta n}$$

and

(36) $$\frac{y_1}{tg y_1} = m.$$

These equations allow us to determine y_1; moreover, they define the interrelationship of m and n.

Between m and n there is still another dependency. They are both coefficients in the equation:

(11) $$I = m(C_1 + A) - nK$$

which must be true for one-cycle-averages of I and A equal to U, and for the average value of K equal to K_0:

$$U = m(C_1 + U) - nK_0.$$

Hence:

(37) $$n = (m - 1) \frac{U}{K_0} + m \frac{C_1}{K_0}.$$

Thus, if values of U/K_0 and C_1/K_0 were given, we could determine m and n from (35), (36), and (37). U/K_0 is nothing else but the rate of amortization, as U is equal to the demand for restoration of equipment, and K_0 is the average volume of that equipment. C_1 is the constant part of the consumption of capitalists. U/K_0 and C_1/K_0 may be roughly evaluated on the basis of statistical data. If we also knew the average gestation period of investments θ, we could determine y_1 and the duration of the cycle $T = 2\pi\theta/y$.

We evaluate the *gestation period of investments* θ on the basis of data of the German *Institut fuer Konjunkturforschung.* The lag between the curves of beginning and termination of building schemes (dwelling houses, industrial and public buildings) can be fixed at 8 months; the lag between orders and deliveries in the machinery-making industry can be fixed at 6 months. We assume that the average duration of θ is 0.6 years.

The rate of amortization U/K_0 is evaluated on the basis of combined German and American data. On that of the German data, the ratio of amortization to the national income can be fixed at 0.08. With a certain approximation, the same is true for U.S.A. Further, according to official estimates of the wealth of U.S.A. in 1922, we set the amount of fixed capital in U.S.A. at $120 milliards (land excepted). The national income is evaluated at $70 milliards for 5 years about 1922. The rate of amoritization would thus be $0.08 \cdot 70/120$, i.e., *ca.* 0.05.

Most difficult is the *evaluation of* C_1/K_0. K_0 was fixed at $120 milliards, C_1 is, as we know, the constant part of the consumption of capitalists. Let us evaluate first the average consumption of capitalists in U.S.A. in the period 1909–1918. The total net profit in that period averaged, according to King, $16 milliards deflated to the purchasing power of 1913. The average increment of total capital in that period is estimated by King at $5 milliards. That figure includes savings of workpeople, but, on the other hand, 16 milliards of profits cover also "capitalistic" incomes of workpeople (use of own houses, etc.). Thus, the difference, $16-5=11$ millards of 1913-dollars, represents with a sufficient degree of accuracy the consumption of capitalists (farmers included). The average national income amounted in the period 1909–1918 to $36 milliards with the purchasing power of 1913 (King). The ratio of the consumption of capitalists to the national income would thus be 0.3. As, further, the average income during 5 years around 1922 amounted, as mentioned, to $70 milliards of current purchasing power, the consumption of capitalists in these years may be estimated at $21 milliards. Now, we have to determine the constant part of that consumption. In order to do that, we assume that when the volume of capitalists' gross profits deviates from the average by, say, ± 20 per cent, the corresponding relative change in their consumption is but 5 per cent, i.e., 4 times smaller. That assumption is confirmed by statistical evidence. Accordingly, the constant part of the consumption of capitalists, equal to $C_1+\lambda B$ (see above, λ is a constant fraction, B—the total gross profit), amounts to 3/4 of $21 milliards, i.e., to $16 milliards. The ratio C_1/K_0 would then be 16/120 or *ca.* 0.13.

Equations (35), (36) and (37), if we put:

$$\theta = 0.6; \qquad \frac{U}{K_0} = 0.05; \qquad \frac{C_1}{K_0} = 0.13;$$

give:

$$\cos y_1 = \frac{m}{m + 0.6n}$$

$$\frac{y_1}{tg y_1} = m$$

$$n = 0.05(m - 1) + 0.13m.$$

The solution of these equations gives:

$$m = 0.95; \qquad n = 0.121; \qquad y_1 = 0.378.$$

Thus, the duration of the cycle is:

$$T = \frac{2\pi}{y_1}\theta = \frac{2\pi}{0.378} \cdot 0.6 = 10.0.$$

The figure of 10 years thus obtained as the time of duration of a cycle is supported by statistical evidence: 8 to 12 years.[7] It may be objected that values θ_1 U/K_0, C_1/K_0, on which our calculation was based, were but roughly estimated, and that the conformity between facts and theory can be merely a coincidence. Let us calculate T for such values of θ_1 U/K_0, C_1/K_0 as would be quite different from those previously taken:

θ	$\frac{U_0}{K_0}$	$\frac{C_1}{K_0}$	T
0.6	0.05	0.13	10.0
0.6	0.03	0.13	10.0
0.6	0.07	0.13	10.0
0.6	0.05	0.07	13.2
0.6	0.05	0.19	8.5
0.3	0.05	0.13	7.1
0.9	0.05	0.13	12.5

We see that the value of U/K_0 plays no great rôle with respect to the result of our calculation. We see further that when values of C_1/K_0 and θ differ by almost 50 per cent from those adopted before ($C_1/K_0 = 0.13$ and $U/K_0 = 0.05$) solutions for T move between 7 and 13 years. The actual duration of the cycle being, as already mentioned, 8 to 12

[7] Shorter cycles can be considered as "short-wave" fluctuations.

years, we can safely say that, irrespective of the degree of accuracy in estimating θ, U/K_0, C_1/K_0, there is no flagrant incongruity between the consequences of our theory and reality.

There is one more question to be dealt with. During the whole time we considered, as stated at the very beginning of the study, an economic system free of secular trend. But a case when the trend is uniform, and when gross accumulation, consumption of capitalists, and the volume of industrial equipment, show the same rate of development, can be easily reduced to a state "free of trend" simply by dividing all these values by the denominator of the trend. Interrelationships stated in our chapter I will remain true for these quotients, with the following changes: (1) The value U will be no longer equal to the demand for restoration of the used-up equipment, but it will cover as well the steady demand for the expansion of the existing equipment as a result of the uniform secular trend. Thus U/K_0 will be equal not to the rate of amortization 0.05, but, assuming the rate of net accumulation equal, say, to 3 per cent, to 0.08. (2) Also stocks of goods, previously considered constant, will increase in the same proportion under the influence of the trend. That steady increment of stocks per unit of time—let us call it C_2—will be a component of the gross profit B, now equal to $C+C_2+A$, where C is the personal consumption of capitalists, C_1 the steady increment of stocks, and A the production of capital goods. If we now consider that, according to equation (2), the consumption of capitalists C is equal to $C_1+\lambda B$, we see that B is proportionate to C_1+C_2+A. The constant C_1+C_2 will play in our considerations the same rôle as C_1 previously did. According to the official estimate of the national wealth of the U.S.A., the volume of stocks of goods amounts to 0.3 of the volume of the industrial equipment, i.e., to $0.3 \cdot K_0$. If the rate of net accumulation be 3 per cent, C_2 will be $0.03 \cdot 0.3 \cdot K_0$. Hence, instead of $C_1/K_0=0.13$ we must take $(C_1+C_2)/K_0=0.14$. From the above table we may easily see that both modifications—0.08 instead of 0.05 for U/K_0 and 0.14 instead of 0.13 for C_1/K_0,—will have but little effect on the result of the calculation of T.

We shall now determine, on the basis of (31), (32), (33), and (34), equations of curves I, A, L, and K, with $\theta=0.6$ and $T=10.0$:

$$I-U=a\sin 0.63t$$
$$A-U=0.98a\sin 0.63\,(t-0.3)$$
$$L-U=a\sin 0.63\,(t-0.6)$$
$$K-K_0=-1.59a\cos 0.63\,(t-0.6).$$

Assuming, in conformity with the above estimate, $U/K_0=0.05$, we find the following formulae for the relative deviations from the state of equilibrium:

$$\frac{I - U}{U} = \frac{a}{U} \sin 0.63t \tag{38}$$

$$\frac{A - U}{U} = \frac{a}{U} \cdot 0.98 \cdot \sin 0.63(t - 0.3) \tag{39}$$

$$\frac{L - U}{U} = \frac{a}{U} \sin 0.63(t - 0.6) \tag{40}$$

$$\frac{K - K_0}{K_0} = - \frac{a}{U} \cdot 0.08 \cos 0.63(t - 0.6). \tag{41}$$

IV

Figure 2 represents the curves of investment orders I, of production of capital goods A, of deliveries of industrial equipment L, and of the volume of industrial equipment K, which correspond to the formulae (38), (39), (40), and (41).

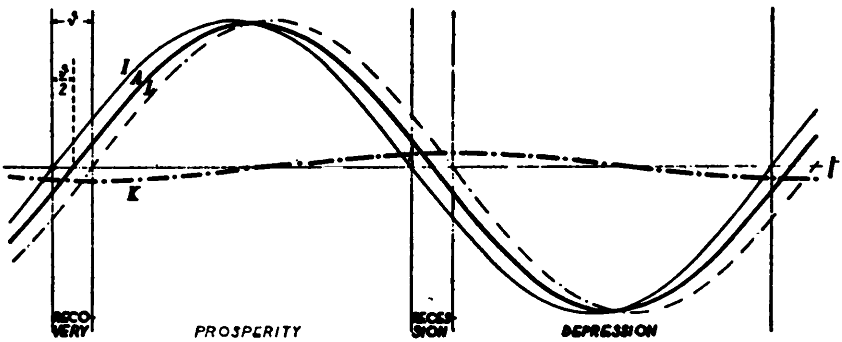

Figure 2

Let us recall the dependence (11), $I = m(C_1 + A) - nK$, wherefrom it follows, if m and n are positive (see p. 333), that the volume of investment orders is an increasing function of the gross accumulation equal to the production of capital goods, and a decreasing function of the volume of the existing industrial equipment. Having these in mind, we can, on the basis of the Figure 2, explain the mechanism of the business cycle.

Recovery is the phase of the cycle of a duration θ, when the volume of investment orders begins to exceed the volume of the demand for restoration of the industrial equipment. But the very volume of the existing industrial equipment is not yet increasing, as deliveries of new equipment still remain below the demand for restoration of equipment.

The output of capital goods A, equal to the gross accumulation, is on the increase. Meanwhile, the volume of the existing industrial equipment K is still on the decrease, and, as a result, investment orders rise at a rapid pace.

During *prosperity* also deliveries of equipment exceed the demand for restoration of the equipment, thus the volume of the existing equipment is increasing. The rise of K at first hampers the rise of investment orders, and at last causes their drop. The output of capital goods follows suit, and begins to fall off in the second part of prosperity.

During *recession* investment orders are below the level of the demand for restoration of the industrial equipment, but the volume of the existing industrial equipment K is still on the increase, since deliveries are still below the demand for restoration. As the volume of production of capital goods, equal to the gross accumulation A, continues to fall off, the volume of investment orders I is decreasing rapidly.

During *depression* deliveries of equipment are below the level of the demand for restoration of the equipment, and the volume of the existing equipment is falling off. The drop in K at first smoothes the downward tendency in investment orders, and then calls forth their rise. In the second part of depression the production of capital goods, too, begins to increase.

* * *

We have seen a plot of investment orders, gross accumulation, and existing industrial equipment. But the fluctuations in the volume of the gross accumulation, which appear as a result of the functioning of the business cycle mechanism, must necessarily affect the movement of prices and the total volume of production. Indeed, the real gross profit B is, on the one hand, an increasing function of the gross accumulation A (B being proportionate to C_1+A, where C_1 is the constant part of the consumption of capitalists, see above) and, on the other hand, it can be represented as a product of the general volume of production and of the profit per unit of production. In that way, the general volume of production and prices (or rather the ratio of prices to wages determining the profit per unit) rises in the upward part of the cycle as the gross accumulation increases.

The interdependence of gross accumulation, equal to the production of capital goods, and of the general movement of production and prices, is realized in the following way. While the output of capital goods increases by a certain amount, in the general volume of production, beside that increment, there is another increment because of the increased demand for consumers' goods on the part of workers recently hired by industries making capital goods.[8] The consequent increase in employment in industries making consumers' goods results, in its turn, in an increase in the demand for consumers' goods on the part of workers. As simultaneously there is an advance of prices, the new demand is but partly met by the new production. The remaining part of that demand is satisfied at the expense of the "old" workers, whose real earnings suffer a reduction. The general level of production and prices must eventually rise, so as to provide for an increment of the real profit equal to the increment of the production of capital goods.

That description is incomplete in so far as it does not reckon with changes in the personal consumption of capitalists. That consumption—C—is dependent, to a certain extent, on the proportions of the total profit B, and increases in accordance with the gross accumulation A (from equations (2) and (3) it follows that $C=(C_1+\lambda A)/(1+\lambda)$, where λ is a constant fraction). The increase in the consumption of capitalists has the same effect as the increase in production of capital goods: there is an increase in the volume of production of consumers' goods for the use of capitalists; as a result, employment increases, hence an additional demand for consumers' goods for the use of workers, and, eventually, a further rise of production and prices.

The general level of production and prices must rise, eventually, so as to provide for an increment of the real profit equal to the increment of the production of capital goods and of the consumption of capitalists.

* * *

The question may arise wherefrom capitalists take the means to increase at the same time the production of capital goods and their own consumption. Disregarding the technical side of the money market such as, e.g., the variable demand for means of payment, we may say that these outlays are "financing themselves." Imagine, for instance, that some capitalists withdraw during a year a certain amount from their savings deposits, or borrow that amount at the Central Bank, in order to invest it in the construction of some additional equipment. In the course of the same year that amount will be received by other capitalists under the form of profits (since, according to our assumptions, workers do not save), and put again into a bank as a savings deposit or used to pay off a debt to the Central Bank. Thus, the circle will close itself.

Yet in reality, just because of the technical side of the money market, which, as a matter of fact, forms its very nature, a credit inflation becomes necessary for two reasons.

[8] We take for granted that there is a reserve army of unemployed.

The first is the fact of the curve I of investment orders not coinciding exactly with that of production of capital goods A, equal to the gross accumulation. When giving an investment order, the entrepreneur has to provide first some corresponding fund, out of which he will currently finance the filling cf that order. At any time the corresponding bank account will be increased (per unit of time) by the amount I equal to the volume of orders allocated, and simultaneously decrease by an amount A spent on the production of capital goods.[9]

In that way, at any time the investment activities require an amount I (per unit of time), notably: $I-A$ to form new investment reserves, and A to be spent on the production of capital goods. The actually spent amount A "finances itself," i.e., comes back to the bank under the form of realized profits, while the increment of investment reserves $I-A$ is to be created by means of a credit inflation.

Another reason for the inflation of credit is the circumstance that the increase in the production of capital goods or in the consumption of capitalists, i.e., increased profits, calls forth a rise of the general level of production and prices. This has the effect of increasing the demand for means of payment under the form of cash or current accounts, and to meet that increased demand a credit inflation becomes necessary.

M. KALECKI

Polish Institute for Economic Research
Warsaw, Poland

[9] The values concerned are not exactly the real values of I and A but corresponding amounts of money, calculated at current prices.