

ABSTRACT. Dislocation motions are related to the concept of fluid viscosity. In this way it is possible to take advantage of existing viscosity theory to interpret dislocation behavior and to unify the description of dislocation mobility. The resulting viewpoint places the behavior of plastic solids between elastic solids at one extreme and fluids at the other.

## *I. A Unified View of Flow Mechanisms in Materials*

J. J. GILMAN

### *1.1 Introduction*

Deformational flow in solids is often viewed as a special process because it occurs by means of the movements of dislocations. This is not a necessary viewpoint, however. If deformation is considered as a transport process, the flow mechanisms in solids can be unified with those in gases and liquids. There is considerable advantage in this concept because it allows a more free exchange of ideas among persons who study these various material types.

The idea that flow in crystalline solids is microscopically inhomogeneous and can be described in terms of the movements of specific configurations (crystal dislocations) has been crucial in developing understanding of the process. It is equally important to realize that flow in noncrystalline solids does not, in general, result from a sequence of random local shears between molecules (or atoms). The molecular shear events tend to be correlated, and if the correlation is high the process can be described in terms of generalized dislocation lines. That is, if a shear event occurs at a certain place in a solid, the probability that another event will occur adjacent to the first is considerably greater than the probability that the second event will occur at some random position. In other words, once a region of shear

is nucleated it tends to grow, and as it grows it is surrounded by a dislocation line-loop.

On the other hand, the overall behavior of a plastic crystal is intermediate between that of a solid and that of a liquid. This is because the perfect regions of a crystal have substantial shear rigidity, but the centers of the dislocated regions do not. These central regions (cores) behave essentially like liquids, sometimes with high viscosity and sometimes with low. The viscosity is not necessarily constant along the length of a dislocation if the material is heterogeneous.

The above comments indicate that high-viscosity "liquids" have some of the flow characteristics of solids, and plastic "solids" behave in part like liquids. Therefore, certain connecting bridges exist between these differing materials, and the following paragraphs are intended to describe some of their common features

### 1.2. Dislocations in High-Viscosity Liquids (Noncrystalline Solids)

The description of flow in crystals is greatly simplified by the fact that Burgers vectors are constant and well conserved in them for given glide systems. However, the concept of a dislocation line remains useful even if the Burgers vector does not have a fixed value, as may be the case for noncrystalline solids such as glasses.

In a noncrystalline solid a dislocation line will have a somewhat variable

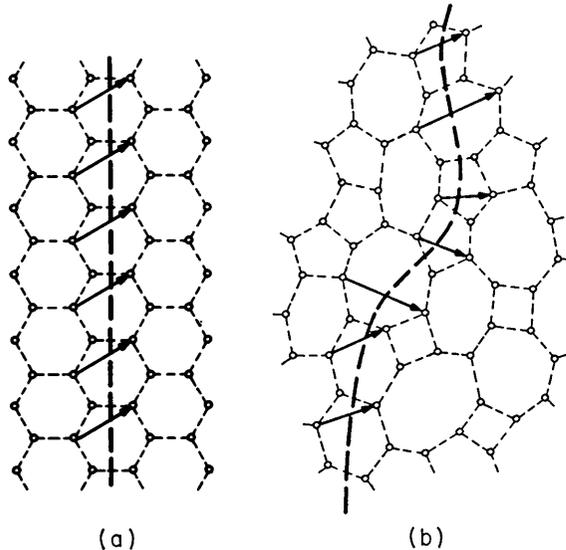


FIGURE 1.1. Dislocation lines in crystalline and noncrystalline silica compared (only silicon atoms are drawn). (a) plane of crystal structure; (b) projection of sheet of glassy network.

Burgers vector along its length, as suggested in Figure 1.1 for the prototype material: silica glass. Projections onto the plane of the drawing of the positions of the silicon atoms of a single sheet in the structure are shown. The oxygen atoms are not shown, but each silicon atom is bonded to one oxygen atom that lies just above it, plus to three others that lie just below it parallel to the plane of the drawing. If parts of the upper oxygen layers are translated while their positions are not, dislocation lines can be formed at positions indicated by the dashed lines in Figure 1.1. The arrows represent the translations that move oxygen atoms in the next higher layer from initial sites to equivalent final sites during an elementary motion of the dislocation line.

It may be seen that by allowing the magnitude and direction of the Burgers vector to fluctuate about mean values, the concept of a dislocation line can be retained for glassy structures (Gilman 1968a). This analogy is not forced but is desirable because it allows the flow properties of these structures to be discussed in a more organized way than is otherwise possible. That is, it provides a simple means for describing the correlations that must exist between adjacent elementary shear processes. In existing treatments of the flow of glasses (Eyring et al. 1964), it is assumed that the elementary flow events are independent of one another. This is not justified except at very high temperatures.

If an elementary flow event occurs somewhere locally, both the chemical and the stress state become changed in the immediate vicinity. Therefore, at ordinary temperatures, the probability that another event will occur in that vicinity is enhanced. That is, a dislocation line is created and continues to exist and move until it becomes annihilated by another line, or by a free surface.

The Burgers vector of a dislocation in a glass will not have a constant value but will fluctuate about a mean value that is determined by the average network dimensions. However, for small fluctuations of  $\mathbf{b}$  the increase in the self-energy is small. Suppose a unit length of dislocation has a Burgers displacement of  $(b + \delta)$  along one-half of its length, and  $(b - \delta)$  along the other half, where  $\delta$  is a small increment. Then, since the self-energy is proportional to the square of  $b$ , the ratio of its energy to that of the same length without the fluctuations is  $1 + \delta^2/b^2$ . Thus fluctuations as large as 30 percent cause only a 10 percent energy increase.

In the noncrystalline case of Figure 1b, the mean Burgers displacement has a definite value that is determined by the network dimensions, but there are fluctuations in both its magnitude and its direction along the line. In order to minimize the energy of such a dislocation, it is necessary for the mean  $\mathbf{b}$  to be conserved over long distances; so although the local  $\mathbf{b}$ 's may fluctuate, there are long-range correlations (occasional large energy densities may cause this condition to be relaxed). Furthermore, there will be little tendency for the line to lie on a single plane, and its local structure

will change as it moves. Nevertheless, it is expected that such dislocations will exist in noncrystalline solids, especially under flow conditions. When they are viewed with a somewhat fuzzy microscope (resolution of approximately 10Å), their behavior should resemble that of dislocations in crystals.

Most of the usual techniques for observing dislocations in solids are ineffective for noncrystals. One that might be used is the high-resolution observation of surface steps which would reveal the egress of dislocations from a material (Gilman 1968a). An effective method for observing monomolecular surface steps is the gold decoration technique discovered by Bassett and developed by Bethge (1962).

Evidence that the ideas of dislocation dynamics that have been developed to describe crystalline solids can be applied to noncrystals (or partial crystals) has been obtained by Dey (1967) for the case of flow in nylon. He measured the velocities of reorientation (Luders) fronts as a function of stress and temperature, and showed that the behavior is consistent with the behavior of crystals.

### 1.3 Viscous Resistance to Dislocation Motion

It is well known that dislocation motion is a very dissipative process with most of the plastic work being converted into heat and some of it into structural defects within the material. Because so much dissipation occurs, there is no unique way of describing the details of the process, but there is a distinct advantage in using the language and ideas associated with the behavior of fluids. One reason is that this tends to unify discussions of crystalline and noncrystalline solids. Another reason is that the theory of transport in fluids has deep traditions and a highly developed status.

Since dislocations have both micro-aspects and macro-aspects, it is necessary to describe the viscous resistance to their motion in at least two stages: first in terms of semi-macromechanics, where viscous effects are described by means of a viscosity coefficient that is treated as a continuous parameter, and second, in terms of the molecular mechanisms that determine the local value of the viscosity coefficient. A complication is introduced by the special structure at the core of a dislocation. This structure can be expected to have a different viscosity coefficient than the remainder of the material. Fortunately, the core region actually plays a dominant role, as will be demonstrated shortly, so attention can be focused on it and its local viscosity.

The net effect of various linear loss-mechanisms integrated over the entire flow field of a dislocation is described by means of a "damping constant" which conventionally has the symbol  $B$ . Oftentimes nonlinear losses are larger than the linear ones, and the damping depends on the stress and/or the velocity in a nonlinear way.

Another source of drag on moving dislocations is the anelastic relaxation that can occur if impurities or other defects are present and can move to cause stress relaxation. This type of drag depends strongly on the dislocation velocity (Schoeck and Seeger, 1959) and has a relatively small magnitude. It will not be discussed further here because it is absent in pure materials, and point defects usually cause other effects that are larger in magnitude.

As a dislocation moves along its glide plane the elastic strains at points remote from its center undergo changes. Thus the moving dislocation is surrounded by a strain-rate field. In addition, at the very center the atoms on the top side of the glide plane slide over those on the bottom side. Thus a velocity gradient exists across the glide plane. For a narrow core, its magnitude can be very large compared with the other velocity gradients (strain-rates) in the system. Whenever a velocity gradient exists in a material (gas, liquid, or solid) it tends to become decreased as momentum is transported from the higher velocity regions to those with lower velocities. The viscosity coefficient measures the efficiency of this transport.

Mason (1960) first emphasized the usefulness of this viewpoint in considering dislocation losses, and showed how to calculate the power loss in the strain-rate field. However, he arbitrarily excluded the core region when he integrated over the field. Gilman (1968b) showed how the core region can be included in the calculation and that most of the loss occurs there for a given viscosity coefficient.

Consider a screw dislocation line that lies parallel to the  $z$  axis and moves with velocity  $v_x$  along the  $xz$  glide plane. Outside the core region, the strain-rate field is given by

$$\dot{\epsilon}_{13} = \left(\frac{b}{2\pi}\right) \frac{v_x \cos \theta}{r^2} \quad (1.1)$$

and if the viscosity coefficient is called  $\eta$ , the power loss  $dP$  in a differential volume  $dV$  is

$$dP = \eta(\dot{\epsilon}_{13})^2 dV. \quad (1.2)$$

If the separation distance at the glide plane is  $a$ , then the elastic approximation can be used in the regions ( $x_2 < -a/2$ ) and ( $x_2 > +a/2$ ), which lie outside a slab of thickness  $a$  centered on the glide plane. The power loss in these regions is

$$P_0 = 2\eta \left(\frac{bv_x}{2\pi}\right)^2 \int_{a/2}^{\infty} \int_{-\infty}^{+\infty} x_1^2 (x_1^2 + x_3^2)^{-3} dx_1 dx_3 = \frac{\eta}{2\pi} \left(\frac{bv_x}{2a}\right)^2, \quad (1.3)$$

and this is essentially the same as Mason's previous result.

In the region within the slab ( $-a/2 < x_2 < +a/2$ ), the velocity gradient can be obtained from the rate of relative displacement (sliding) across the

glide plane. To a good approximation (and exactly in some cases), the relative displacement is given by

$$u_3(x_1) = -\frac{b}{\pi} \tan^{-1}\left(\frac{2x_1}{w}\right), \quad (1.4)$$

where  $w$  is the “width” of the dislocation core. Then the velocity gradient in the glide plane region is

$$\dot{\epsilon}_g = \frac{1}{a} \left( \frac{du_3}{dt} \right) = \frac{v_x}{a} \left( \frac{\partial u_3}{\partial x_1} \right). \quad (1.5)$$

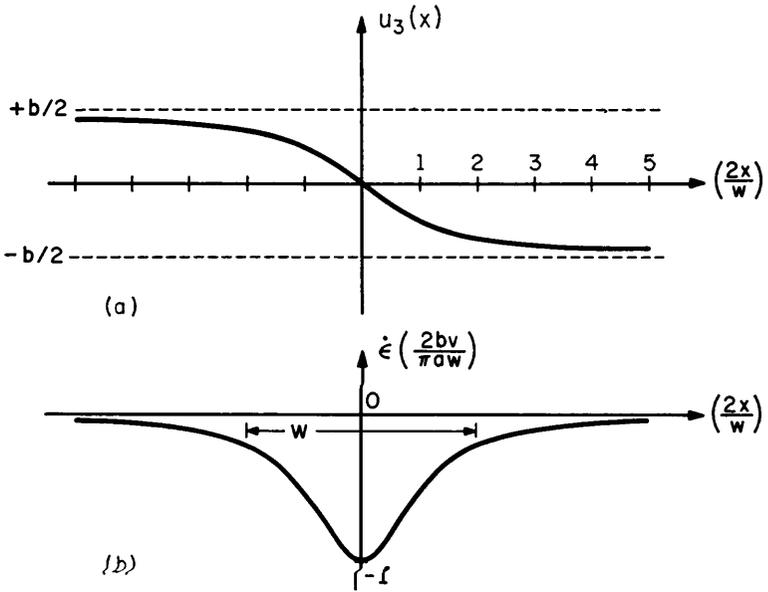


FIGURE 1.2. Relation of velocity gradient ( $\dot{\epsilon}$ ) to displacement along the glide plane of a dislocation. (a) displacement function of stationary dislocation; (b) velocity gradient for moving dislocation (neglects perturbations caused by local atomic interactions)

The relations just above are illustrated by Figure 1.2. Substitution of this into Equation 1.2 followed by integration yields the power loss

$$P_g = \frac{\eta v_x^2 b^2}{\pi a w} \quad (1.6)$$

Equations 1.3 and 1.6 combined give the total power loss

$$P \simeq \frac{\alpha \eta v_x^2 b^2}{\pi a w} \left[ 1 + \frac{1}{8} \left( \frac{w}{a} \right) \right], \quad (1.7)$$

where  $\alpha$  is a numerical factor that equals unity for a screw dislocation; and  $3/4(1 - \nu)$  for an edge dislocation. The width is usually comparable with  $a$  in size so the second term in Equation 1.7 is small compared with the first. Thus most of the power loss occurs on the glide plane near the core.

Because of the localization of the power loss, viscous processes that occur at the glide plane are most important in determining the dislocation damping constant,  $B$ , which can be written approximately as

$$B \simeq \frac{\eta}{\pi} \left( \frac{b^2}{aw} \right), \quad (1.8)$$

where  $\eta$  is the local viscosity coefficient. This has the fortunate consequence of considerably reducing the number of important mechanisms. Furthermore, if dislocation-line motion is decomposed into a series of kink motions, then the possible sources of viscosity become even more localized.

#### 1.4 Sources of Viscosity

In order to smooth out differences in velocities within a medium, momentum must be transferred from regions that are moving fast to slower ones. The means for this to occur were analyzed long ago by Maxwell (1867); modern developments began with the work of Frenkel (1926) and Andrade (1934). There are two general categories of viscous mechanisms: "gas-like" and "solid-like" (Green 1952). In the gas-like mode particles (or quasiparticles) are free to traverse relatively long distances between collisions. As the particles cross an imaginary plane perpendicular to a velocity gradient, they carry more momentum down the gradient (on the average) than they carry up it. Thus the velocity of the slower material tends to increase, while that of the faster tends to decrease. The net velocity relative to some fixed reference tends toward zero.

In the case of solids, the main "gases" are formed by free electrons, and phonons. Other excitations that create mobile quasiparticles may also act in this fashion, but their densities may be too small to cause significant viscosity.

In the solid-like mode, direct interactions between sliding molecules tend to smooth out velocity differences. The molecules may be constrained to remain in their own layers, but a faster moving layer sliding over a slower one exerts a dragging force that tends to speed up the latter. At the same time the faster layer tends to slow down. On the average one can think of the sliding molecules as being temporarily coupled together by a force, or per unit area by a coupling shear stress,  $\sigma_c$ . If the mean time that the coupling lasts is called  $\tau$ , then the viscosity coefficient is (Maxwell 1867)

$$\eta = \sigma_c \tau, \quad (1.9)$$

and if a single process causes most of the loss, the damping constant becomes

$$B \simeq \left( \frac{b^2}{aw} \right) \frac{\sigma_c \tau}{\pi}. \quad (1.10)$$

The coupling relaxation time may be a function of such factors as the applied stress and the temperature, depending on what particular loss mechanism operates.

For dislocations moving through otherwise perfect crystals, the Peierls stress couples the molecules across the glide plane. If it is small, as for close-packed glide planes in pure metals, then since  $\tau$  is roughly the reciprocal Debye frequency (say  $10^{-12}$  sec), and  $\sigma_c$  is certainly less than about  $10^5$  dyn/cm<sup>2</sup>, the local viscosity coefficient may be as small as  $\sim 10^{-7}$  P, which is very small compared with the viscosity of a typical liquid metal ( $\sim 10^{-2}$  P).

On the other hand, if the Peierls stress is large as in covalent crystals such as Ge and Si (say  $\sim 10^{10}$  dyn/cm<sup>2</sup>), and the coupling force is localized so that the velocity is directly related to the coupling time, which is given by  $b/V \simeq 5 \times 10^{-4}$  sec, for a velocity of one micron per second; then the local viscosity coefficient is  $\sim 5 \times 10^6$  P, which is moderately large.

In imperfect crystals the viscosity is heterogeneous. Dislocations may move quite freely over glide-plane areas that are free of imperfection, impeded only by electron, or phonon, gas viscosity. At imperfections, strong local bonding may create a strong coupling force across the glide plane, or weak bonding may destroy the local periodicity and thereby raise the effective Peierls stress. Nonviscous drag will also result if the moving dislocation intersects another one and acquires a jog so that it leaves a dipole in its wake. This constitutes a net change in the internal structure and leads to strain-hardening, but is qualitatively different from a viscous loss mechanism.

### 1.5 Viscosity at High Velocities

A linear dependence of dislocation velocity on applied stress has been observed in several pure crystals: copper (Greenman, Vreeland, and Wood, 1967); zinc (Pope, Vreeland, and Wood, 1967); and germanium (Schafer, 1967). It is apparent, however, that the velocity cannot continue to be proportional to stress indefinitely because it would soon exceed sonic velocities. Therefore, it is legitimate to wonder how the damping constant depends on velocity. Taylor (1968) has suggested that relativistic effects cause the damping constant to take the form (screw dislocation)

$$B = B_0(1 - v^2/c_t^2)^{-1} \quad (1.11)$$

where  $B_0$  is the constant at low velocities,  $v$  is the instantaneous velocity, and  $c_t$  is the transverse elastic-wave velocity. According to this expression, the drag force increases without limit as  $v$  approaches  $c_t$ . Therefore, for any feasible applied force,  $v$  cannot exceed  $c_t$  (in a linear system).

The present author offers the following simple justification of Taylor's suggestion. The damping constant has the form  $B = p_x/A_y$  where  $p$  is momentum and  $A$  is area. That is, it measures momentum transfer per unit area. It is well known that the Lorentz transformation for a momentum component is  $p_x = p_x^0/\beta$  where  $\beta = (1 - v^2/c^2)^{1/2}$ ; and since sliding occurs in one direction, the Fitzgerald contraction of the area is given by  $A_y = \beta A_y^0$ . Thus Equation 1.11 follows.

For steady-state motion, the applied force  $\sigma_s b$  on a dislocation equals the drag force  $B_0 v/\beta^2$  so that the velocity is explicitly related to the applied stress

$$v = \frac{c}{x} [(1 + x^2)^{1/2} - 1], \quad (1.12)$$

with  $x = \text{reduced stress} = \sigma_s(2b/B_0c)$ . This equation is plotted in Figure 1.3 to display its linear and saturation limits.

### 1.6. Nonlinear Viscous Drag

When localized coupling forces exist across glide planes, and the temperature is low (that is, low compared with the Debye temperature associated with the local coupling force) the flow velocity is not proportional to the applied stress. Instead, it is observed that the flow rate is very small until some critical stress is reached, and then "yielding" occurs. In other words the flow is stress-activated. In terms of Equation 1.9, the mean coupling time is a function of stress and it rather suddenly decreases when a critical applied stress is reached. A simple analytic form that relates velocity and stress in this case has been proposed by the author:

$$v = v^* e^{-D/\sigma_s}, \quad (1.13)$$

where  $v^*$  is a terminal velocity and  $D$  is a drag stress. This form is followed by several sets of data (Gilman 1965), and is consistent with the idea that stress can activate dislocation motion via quantum-mechanical tunneling (Gilman 1968a). It is not the purpose here to discuss these matters, however, so Equation 1.13 will simply be asserted as a reasonable form.

At steady state the work done on a moving dislocation equals the power dissipated in the form of both heat and structural defects. Assuming that heat production dominates, the effective viscous damping constant is the ratio of the driving force per unit length to the velocity

$$B_{\text{eff}} = \frac{\sigma_s b}{v} = \left(\frac{b}{v^*}\right) \sigma_s e^{D/\sigma_s} \quad (1.14)$$

Thus the effective damping constant is infinite when the stress is zero (the dislocation is "pinned"). Then the damping decreases rapidly with increasing stress until a minimum value is reached when  $\sigma_s = D/2$ ; and then it increases nearly linearly with further decreases of the stress. The corresponding velocity-stress curve for this "solid-like" viscosity is compared with the curve for the "gas-like" viscosity in Figure 1.3.

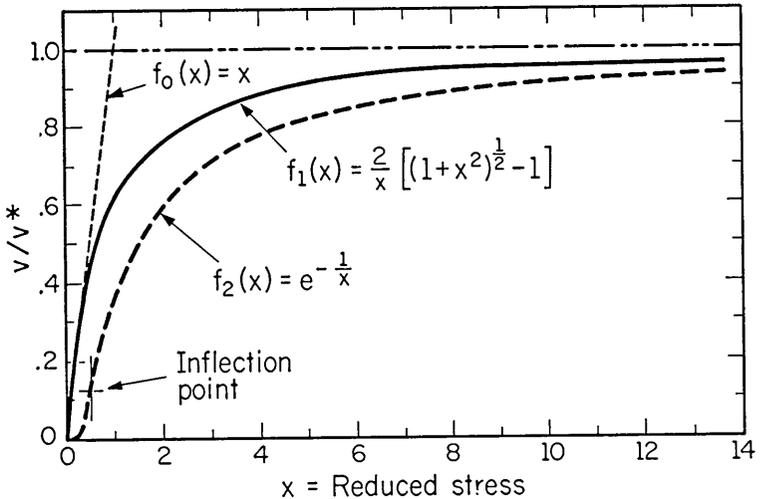


FIGURE 1.3. Comparison of velocity-stress functions.  $f_0$  = linear viscosity (Newton);  $f_1$  = relativistic viscosity (Taylor);  $f_2$  = yielding function (Gilman).

### 1.7. Amplification of Viscous Resistance by Internal Stresses

In addition to the complications caused by the heterogeneous nature of real materials, the internal stress fluctuations that are usually present in them can have important effects, especially for nonlinear flow.

For linear viscosity if the internal stresses fluctuate about some mean value,  $\sigma$ , with an average amplitude,  $\Delta\sigma$ , and an average wavelength,  $\lambda$ , it can be readily shown that the effective damping constant is

$$B'_{\text{eff}} = B_0 \left[ 1 - \left( \frac{\Delta\sigma}{\sigma} \right)^2 \right]^{-1}, \quad (1.15)$$

so that for small fluctuations there is a negligible effect.

For the nonlinear case, since  $e^{1/1+\delta}$  is small compared with  $e^{1/1-\delta}$  for moderate values of  $\delta$ , the effective damping constant has the approximate value

$$B''_{\text{eff}} \simeq B_{\text{eff}} e^{(\sigma/\sigma - \Delta\sigma)}, \quad (1.16)$$

so that the effective viscosity increases very rapidly with increasing stress fluctuation amplitude. A more detailed discussion of this effect was first given by Chen, Gilman, and Head (1964); more recently it has been considered by Li (1968) and by Argon (1968).

### 1.8. Summary

It has been shown that by relating dislocation motion to the concept of fluid viscosity, it is possible to unify the description of mobility, and to take advantage of the existing theories of fluids in interpreting dislocation behavior. Also, this viewpoint places plastic solids between elastic solids and fluids in a well-defined way. This is their natural position.

### REFERENCES

- Andrade, E. N. da C., 1934, *Phil. Mag.*, **17**, 497, 698.  
Argon, A. S., 1968, *Mat. Sci. Eng.*, **3**, 24.  
Bethge, H., 1962, *Phys. Stat. Sol.*, **2**, 3.  
Chen, H. S., J. J. Gilman, and A. K. Head, 1964, *J. Appl. Phys.*, **35**, 2502.  
Dey, B. N., 1967, *J. Appl. Phys.*, **38**, 4144.  
Eyring, H., D. Henderson, B. Stover, and E. Eyring, 1964, *Statistical Mechanics and Dynamics*, New York: Wiley Chapter 16.  
Frenkel, J., 1926, *Z. Physik*, **35**, 652.  
Gilman, J. J., 1960, *Australian J. Phys.*, **13**, 327.  
Gilman, J. J., 1965, *J. Appl. Phys.*, **36**, 3195.  
Gilman, J. J., 1968a, in A. R. Rosenfield et al. (eds.), *Dislocation Dynamics*, New York; McGraw-Hill, p. 3.  
Gilman, J. J., 1968b, *Phys. Rev. Letters*, **20**, 157.  
Green, H. S., 1952, *The Molecular Theory of Fluids*, New York: Interscience.  
Greenman, W. F., T. Vreeland, and D. S. Wood, 1967, *J. Appl. Phys.*, **38**, 3595.  
Li, J. C. M., 1968, in A. R. Rosenfield et al. (eds.), *Dislocation Dynamics*, New York: McGraw-Hill, p. 87.  
Mason, W. P., 1960, *J. Acoust. Soc. Amer.*, **32**, 458.  
Maxwell, J. C., 1867, *Phil. Trans. Roy. Soc.*, **A157**, 49.  
Pope, D. P., T. Vreeland, and D. S. Wood, 1967, *J. Appl. Phys.*, **38**, 4011.  
Schafer, S., 1967, *Phys. Stat. Sol.*, **19**, 297.  
Schoeck, G., and A. Seeger, 1959 *Acta Met.*, **7**, 469.  
Taylor, J. W., 1968, private communication.