

## Subject Index

- a priori expected codelength, 555
- absolute singularity, 507
- accidental information, *see*  
information, accidental
- action, 574
- adaptive criterion, 417
- adaptivity, 193
- admissible, 527
- affinity, 499, **645**
- agnostic, 579
- AIC, 417, 532, 541, 549  
and BIC and MDL, 552  
regression, 450
- Akaike information criterion, *see*  
AIC
- algorithmic  
MDL, *see* MDL, idealized  
randomness, 11  
statistics, 11
- almost in-model, 493
- $\alpha$ -factor, 479, 512
- $\alpha$ -two-part code MDL, 477, 503,  
512, 645
- alphabet, **6**, 79, 80
- annealed entropy, 583
- apology, 561
- approximation, 376
- AR process, 214
- arbitrariness, 29, 152
- ARMA process, 214, 508
- astronomy, 545
- asymptotic, 429  
Bayesian code regret, 232, 244  
CNML regret, 323  
distinguishability, *see*  
distinguishability,  
asymptotic
- linear model regret, 366
- LNML regret, 312
- parametric complexity, 211
- plug-in code regret, 260
- two-part code regret, 273
- average, 631  
and approximation, 453  
Césaro, 474  
case vs. worst-case, 451
- averaging, 26, 341, 346
- ball, 44  
distinguishability, 221  
Kullback-Leibler, 220  
Mahalanobis, 121
- Barron, A.R., 242
- basis functions, 336
- batch, 419
- Bayes  
factors, 418, 539, 540, 549  
predictive interpretation,  
541  
formula, **50**, 75, 192  
generalized, 317  
nonparametric, 543  
vs. MDL, 533
- Bayesian  
brands, 544  
confirmation theory, 545  
estimator, *see* estimator,  
Bayesian  
evidence, 77, 175  
hypothesis testing, 650  
inconsistency, 543

- Information Criterion, *see*
  - BIC
  - interpretation, 418
  - linear model, *see* linear model, Bayesian
  - MAP, *see* MAP
  - marginal likelihood, 77
  - mixture, 77, 175
  - predictive distribution, *see* distribution, predictive
  - principles, 532
  - regret, 232
  - statistics, *see* statistics, Bayesian
  - universal model, *see* universal model, Bayesian
- Bernoulli family, *see* model, Bernoulli
- beta prior, 258
- BIC, 417, 532, 549
  - and AIC and MDL, 552
  - regression, 450
- binary tree, 92
- biological applications, 492
- Booleans, 42
- bound
  - Chernoff, 634, 636
  - generalization, 586
  - grand Cramér-Rao, 455
  - Hoeffding, 634, 636
  - Rissanen, 454
- boundary, 44
- bounded set, 44
- Brandeis dice, 638, 643
- Césaro
  - average, 474
  - consistency, *see* consistency, Césaro
  - KL risk, *see* KL risk, Césaro
  - universal model, 474
- calculation, *see* computation
- canonical
  - parameterization, *see* parameterization, canonical
  - prior, *see* prior distribution, canonical
- carving up models, 160
- central limit theorem, 56, 113, 220, 247
  - and relative entropy, 129
  - multivariate, 633
  - uniform, 289
- chain rule, *see* probability, chain rule
- cheating, 571, 572
- Chernoff
  - bound, 634, 636
  - divergence, 646
  - information, 650
- $\chi^2$ -divergence, 517
- choose function, 127
- classification, 72, 439
- Clinton, W.J., 437
- closure, 44
- CLT, *see* central limit theorem
- clustering, 229, 407
- CNML, *see* conditional NML
- code, 7, **80**
  - comma-free, 86
  - complete, 90, 94, 99
  - concatenation, 84, 86
  - conditional, 86, 290, 296, 302, 316, 433, 572
  - Liang and Barron, 446

- Liang-Barron, 325, 449, 470
  - design, 152, 157, 158
  - efficient, 90, 94
  - fixed-length, 88
  - for Markov chains, 137
  - inefficiency, 161
  - instantaneous, **84**
  - meta-two-part, *see*
    - meta-two-part code
  - metauniversal, *see*
    - metauniversal code
  - minimax, 99
  - NML, *see* NML
  - one-part, 152
  - optimal, 135
  - partial, 80
  - predictive, 152
  - predictive MDL, 198
  - prefix, 83, **84**
  - quasi-uniform, 90, 100, 159, 187, 422, 425
  - Shannon-Fano, **95**, 110, 136
  - Shtarkov, *see* NML
  - standard for integers, 101, 137, 186
  - trivial, 89
  - two-part, *see* two-part code
    - design, 161
  - uniform, 87, 99, 106, 137, 285
  - uniquely decodeable, 7
  - universal, *see* universal code
  - word, 80, 82
- codelength
  - absolute, 305, 568
  - as probability, 96
  - difference, 411
  - excess, 177
  - expected, 625, 630, 642
  - function, **99**
  - ignoring, 439
  - interpretation, 433
  - invariance, 98
  - minimax, 87, 106, 624, 637, 641
  - minimax relative, 643
  - noninteger, 95
  - observed average, 630
  - relative, 305, 568
  - Shannon-Fano, 114
  - worst-case, 87
- coding
  - by index, 284
  - model index, 424
  - system, 7, **80**
    - lossy, 80
    - partial, 80
    - singular, 80
  - with help of, 136
- coherent, 544
- compactness, 44
- compatibility condition, 53, 194
- compatible
  - prior distribution, 232
- complete code, *see* code, complete
- complexity, 135, 143, 180, 411, 416
  - adjusted, 427
  - and volume, 216, 222
  - Bernoulli model, 212, 227
  - classification, 582
  - computation, *see*
    - computation, parametric
    - complexity
  - conditional, 302
  - constrained, 302, 440
  - data-dependent, 584
  - distinguishability, 224
  - exponentiated asymptotic, 216

- for exponential families, 226
  - histogram, 378
  - histogram model, 228
  - infinite, 215, 295, 420
  - Kolmogorov, 8, 546, 570
  - learning theory, 582
  - multinomial model, 228
  - nonasymptotic, 228
  - of hypotheses, 30
  - of models, 30
  - PAC-Bayesian, 589
  - parametric, 208, 210
    - asymptotic, 211
  - Rademacher, 583
  - renormalized, *see* RNML
  - second order, 307
  - simplification, 226
  - stochastic, *see* stochastic complexity
- composite hypotheses, 421
- composition of experiments, 345
- compression, 73, 413, 595
  - and fast learning, 469, 482
  - and regularity, 103
  - game, 643
  - interpretation, 415
  - lossy, 571
  - schemes, 584
- computable distribution, 546, 571
- computation
  - and Bayes, 538
  - parametric complexity, 226, 422
  - plug-in codelength, 269
  - two-part code, 150
  - universal codelengths, 428
- computational learning theory, *see* learning theory, computational
- concave function, 42
- concentration, 512
- concept learning, 72
- conditional
  - code, *see* code, conditional
  - description method, 86
  - distribution, 49
  - exponential family, 619
  - limit theorem, 127, 569
  - NML, 320, 470
    - and Jeffreys, 323
    - and luckiness NML, 322
  - asymptotic regret, 323
  - linear model, *see* linear regression, NML
  - model selection, 431, 446
  - NML-1, 322, 420, 426
  - NML-2, 321, 420, 426
  - NML-3, 323
  - probabilistic source, *see* probabilistic source, conditional
  - regret, 365
  - two-part, *see* two-part code, conditional
  - universal code, *see* code, conditional
  - universal model, *see* code, conditional
- confidence, 411, 421, 535
  - level, 413, 585
- consistency, 71, 73, 143, 145, 153, 425, 449, 477
  - AIC, 550
  - and measure 0, 506
  - BIC, 550
  - Césaro, 467, 472
  - discussion, 501
  - essential, 468, 472

- information, 467
- KL, 467, 472
- linear regression, 508
- MDL
  - general, 514
  - MDL model selection, *see*
    - MDL model selection,
    - consistency
  - misspecification, 504
  - model selection, 74
  - of ML estimator, 59
  - peculiarities, 511, 593
  - prequential MDL, *see*
    - prequential MDL,
    - consistency
  - scenarios, 501
  - theorem, 155, 467, 478, 506
  - trivial, 504
  - two-part MDL, *see* two-part
    - MDL, consistency
  - weak, 479, 503
- constrained parametric
  - complexity, *see*
  - complexity, constrained
- constraints, 568
- continuum limit, 488, 497
- convergence rate, 59, 158, 161,
  - 387, 425, 467, 477, 501,
  - 506, **515**
- AIC, 550
- BIC, 550
- essential, 473
- MDL, 520
  - model selection, 522
- minimax, **519**
- parametric, 469, 483
- prequential MDL, 467
- scenarios, 501
- two-part MDL, 478
  - uniform, 516
- convex, 42
  - analysis, 604
  - duality, 613
  - function, 42
  - set, 42
- correlated prediction error, 567
- countable, 423
  - universal model, 184
- counting interpretation, 416
- covariance, 48, 606, 634
  - linear model, 353
- covering number, 222, 281
- Cramér's Theorem, 634
- Cramér-Rao bound, 455
- critical value, 412
- cross-product matrix, *see* Fisher
  - information, regression
  - matrix
- cross-validation, 541, **565**
- crude MDL, *see* MDL, crude
- Csiszár-Shields, *see* inconsistency,
  - Csiszár-Shields
- cumulant, 605
- cumulative risk, 467
- CUP, 65
  - codes, *see* universal model,
  - CUP
  - model class, *see* model class,
  - CUP
- curvelength, 218
- CV, *see* cross-validation
- data, **6, 69**
  - alphabet, *see* alphabet
  - item, 6
  - nondegenerate, 142
  - sample, 6
  - sequence, 6

- unseen, 419, 567
- virtual, 347
- Dawid, A.P., 190, 261, 562
- De Finetti, B., 548
- decision
  - function, 69
  - theory, 594
- decoder, 83, 172
- decoding function, 83
- defective, 94, 103, 149
- degree-of-belief, 539
- denoising, 349, 407, 423
- density
  - differentiable, *see*
    - differentiable density
  - estimation, 407, 460, 464
    - histogram, *see* histogram
    - density estimation
  - nonparametric, *see*
    - nonparametric density
    - estimation
  - function, 46
- description method, 7, 80
  - conditional, 86
  - prefix, 83, **84**
  - quasi-uniform, 90, 100
- descriptions, 83
- design
  - matrix, 340, 391
  - principle, 29, 525, 591
- determinant, 43
- deviance, 539
  - information criterion, 539
- DIC, 539
- dice, 638, 643
- diffeomorphism, 611
- differentiable density, 370, 371, 503, 520, 525
- differential entropy, 104
- dimensionality, 219
- Dirichlet distribution, 263
- discrete estimator, 489
- discretization, 104, 137, 278, 485
- distance
  - Euclidean, *see* Euclidean distance
  - Hellinger, *see* Hellinger distance
  - integrated squared error, 517
  - Mahalanobis, *see*
    - Mahalanobis distance
    - relations, 517
- distinguishability, 153, 182, 216, 339, 416
  - and complexity, 224
  - and conditional two-part code, 290
  - and Jeffreys' prior, 236
  - and KL divergence, 219
  - and phase transition, 290
  - asymptotic, 153, 507
  - ball, 221
  - level, 224, 279
  - number of distributions, 224
  - region, 220
- distinguishable distributions, *see* distinguishability
- distribution
  - Bernoulli, *see* model, Bernoulli
  - beta, 258
  - computable, 546, 571
  - conditional, 49
  - Dirichlet, 263
  - distinguishable, *see* distinguishability
  - gamma, 362, 445
  - generating, 71

- joint, 49
- marginal, 49, 55
  - Bayesian, 77
  - multivariate normal, 50, 634
  - normal, *see* model, normal
  - posterior, *see* posterior distribution
  - predictive, 77, 391, 460, 461
    - linear model, 356
  - prior, *see* prior distribution
  - product, 51, 55
  - square-root inverted gamma, 362, 445
  - true, 20, 525
  - uniform, 99, 638
- divergence
  - $\chi^2$ , 517
  - KL, *see* KL divergence
  - Rényi, *see* Rényi divergence
  - relations, 517
- DNA, 492
- d*-risk, 515
- Dutch book, 544
- early stopping, 490
- efficient code, *see* code, efficient
- eigenvalues, 43
- eigenvector, 120, 278
- ellipsoid, 120, 278
- Ellsberg paradox, 545
- EM algorithm, *see* expectation-maximization
- empirical
  - error, 581
  - loss, 582
  - Rademacher complexity, 583
  - risk minimization, 580
- encoder, 83, 172
- end-of-input marker, 86
- ensemble, 631
- entropification, 574, 588
- entropy, 103, 615
  - annealed, 583
  - coding interpretation, 104
  - combinatorial interpretation, 127
  - differential, 104
  - for exponential family, 606
  - maximum, 106, 600, 624, 637
    - principle, 567
    - prior, 569
  - minimum relative, 624
  - of normal distribution, 640
  - Rényi, 583
  - Rényi vs. Shannon, 650
  - relative, *see* KL divergence
  - Shannon, 104
- epigraph, 42
- equalizer strategy, 181
- ERM, 580
- essential
  - consistency, *see* consistency, essential
  - convergence, *see* convergence rate, essential
- estimation
  - density, *see* density estimation
  - maximum likelihood, *see* likelihood, maximum
  - nonparametric, *see* nonparametric density estimation
  - parameter, 70, 476
    - MDL, *see* MDL parameter estimation
  - predictive MDL, *see* prequential MDL

- estimator, 57
  - $\alpha$ -two-part MDL, 477, 503, 512, 645
  - Bayes MAP, *see* MAP
  - Bayes mean, 354, 495, 556
  - Bayesian, 268
  - discrete, 489
  - in-model, 263, 268, 462, 463, 491, 576
  - Krichevsky-Trofimov, 258
  - Laplace, 258, 527
  - least-squares, *see* least-squares
  - maximum likelihood, *see* likelihood, maximum
  - model selection-based, 509
  - out-model, 263, 268, 463, 485, 491
  - SMML, *see* minimum message length, strict
  - superefficient, 455, 527
  - two-part MDL, *see* two-part MDL
  - unbiased, 268
  - Wallace-Freeman, 497, 560
- Euclidean distance, 43, 159, 515, 517
  - rescaled, 120
- evolutionary trees, 492
- exchangeable, 434
- expectation, 48, 52
  - vs. hope, 533, 535
- expectation-based MDL, *see* MDL, expectation-based
- expectation-maximization, 151, 427, 490
- expected
  - codelength, 625
  - redundancy, *see* redundancy, expected
- experiment composition, 345
- experimental design, 32
- experts, 574
- exponential family, 66, 67, 599, 600
  - and maximum entropy, 638
  - canonical parameterization, 287
  - conditional, 619
  - discrete, 286
  - entropy, 606
  - Fisher information, 606
  - i.i.d., 603, 619
  - linear model, 352
  - mean, 606
  - minimal, 66, 601
  - ML estimator, 629, 630, 632
  - of general sources, 617
  - robustness property, 208, 240, 266, 605, **624**, 641
  - variance, 606
- exponentiated asymptotic complexity, 216
- extended KL divergence, 625
- family
  - exponential, *see* exponential family
  - likelihood ratio, 647
  - normal, *see* model, normal
- feature vector, 336
- Fechner's model, *see* model, Fechner's
- Fisher information, 607
- Fisher information, 120, 121, 221, 275, 606, **619**, 634
  - cancellation, 488



- determinant, 211
- empirical, 112
- expected, **119**, 241
- normal models, 300
- Observed, 620
- observed, **112**, 241
- regression matrix, 343, 350
- Fisher, Sir Ronald, 117, 145
- fitted sum of squares, 344
- footing, 416, 425
- forward validation, 448, 563
- Foster and Stine approach, 329
- France, 75
- F*-ratio, 446
- free parameters, 212
- frequency, 59, 61, 285
  - in sample, 53
- frequentist statistics, *see* statistics, frequentist
- function
  - concave, 42
  - convex, 42
  - decoding, 83
  - encoding, 80
  - partial, 47
- functional form, 24, 216
- gambling
  - Kelly, 98, 191
- game
  - compression, 643
  - theory, 573
  - zero-sum, 637
- gamma
  - distribution, 362, 445
  - function, 45
- gappy sequence, 473
- Gauss curve, 113
  - multivariate, 117
- Gauss, C.F., 145
- Gaussian, *see* model, normal
  - prior, *see* prior distribution, Gaussian
  - process, 364, 371, 390, 394, 472, 504, 542
  - as universal model, *see* universal model, Gaussian process
  - finite-dimensional, 399
  - regression, 395
- generalization
  - bound, 586
  - error, 579
  - performance, 73, 581
- generalized
  - Bayes, 317
  - linear model, *see* linear model, generalized
  - NML, *see* luckiness NML-1
- geometric family, *see* model, geometric
- Germany, 75
- Gibbs' inequality, 105
- global
  - maximum, 151
  - ML principle, 149, 532
- gMDL, 443, 449
- goals of inductive inference, 71
- goodness-of-fit, 135, 143
- g*-prior, 444
- gradient, 116
  - descent, 427
- gzip, 537, 538
- heavy tails, 451
- Hellinger
  - affinity, 499, 645, 646

- distance, 71, 478, 491, 510, 512, 515, 517, 645
- risk, 490, 504
- Hessian matrix, *see* matrix, Hessian
- Hilbert space, 394, 396
- hill-climbing, 151
- hindsight, 174, 419
- histogram, 482
  - Bayes, 378
  - density estimation, 376, 471, 510
    - alternative, 471
  - irregular, 471
  - minimax optimal, 382
  - model, 376
  - NML, 378
  - regular, 377
  - universal model, 378
- Hoeffding bound, 634, 636
- honest, 161, 567
- hope, 533, 535
- horizon, 190, 194, 563
- hypercube, 42, 222
- hyperparameter, 441
- hypothesis, **13**
  - complexity, 30
  - composite, **69**, 72
  - individual, 69
  - point, **13**, **69**
  - selection, 70, 406
  - simple, 70
  - singleton, 69
  - testing, 412
    - and luckiness, 421
- I-divergence, 646
- i.i.d., 54, 62
- idealized MDL, *see* MDL, idealized
- ignoring codelength, 439
- iMDL, 448, 449
- improper prior, 317
- in-model estimator, *see* estimator, in-model
- incompleteness, 274
- incompressibility, 103
- inconsistency, 146, 504, 530, 590, 593
  - Bayesian, 543
  - Csiszár-Shields, 506, 513
- independence, 51, 54, 638
- index coding, 424
- index of resolvability, 483
- indicator function, 52
- individual-sequence, 178, 564
  - MDL, *see* MDL, individual sequence
  - prediction, 573
  - prequential MDL estimator, 464
- inductive inference, 69
  - goal of, 71
- ineccsi, 301
  - model, 209
  - sequence, 210
  - subset, 209
- inefficiency of codes, 161
- inequality
  - Gibbs', 105
  - information, 101, 103, 105, 118, 452
  - Jensen's, 105, 475, 500
  - Kraft, 91, 500
  - Markov's, 56
  - no-hypercompression, **102**, 155, 413, 535

- triangle, 480
- infimum, 44
- infinite
  - complexity, *see* complexity,
  - infinite
  - horizon, 563
- infinite-dimensional, 394
- infinitely many models, 422
- infinity problem, 295
- information
  - accidental, 416
  - Chernoff, 650
  - closure, 376, 471
  - consistency, 467
  - criterion
    - AIC, *see* AIC
    - BIC, *see* BIC
  - inequality, *see* inequality,
  - information
  - matrix, *see* Fisher information
  - meaningful, 416
  - observed, 112
  - source, 53
  - theory, 72, 79
- inner product, 43
- input vector, 336
- insight, 492
- integers, 42
- integral
  - Gaussian, 240
- integrated squared error, 517
- integration
  - Laplace method, 239
  - saddle point method, 239
- interior, 44
- interval
  - closed, 42
  - open, 42
  - unit, 42
- invariance theorem, 10
- irregular histogram, 471
- Jacobi matrix, 611
- Jeffreys' prior, *see* prior distribution, Jeffreys'
- Jensen's inequality, *see* inequality, Jensen's
- joint distribution, 49
- Kelly gambling, *see* gambling, Kelly
- kernel, 472, 542
  - density estimator, 371
  - finite-dimensional, 398
  - function, 390
  - Matern, 400
  - Mercer, 400
  - polynomial, 391
  - RBF, 393
  - trick, 393
- kernelization, 390
- KL consistency, 467
- KL divergence, 103, 104, 120, 154, 156, 159, 201, 481, 515, 517, 620, 625, 633, 634, 647
  - and distinguishability, 219
  - ball, 220
  - chain rule, 465
  - coding interpretation, 105
  - extended, 266
  - linear model, 351
  - robustness, *see* exponential family, robustness property
  - Taylor expansion, 117, 240, 266, 276, 518, 636
- KL risk, 466

- Césaro, 467, 474, 479, 520
- Kolmogorov
  - complexity, *see* complexity, Kolmogorov
  - minimum sufficient statistic, 11, 571
  - structure function, 571
- Kolmogorov, A.N., 8
- Kraft inequality, *see* inequality, Kraft
- Krichevsky-Trofimov estimator, 258
- Kullback-Leibler divergence, *see* KL divergence
- label, 579
- Laplace
  - integration, 239
  - rule of succession, 258
- Laplace, P.S. de, 258
- large deviations, 624, 634
- law
  - of iterated logarithm, 247
  - of large numbers, 55, 125
- learning and compression, 469, 482, 595
- learning theory
  - computational, 72, 573, 579
  - luckiness in, 309
  - statistical, 72, 449, 525, 579
- least-squares, 337, **340**
  - and projection, 342
  - penalized, **346**, 355
  - predictive, *see* PLS
- leave-one-out
  - cross-validation, *see* cross-validation error, 566
- Legendre transform, 614, 648
- length function, *see* codelength, function
- level of distinguishability, 224
- Liang-Barron, *see* code, conditional, Liang-Barron
- light tails, 480
- likelihood, 57, 111
  - expected, 117
  - maximum, 57, 111, 124, 527, 624, 629, 632
  - Bernoulli, 258
  - consistency, 59
  - discretized, 224
  - global, 149
  - linear model, 350
  - local, 427
  - luckiness, 311, 419, 484, 487, 494, 497, 498
  - modified, 258, **260**
  - principle, 144
  - vs. MDL, 147
- maximum normalized, *see* NML
- maximum renormalized, *see* RNML
- ratio, 631
  - family, 647
  - test, 412, 417
- linear
  - model, 64, 335, 337, **348**, 423, 428, 589, 617
  - and normal, 338, 349
  - as exponential family, 352
  - Bayesian, 354, 364, 390
  - covariance, 353
  - generalized, 401, 443, 449, 508
  - Jeffreys prior, 359

- KL divergence, 351
- MAP, 354
- marginal density, 355
- ML estimator, 350
- parameterization, 352
- posterior, 355
- predictive distribution, 356
- regret, 350
- regression, 63, 335, 580, 617, 619
  - CNML, 446
  - consistency, 503, 508
  - gMDL, 443
  - iMDL, 448
  - Liang and Barron, 446
  - model selection, 438
  - NML, 363
  - plug-in code, 448
  - RNML, 439
  - universal model, 363
- link function, 401
- LLN, *see* law of large numbers
- LML, *see* likelihood, maximum
  - luckiness
- LNML, *see* luckiness NML
- local maximum, 151
  - and NML, 427
- log
  - likelihood, 57
  - loss, *see* loss, logarithmic
  - score, *see* loss, logarithmic
- log-convex mixtures, 645
- log-likelihood
  - expected, 117
  - surface, 630
- logarithm, 45
- logistic
  - function, 401
  - regression, 630
- LOO, *see* leave-one-out
- loss, 574
  - 0/1, 540, 574
  - classification, 574
  - empirical, 582
  - function, 73
  - general, 461, 464
  - logarithmic, 190, 460, 574
  - simple, 575
  - squared, 337, 574, 579
  - symmetric, 575
- lossy compression, 571
- luckiness, 424, 433, 585
  - and Bayes, 310, 534
  - and hypothesis testing, 421
  - and PAC-Bayes, 586
  - Bayesian universal model, 313
  - coding, 296
  - function, 309, 354, 398, 406, 443, 483
    - choosing, 534
  - Gaussian process, 397
  - in learning theory, 309
- ML, *see* likelihood, maximum
  - luckiness
- NML, 309, 422
  - and conditional NML, 322
  - asymptotic regret, 312
  - linear model, *see* linear regression, NML
- NML-1, 309, 426
  - and RNML, 443
- NML-2, 311, 426, 485
- principle, 92, 159, 305, 420, 425, 449, 532
- rationale, 535
- regret, 311, 397, 422, 484, 493
- tilted Jeffreys' prior, 313, 484

- uniform, 485
- universal model, 308
  - Bayesian, 313
- machine learning, 69, 72
- Mahalanobis distance, 120, 278, 344, 350, 398, 517
- many-to-one, 42
- MAP, 311, 493, 556
  - and LML, 494
  - linear model, 354
- marginal
  - distribution, *see* distribution, marginal
  - likelihood, 175
- Markov chain, *see* model, Markov
- Markov's inequality, 56
- Matern kernel, 400
- matrix
  - cross-product, *see* Fisher information, regression matrix
  - design, 340
  - determinant, 43
  - eigenvalues, 43
  - Hessian, 43, 117
  - information, *see* Fisher information
  - inverse, 43
  - positive definite, 43, 120, 606
  - projection, 342
- MaxEnt, *see* entropy, maximum
- maximum, 44
  - a posteriori, *see* MAP
  - entropy, *see* entropy, maximum
  - likelihood, *see* likelihood, maximum
- probability principle, 95, 149, 532
- MDL
  - algorithmic, *see* MDL, idealized
  - and cheating, 571
  - and MML, 558
  - and prediction, 578
  - and traditional statistics, 469, 482
  - application, 38, 68, 592
  - consistency, *see* consistency
  - convergence rate, *see* convergence rate, MDL
  - criterion, 553
  - crude, 133, 389
  - decision theory, 594
  - expectation-based, 21, 407, 504, 530
  - idealized, 11, 30, 546, 570
  - individual-sequence, 21, 407, 484, 504, 523, 530
  - justification, 152
  - meta-principle, 160
  - model selection, 409
    - and null hypothesis testing, 413
  - Bayesian interpretation, 418
  - compression
    - interpretation, 415
  - consistency, 415, 505, 510
  - consistency theorem, 506
  - convergence rate, 522
  - counting interpretation, 416
  - definition, 427
  - discussion, 448
  - four interpretations, 415

- general, 410, **420**
- gMDL, 443
- iMDL, 448
- infinitely many models, 422
- Liang and Barron, 446
- linear regression, **438**
- nonparametric, 508
- plug-in codes, 431
- prequential interpretation, 419
- refined, **426**
- RNML, 439
- simple, 411
- model selection-based estimator, 509
- nonparametric regression, 448
- nonprobabilistic, 20
- parameter estimation, 483
  - approximation, 486
  - vs. LML, 487
- philosophy, 199, 436, 487, 595
- predictive, *see* prequential MDL
- prequential, *see* prequential MDL
- principle, 595
- probabilistic, 20
- problems, 593
- refined, 17, 406
- two-part code, *see* two-part MDL
- vs. Bayes, 533
- vs. ML, 147
- mean, 48
- mean-value parameterization, *see* parameterization, mean-value
- meaningful information, *see* information, meaningful
- medical testing, 75
- Mercer kernel, 400
- Mercury, 545
- messages, 83, 172
- meta-Bayes code, 374, 379, 389, 406, 471
- meta-MDL principle, 160
- meta-two-part code, 303, 373, 379, 389, 406, 409, 443, 471, 505
- metauniversal code, 296, **301**
  - problems, 308
- meteorology, 572
- minimal representation, 601
- minimax
  - analysis, 451
  - codelength, *see* codelength, minimax
  - convergence rate, *see* convergence rate, minimax
  - nonparametrics, *see* redundancy, nonparametric
  - optimal histogram code, 382
  - regret, *see* regret, minimax theorem, 644
- minimum, 44
  - message length, 493, 555
    - and MDL, 558
    - strict, 497, 556
  - relative entropy, *see* entropy, minimum relative, 640
- misspecification, 265, 502, 504, 530, 590, 593
- mistake probability, 182
- mixability, 576, 590

- mixture, 576
  - Bayesian, 77
  - family, *see* model, mixture
  - model, *see* model, mixture
- ML, *see* likelihood, maximum
- MML, *see* minimum message length
- model, 13, 70
  - Bernoulli, 58, 65, 66, 71, 118, 122, 233, 461, 491, 495, 602, 604, 607, 615, 629, 637
  - and Jeffreys, 236
  - complexity, 212
  - class, 13, 70
    - CUP, 65, 370, 372, 409, 502, 505, 556
    - fully nonparametric, 370
    - Gaussian process, *see* Gaussian process
    - NCUP, 372
    - nonparametric, 369, 468, 471, 492, 503
    - probabilistic, 60, 70
  - complexity, 30, 180, 412
    - Bernoulli, 188, 227
    - multinomial, 228
    - nondecreasing, 189
    - Poisson, 189
  - conditional, 62
  - cost, 412
  - crazy Bernoulli, 217
  - exponential family, *see* exponential family
  - Fechner's, 23, 417, 505
  - functional form, 216
  - Gaussian process, *see* Gaussian process
  - geometric, 428, 603, 605, 607, 628
    - complexity, 299
  - histogram, *see* histogram
  - i.i.d., 62, 433
  - linear, *see* linear model, 566
  - Markov, 60, 65, 66, 71, 133, 185, 423, 513, 617, 618
    - code design, 158
    - hidden, 427, 437
  - meta-selection, 555
  - mixture, 68, 427
  - multinomial, 59, 262, 378, 428, 602
    - complexity, 228
    - histogram, 228
  - naive Bayes, 562
  - nested, 24, 60
  - non-nested, 23, 224, 431
  - normal, 65, 67, 264, 298, 428, 461, 602, 605, 626, 631, 640
    - and linear, 338, 349
    - complexity, 298
    - Fisher information, 300
    - mixture, 68
    - RNML, 306
  - parametric, 57, 369, 375, 483, 519
    - Poisson, 65, 189, 428, 433, 534, 602, 604
      - complexity, 298
  - probabilistic, 57, 70
  - rich, 180
  - selection, 70, 406, 409
    - by cross-validation, *see* cross-validation
  - CNML, 446
  - consistency, 74



- criterion, 509
  - MDL, *see* MDL, model
  - selection
  - meta, 555
  - nested, 141
  - non-nested, 23, 141
  - prequential, *see*
  - prequential model
  - selection
  - warnings, 435
- Stevens's, 23, 417, 505
- time series, *see* time series
- trivial, 414
- true, 29
- universal, *see* universal
- model
- model selection-based estimator, 509
- multinomial, *see* model, multinomial
- multivariate normal distribution, *see* distribution, multivariate, normal
- mutual singularity, 154
  
- naive Bayes, 562
- nats, 110
- natural numbers, 42
- Nature, 644
- NCUP model class, 372
- Neal, R., 542
- nearest neighbor, 371
- nested, *see* model, nested
- neural networks, 72, 583
- Neyman-Scott problem, 527
- NMAP, 311
- nMDL, 439
- NML, 181, 183, 411, 412, 422, 513, 531
- conditional, *see* conditional NML
- generalized, *see* luckiness NML-1
- linear model, *see* linear regression, NML
- luckiness, *see* luckiness NML
- undefined, 183, 296, 298
- geometric model, 299
- normal model, 298
- Poisson, 298
- no-hypercompression inequality, *see* inequality, no hypercompression
- noise, 416, 449, 575
- non-nested, *see* model, non-nested
- nondegenerate data, 142
- nondifferentiable density, 526
- nonineccsi sequences, 238
- noninformative prior, *see* prior distribution, noninformative
- nonmixability, 590
- nonparametric, 65, 369, 525
- Bayes, 543
- density estimation, 470
- model class, *see* model class, nonparametric
- model selection, 508
- rate, 520
- redundancy, *see* redundancy, nonparametric
- regression, *see* regression, nonparametric
- statistics, 35
- nonpredictive description, 492
- nonprequential, 196, 388
- nonuniform

- convergence rate, 516
  - universal model, 185
- normal distribution, *see* model, normal
- normal equations, 342
- normality rules, 357
- normalized
  - MAP, 311
  - maximum likelihood, *see* NML
  - NML
- null
  - hypothesis, 412
  - model, 444, 514, 573
- objective
  - Bayes, 546
  - priors, 547
- observation, 6, 72
- Occam factor, 539
- Occam's razor, 29, 408, 539
  - as a methodology, 35
  - bound, 585
  - hidden, 542
- on-line decoding, 196
- one-to-one, 42
- online decoding, 135
- open problem, 264, 267, 268, 289, 299, 314, 323, 383, 413, 421, 451, 472, 490, 492, 511, 514, 522, 553, 569, 583
- order
  - dependence, 434
  - notation, 45, 83
- order notation, 83
- orthodox statistics, *see* statistics, frequentist
- out-model estimator, *see* estimator, out-model
- outlier, 451
- output vector, 336
- overfitting, 24, 133, 134, 145, 346, 416, 468, 490
- PAC-Bayes, 585
  - and luckiness, 586
  - and square root, 589
  - and SRM, 587
- PAC-MDL, 585
- packing number, 222, 281
- paradigm
  - Bayesian, 531
  - frequentist, 524
- parameter, 57, 284
  - estimation, *see* estimation, parameter, 141
  - precision, 283
  - space, 57
- parameterization, 57
  - canonical, 67, 211, 604, 611
  - dependence, 159, 307, 556
  - independence, 159, 211, 215, 485, 494
  - mean-value, 604, 611
  - properties, 615
  - uniform, 159
- parametric, 65
  - complexity, *see* complexity model, *see* model, parametric rate, 519
- partial
  - code, 80
  - function, 47
  - random variable, 47
- partition function, 601
- pattern recognition, 69, 72
- penalized least-squares, *see* least-squares, penalized

- permutation invariance, 328, 339, 435
- phase transition, 290
- Pitman-Koopman-Darmois, 604
- PLS, 448–450
- plug-in distribution, *see* universal model, plug-in
- point hypothesis, 406, 476
- point null hypothesis, 412
  - and luckiness, 421
- Poisson family, *see* model, Poisson
- polynomial, 70, 337, 341, 348, 391, 416, 437, 438
- positive (semi-) definite, *see* matrix, positive definite
- positive definite kernel, 393
- posterior
  - concentration, 512
  - convergence, 511
  - predictive, 78
- posterior distribution, 73, 75, 532
  - linear model, 355
  - meaning, 537
- pragmatic prior, *see* prior
  - distribution, pragmatic
- precision, 283
- prediction, 70, 191, 406, **459**, 574
  - error, 73
  - individual-sequence, 573
  - probabilistic, 191
  - strategy, 190, 462
  - weather, *see* weather
    - forecasting
  - with expert advice, 574
- predictive
  - distribution, *see* distribution, predictive
  - least-squares, *see* PLS
  - MDL, *see* sequential MDL
- prefix
  - codes, 83
  - description methods, 83
- prequential, 364, 368, 388
  - analysis, 562
  - interpretation, 419
  - model selection, 419
  - parametric MDL, 484
  - plug in model, *see* universal model, plug-in
  - principle, 528
    - infinite horizon, 563
  - universal model, *see* universal model, prequential
- prequential MDL, 198, 460
  - consistency, 465, 502
  - consistency theorem, 467
  - estimation, 483
  - estimator, 461, 472
  - individual-sequence
    - estimator, 464
- prequentialization, 196
- principal axis theorem, 347
- prior, *see* prior distribution, 175
- prior density, *see* prior
  - distribution
- prior distribution, 73, 74, 231, 532, 569, 585
  - g*-prior, 444
  - and luckiness, 534
  - beta, 258
  - canonical, 289, 292
  - compatible, 232
  - Diaconis-Freedman, 543
  - flat, 539
  - Gaussian, 354, 539
  - improper, 317, 420
  - informative, 420

- Jeffreys, 234, 258, 289, 418, 420, 447, 461, 513, 547, 570
  - and Bernoulli, 236
  - and boundary, 237, 244
  - and CNML, 323, 368
  - and distinguishability, 236
  - geometric, 300
  - linear model, 359
  - normal family, 299
  - Poisson, 300
  - Takeuchi-Barron
    - modification, 242
    - tilted, 312
    - undefined, 239, **299**
    - Xie-Barron modification, 239
  - least informative, 235
  - noninformative, 359
  - objective, 547
  - pragmatic, 534
  - reasonable, 233
  - reference, 547
  - subjective, 556
  - uniform, 258, 285, 379
  - universal, integers, 101, 186, 423
  - weakly informative, 539
- probabilistic source, 53, 69, 464, 575
  - and exponential families, 617
  - conditional, 62, 391, 588, 617, 619
- probability
  - 1-statement, 55
  - as codelength, 96
  - chain rule, 54, 465
  - conditional, 54
  - defective, **94**
  - density function, 46
  - mass function, 46
  - posterior, *see* posterior distribution
  - prior, *see* prior distribution
  - sequential decomposition, 54, 60, 465
- probably approximately correct, 586
- product distribution, 51
- projection, 342
- quadratic form, 344
- quasi-uniform, *see* code, quasi-uniform
- Rényi divergence, 478, 491, 512, 645
  - interpretation, 649
  - unnormalized, 645
- Rademacher complexity, 583
- radial basis function, 393
- random
  - process, 53
  - sequence, 103
  - variable, 47
    - dependent, 51
    - independent, 51
  - vector, 47
- randomization, 432
- rate, *see* convergence rate
- RBF kernel, 393
- real numbers, 41
- receiver, 83, 172
- recipe, 504, 524
- redundancy, 177
  - almost sure, 199, 246, 265
  - conditional, 325, 446
  - CUP codes, 384

- expected, 199, 245, 325, 455, 467, 471
    - minimax, 247
  - histogram codes, 380
  - individual-sequence, 199
  - minimax, 202
    - expected, 247
  - nonparametric, 383, 387
  - relative, 265
  - stochastic, 199
  - terminology, 201
  - worst-case, 177
- reference code, 643
- refined MDL, 17, 406
- region of distinguishability, 220
- regression, 25, 63, 72, 336
  - Gaussian process, *see* Gaussian process
  - linear, *see* linear regression
  - nonparametric, 448, 472
  - polynomial, *see* polynomial
  - ridge, 347
  - robust, 451
- regressor, 63, 336
- regret, **179**, 210
  - almost sure, 199, 244, 265
  - and maximum entropy, 568
  - asymptotic
    - Bayesian, 232
    - CNML, 323
    - linear model, 366
    - LNML, 312
    - NML, 211
    - plug-in, 260
    - two-part, 273
  - Bayes and linear model, 364
  - beyond log loss, 577
  - CNML and linear model, 365
  - conditional, 365
  - expected, 199, 202, 245, 260
  - Gaussian process, 397
  - individual-sequence, 199, 202, 273
  - linear model, 350, 363
  - LNML and linear model, 364
  - luckiness, *see* luckiness regret
  - metauniversal coding, 304
  - minimax, 182, 202, 208, 293
    - exponentiated, 216
    - unrestricted, 202
  - nonexponential families, 241
  - stochastic, 199
  - worst-case, 180
- regular histogram, 377
- regularity, 73, 103, 595
- relation, 82
- relations between divergences, 517
- relative
  - entropy, 103
  - redundancy, 265
- renormalized complexity, *see* RNML
- reparameterization, 67, 159
- reproducing kernel Hilbert space, 396
- residual sum of squares, 343
- resolvability, 483
- $\rho$ -divergence, 512
- ridge regression, 347
- Riemann zeta function, 243
- risk, 466, 515, 519, 528
  - cumulative, 467
  - empirical, 580
  - Hellinger, *see* Hellinger risk
  - in learning theory, 579
  - KL, *see* KL risk
- Rissanen lower bound, 454

- Rissanen renormalization, *see* RNML
- Rissanen, J., xxiv, 26
- RKHS, 396
- RNML, **306**, 438, 439, 449
  - as LNML-1, 443
- road ahead, 592
- robust regression, 451
- robustness property, *see*
  - exponential family,
  - robustness property
- rotation, 277
- rule of succession, 258
  
- sacrifice, 320
- saddle point integration, 239
- safe, 535
- sample, 69, 72
  - empty, 53
  - size
    - unknown, 134
  - space, 6, **46**
    - continuous, 46
    - discrete, 46
    - virtual, 258
- sanity check, 29, 413, 514, 525
- Sanov's theorem, 127, 636
- Sauer's lemma, 583
- score, 116
  - logarithmic, *see* loss,
  - logarithmic
- selection-of-variables, 25
- semiprequential, 135, 196, 379
- sender, 83, 172
- separation, 416
- sequence
  - gappy, 473
  - infinite, 53
  - random, 103
  
- sequential decomposition
  - property, *see* probability,
  - sequential decomposition
- set, 41
  - bounded, 44
  - compact, 44
- Shannon, C.E., 95
- Shannon-Fano code, *see* code, Shannon-Fano
- Shtarkov code, *see* NML
- significance level, 412, 421
- simplex, 42, 68
- simplicity, 29
- singular coding system, 80
- singularity
  - absolute, 507
  - mutual, 154
- slack function, *see* luckiness function
- SMML, *see* minimum message length, strict
- smooth, 65
- Solomonoff's approach, 8, 546
- Solomonoff, R.J., 8
- source symbol, 80
- space of observations, *see* sample space
- speech recognition, 437
- squared
  - error, *see* loss, squared
  - loss, *see* loss, squared
- SRM, 581
  - and PAC-Bayes, 587
- start-up problems, 198, 261, 430
- statistic, 284
- statistical learning theory, *see* learning theory, statistical

- statistical risk, *see* risk
- Statistician, 644
- statistics, 69, 72
  - Bayesian, 26, 73, 74, 531
    - objective, 546
    - subjective, 544, 573
  - frequentist, 73, 524, 591
  - nonparametric, 35
  - orthodox, *see* statistics, frequentist
  - sufficient, 286, 301, 441, 568, 603, 630
    - Kolmogorov, 571
    - traditional, *see* traditional statistics
- Stein's lemma, 221
- Stevens's model, *see* model, Stevens's
- Stirling's approximation, 127, 128
- stochastic
  - complexity, 412
    - extended, 578
  - universality
    - of Bayes, 244
- strong law of large numbers, *see* law of large numbers
- structural risk minimization, 581
- structure, 72, 416
  - function, 571
- subjective Bayes, 544, 573
- subjectivity, 160, 533, 548
- sufficient statistics, *see* statistics, sufficient
- sum of squares, 341
  - fitted, 344, 444
  - residual, 343, 450
- superefficiency, 455, 527
- supervised learning, 72
- support, 47
- support vector machine, 401, 583
- supremum, 44
- surface area, 218
- SVM, *see* support vector machine
- Sweden, 75
- symbol, 79
- Taylor expansion, *see* KL divergence, Taylor expansion
- terminology, 69
- test set, 72, 565
- theorem, 644
- $\Theta_0$ -sequence, 210
- time series, 269, 368, 508
  - computation, 431
- topology, 44
- tradeoff, 135, 411, 415, 581
- traditional statistics, 469, 482
- training set, 72, 73, 565
- transpose, 43
- triangle inequality, 480
- trigonometric functions, 337
- trivial model, 414
- true, 525
- two-part code, 174, 183
  - behavior, 142
  - computation, 430
  - conditional, 284
    - modified, 292
  - countable, 184
  - crude, 274
  - design, 152, 157
  - discretization, 272
  - incomplete, 274, 291
  - MDL, *see* two-part MDL
  - meta, *see* meta-two-part code
  - regret
    - asymptotic, 273

- sample size-dependent, 196, 272
- simplistic, 139
- simplistic, for Markov chains, 138
- stupid, 157
- two-part MDL, 132, 136, 476, 477, 512, 590
  - approximating, 150
  - code, *see* two-part code
  - computing, 150
  - consistency, 143, 153, 157, 502
  - consistency theorem, 478
  - convergence rate, 478
  - estimator, 645
  - for Markov chains, 133
  - parameter estimation, 485
- type, 127
- umbral calculus, 228
- underfitting, 143, 147
- uniform
  - central limit theorem, 289
  - code, *see* code, uniform
  - convergence, 232
  - convergence rate, 516
  - law of large numbers, 580
  - luckiness, 485
- uniformly universal model, 183
- union bound, 55, 164, 637
- unit simplex, 42, 68
- universal
  - code, *see* universal model
  - computer language, 8
  - Turing machine, 8
- universal code
  - computation, 428
- universal model, 172, 175, **178**
  - adaptive, 193
- Bayesian, 175, 183, 192, 231, 443, 452, 519
  - approximation, 197
  - Bernoulli, 233, 258, 285
  - boundary, 234
  - computation, 429
  - countable, 184
  - for normal family, 264
  - histogram, 378
  - minimax, 234
  - multinomial, 262
  - regret, 232
  - stochastic
    - regret/redundancy, 244
    - surprise, 472
- Césaro, 474
- conditional, *see* code, conditional
- countable  $\mathcal{M}$ , 184
- CUP, 370, 505, 509
  - Bayes, *see* meta-Bayes code, 379
  - redundancy, 384
  - two-part, *see* meta-two-part code, 379
- finite, 178
- Gaussian process, 396, 542
- Liang-Barron, *see* code, conditional, Liang-Barron
- linear regression, *see* linear regression, universal model
- luckiness, *see* luckiness universal model
- minimax, 180, 181
- NML, *see* NML histogram, 378
- nonuniform, 185, **186**, 187



- optimal, 180, 181, 202
- plug-in, 197, 462
  - asymptotics, 260
  - computation, 430
  - for exponential families, 260
  - for regression, 448
  - model selection, 431
  - multinomial, 262
  - redundancy, 265
  - regret, 260, 265
  - start-up problem, 261
- prequential, 190, 194, 195, 419, 461, 493
- semiprequential, 196
- stochastic, 201
- top-level, 406
- two-part, *see* two-part code
- uniform, 187
- uniformly, **183**
- universality
  - almost sure, 199
  - expected, 199
  - individual-sequence, 199
  - stochastic, 199
- utility, 532, 535, 540
  
- variance, 48
- VC-dimension, 582
- vector, 43
- virtual data, 347
- volume, 453
  - and complexity, *see* complexity, and volume
  - relative to divergence, 222
- Vovk, V., 576
- Vulcan, 545
  
- Wallace, C., 556
- Wallace-Freeman estimator, 497, 560
- warning, 523
- wavelets, 337
- weak
  - frequentist paradigm, 526
  - prequential principle, 528, 563
- weather forecasting, 529, 572
- Webb, G., 30
- weighted averaging, 26
- worst case
  - vs. average case, 451
- worst-case codelength, *see* codelength, minimax
- WPP, *see* weak prequential principle
- wrong-yet-useful principle, 33
  
- zero prior problem, 545
- zero-sum game, 637
- zeta function, 243