Chapter 1

INTRODUCTION

On the basis of the laws of conservation of mass, momentum, and energy and Maxwell's electromagnetic equations, several types of discontinuities can exist in ideal electrically conducting fluids in the presence of magnetic fields.[1] The discontinuities characterized by the condition that both the mass flow and density change across them are different from zero are called shock waves. This monograph is primarily concerned with one basic question related to these so-called "magnetohydrodynamic" shock waves: Can one expect to find them in nature? This question is too broad and complex to be treated exhaustively at the present time; hence, to make the subject of this monograph tractable, we shall restrict it in two ways: (1) by considering the shock wave as a local phenomenon away from physical boundaries and (2) by considering only those shock waves lying in a specified region of the phase space of the temperature, density, and magnetic-field variables.

We take into account the first restriction by assuming that the steady-state shock wave lies in a plane of infinite extent, with the properties of the flow varying only in the direction normal to that plane. This assumption, which says essentially that the local radius of curvature of the shock wave is large compared with its thickness, restricts the discussion to local properties of shock waves and leaves out of account the problem of stability in the large.

The second restriction is defined by the following assumptions;

1. The density of the gas is high enough and the magnetic field low enough so that the shock wave is collision-dominated; i.e., the collision frequency is large compared with the cyclotron frequency of the electrons. Use of this assumption is justified by the fact that there is a range of shock waves of experimental interest for which it is valid. It is manifested in the fact that the gas pressure and electrical conductivity are assumed to be scalar quantities, Hall currents are neglected, and the Navier-Stokes approximation is used for the pressure tensor and heat-flux vector.

2. The temperature and shock velocity are both low enough so that relativistic effects are unimportant, and the radiation

pressure and energy density are both small compared with the corresponding gas and magnetic quantities. These assumptions are justified both because shock waves in the broad region in which they are valid are not yet completely understood and because inclusion of these effects would make the theory considerably more difficult.

Moreover, in some of the discussions in Chapter 2 and in Chapters 5 and 6 it is assumed that the perfect-gas law holds. The reason for this assumption is that it enables one to visualize the form of certain integral surfaces and thus simplifies the proofs of the existence of shock waves. These existence proofs are of such a nature, however, that there is good reason to believe that the results will be valid for a wider range of gases, and in any case, an analytical framework has been established which will simplify the analysis of shock waves in real gases.

The study of magnetohydrodynamic shock waves was begun in 1950 with the paper of de Hoffmann and Teller.[2] Since then, continued interest inspired by astrophysics, by the possibilities of thermonuclear power, by flight at the outer edges of the atmosphere, etc., has produced many papers describing shock wave properties. The basic properties of magnetohydrodynamic shock waves as determined by the conservation laws (the Rankine-Hugoniot relations) have been developed further by Friedrichs,[3] Helfer,[4] Lüst,[5,6] Bazer and Ericson,[7] Napolitano,[8] and others, and they are now well understood; but the more complex question of their existence in nature has yet to be exhaustively treated.

The first efforts in this direction are due to several Russian authors, whose works are acknowledged and discussed in detail in Chapter 3. In their papers, the shock wave is considered to be a plane discontinuity in a perfect fluid, and the problem posed is to determine the stability of this configuration with respect to disintegration resulting from small disturbances in the flow. They have found quite simple criteria for the stability boundaries of shock waves and have also computed possible modes of disintegration of unstable shock waves. The main improvement that could be made on their work would be to provide greater physical insight into the conclusions reached.

There is another problem of fundamental importance to the present study: Can the nonlinear wave steepening processes which produce shock waves balance the diffusive processes in the fluid to such an extent that a steady-state shock wave will be maintained? This is the problem of the existence of the steady-state shock layer. It has had a more international history, reviewed at the beginning of Chapter 4, and should really come before study of the effects of small disturbances on shock waves. It is treated second in this monograph only because it is more difficult, requiring much more detailed knowledge of the equations of the flow, since all dissipative effects must be included. It has been found, in fact, that the basic equations of magnetohydrodynamics must be rederived for the shock-layer problem because the occurrence of finite gradients over distances of the order of the mean free path causes certain terms, neglected in the derivation of the usual macroscopic equations of magnetohydrodynamics, to become important. All previous papers on collision-dominated magnetohydrodynamic shock waves, to the author's knowledge, have used the latter equations; hence, the results are open to question.

The problems treated in the present monograph, as mentioned above, are: (1) the stability of shock waves considered as discontinuities in a perfect fluid and (2) the existence of the steady-state shock layer. It must be mentioned that both of these problems have been solved for ordinary gas dynamics, and the solutions have furnished essential background material and inspiration for the various attacks on the corresponding problems in magnetohydrodynamics. These problems are discussed in the introductions to Chapters 3 and 4, respectively, and also, for example, by Hayes.[9] It has been found by Gilbarg,[10] and in more detail by Gilbarg and Paolucci,[11] that normal gas-dynamic shock waves have a steady-state structure if and only if the flow upstream of the shock wave is supersonic and the flow downstream is subsonic. Under the same conditions, it has been found by Burgers[12] that these shock waves are also stable with respect to small disturbances in the flow. The situation in magnetohydrodynamics is far more complex; it is found there, for example, that certain shock waves which possess steady-state structure are not stable with respect to arbitrary small disturbances.

At each stage of development of the theory of this monograph, the basic equations are given only in the generality required at that stage, in order to avoid needless complication. Throughout this study, MKS units are used.

In Chapter 2, the magnetohydrodynamic Rankine-Hugoniot relations are briefly reviewed under the assumption that the reader has some familiarity with magnetohydrodynamic shock waves, such as can be obtained, for example, from Reference 1. The development leans toward those concepts which will be useful in later chapters, and has two novel features. One is the representation of the end states of shock waves as the intersection of three surfaces in the phase space of temperature, specific volume, and the component of the magnetic field parallel to the plane of the shock wave; the other is a derivation and presentation of the shock-wave adiabatic curve in a particularly illuminating form.

In Chapter 3, the existing literature on the problem of the reaction of magnetohydrodynamic shock waves to arbitrary small-flow perturbations is reviewed, and the ideas contained therein are somewhat augmented. This is the first problem mentioned above, and in it the shock wave is treated as a plane discontinuity in an infinite domain of perfect fluid. The concept of group velocity, essential in one part of the proof, is discussed exhaustively for magnetohydrodynamic and magnetoacoustic waves, and some arguments which give insight into the physical causes of instability of certain shock waves are presented.

In Chapter 4, a new derivation of the macroscopic equations of magnetohydrodynamics, valid for collision-dominated non-relativistic shock waves, is presented. These equations, based on the kinetic theory of fully ionized gases, have been closed by expressing the pressure tensor and heat-flux vector in terms of lower-order dependent variables by use of the phenomenological Navier-Stokes approximation. In the heat-flux vector, the cross-coupling effect of electromagnetic forces is included. By means of a dimensional analysis of the one-dimensional steady-state equations, it is deduced that the current-inertia terms in the generalized Ohm's law and the electric-force term in the momentum equation are by no means negligible within the shock wave. Each of these terms can cause important modifications in the structure of the shock wave.

In Chapters 5 and 6, the existence and uniqueness properties of shock waves are deduced. The analysis of these chapters is based for the most part on the work of Germain;[13] the inclusion of the current-inertia effect, however, has made the present analysis considerably more complex, but at the same time more nearly correct. Germain found that the methods he and others had used to prove existence and uniqueness of the shock layer were not sufficiently powerful to say much in general about slow and intermediate shock waves; inclusion of the current-inertia effect has made those methods even less conclusive. It has been found, however, that there are powerful arguments related to the topological behavior of integral curves between singular points which can be used to arrive at positive conclusions in all cases. In Chapter 5, the existence and uniqueness properties of shock waves which satisfy the equations derived in Chapter 4 are studied without making further approximations. The problem presented there is five-dimensional and as a result is difficult to visualize; hence, a number of reduced cases in two and three dimensions are solved in Chapter 6 to obtain a greater understanding of the topological behavior of the integral curves of the five-dimensional problem and also to study the effects of current inertia in a simpler context. Moreover, the special case in which the magnetic field has no component normal to the plane of the shock wave, and the cases of "switch-on" and "switch-off" shock waves, are given special consideration. As a byproduct of the above analysis, the qualitative profile of the shock layer becomes clear, and a formula for the thickness of the shock layer results. The shock thickness based upon this formula is not sensitive to local variations in the shock profile and can be calculated numerically with relative ease.

In Chapter 7, results and conclusions obtained from the entire study are discussed. Several suggestions are made for further theoretical work and for means by which experimental confirmation of the principal results of the theory can be obtained.