1

Basic Concepts

1.1 Introduction

In this chapter we will develop in the simplest manner possible the central concepts of the theories we will be using in this book. It is actually convenient to start the exposition with the polar case of a Walrasian market-clearing model. This will allow us to identify a number of important elements that are obviously missing if one wants to extend the analysis to more general environments. We will then see how the concepts can be generalized to nonclearing markets and imperfect competition. We will look at how transactions are organized when demands and supplies do not match (section 1.3), how quantity signals are formed in the process (section 1.4), how demands and supplies themselves react to market imbalances (section 1.5), and finally how prices are determined in such an imperfectly competitive environment (section 1.6). So before venturing any further, let us scrutinize the Walrasian model in order to precisely identify the missing elements.

1.2 Walrasian Theory: The Missing Parts

We will first briefly describe the characteristics of the Walrasian model, and then outline how it has to be generalized to deal with nonclearing markets and imperfect competition.

1.2.1   The Walrasian Paradigm

Consider an economy where goods indexed by $h = 1, \ldots, \ell$ are exchanged among agents indexed by $i = 1, \ldots, n$. Call p_h the price of good h, and p the price vector:

$$p = (p_1, \ldots, p_h, \ldots, p_\ell) \tag{1}$$

All private agents receive the same price signal, the vector of prices p, and assume that they will be able to exchange whatever they want at this price system (a belief which will actually be validated ex post). Denote by d_{ih} and s_{ih} the demand and supply of good h by agent i. Each agent i sends to the market his Walrasian demands and supplies, obtained through maximization of his own objective function. Of course, demand and supply depend on the price system. So we denote them as

$$d_{ih} = d_{ih}(p), \quad s_{ih} = s_{ih}(p) \tag{2}$$

In this economy there is an "auctioneer" who changes the price system by some unspecified mechanism (the famous "tâtonnement" process) until a Walrasian equilibrium price vector p^* is reached. This equilibrium price p^* is characterized by the equality of aggregate demand and aggregate supply in all markets:

$$\sum_{i=1}^{n} d_{ih}(p^*) = \sum_{i=1}^{n} s_{ih}(p^*) \qquad \text{for all } h = 1, \ldots, \ell \tag{3}$$

Transactions are equal to the demands and supplies at this price system. No quantity constraint is experienced by any agent, since demands and supplies match on all markets.

1.2.2   The Missing Elements

The Walrasian story is a good description of reality for the few real world markets, such as the stock market which inspired Walras, where the equality between demand and supply is ensured institutionally by an actual auctioneer. For all other markets with no auctioneer in attendance, the Walrasian story is clearly incomplete, something pointed out by Arrow (1959) himself. Two

important characteristics of the Walrasian model deserve to be stressed here:

• All agents receive price signals (actually the *same* price vector) but no agent actually sends any price signal, as price setting is left to the implicit Walrasian auctioneer.
• Though all agents send quantity signals (their Walrasian demands and supplies), no agent makes any use of the quantity signals available on the market.

Our purpose will be to fill these gaps, and to build a consistent theory of the functioning of decentralized market economies when no auctioneer is present, market clearing is not axiomatically assumed, and quantity signals have to be considered in addition to price signals.

1.2.3 The Generalization

The consequences of abandoning the assumption that all markets clear at all times turn out to be quite far-reaching:

• Transactions cannot be all equal to demands and supplies expressed on markets. As a consequence some agents experience rationing, and quantity signals are formed in addition to price signals.
• Demands and supplies must be substantially modified on account of these quantity signals. Walrasian demand, which takes only prices into account, must be replaced by a more general *effective demand,* which takes into account *both* price and quantity signals.
• Price theory must also be amended in a way that integrates the possibility of nonclearing markets, the presence of quantity signals, and makes agents themselves responsible for rational price-making decisions. As we will see, the resulting framework is reminiscent of the traditional theories of imperfect competition.

Full general equilibrium concepts embedding these features will be developed in chapter 3. Later in this chapter we will study in a simpler setting the essential microeconomic elements of the theory. Notably we will be concerned with quantity signals, demand-supply theory, and price setting. Before that we must clarify which institutional structure we are dealing with.

Goods	1	2	3
1	▨	×	×
2	×	▨	×
3	×	×	▨

(a)

Goods	1	2	3	M
1	▨			×
2		▨		×
3			▨	×
M	×	×	×	▨

(b)

Figure 1.1   Barter and money

1.2.4   The Organization of Markets: Monetary Exchange

A relatively neglected issue in Walrasian general equilibrium models is the problem of the actual institutions of exchange. In his initial model Walras (1874) referred to a barter economy, with a market for each pair of goods. Other authors have assumed that all exchanges are monetary.

The difference between these two systems appears quite clearly in figure 1.1, adapted from Clower (1967), which shows which markets for various pairs of goods may be open or closed in each system. In the figure the existence of a market for the exchange between two goods is indicated by a cross in the corresponding box.[1] In a barter economy (panel a) a market exists for every pair of goods. So, for ℓ goods, there are $\ell(\ell - 1)/2$ markets. In a monetary economy, on the contrary (panel b), all exchanges must go through a medium of exchange, money, which we assume to be an additional good denoted M, so that there are ℓ markets.

Clearly, for a large number of goods, the cost of operating a barter structure would be prohibitive, and this explains why barter is almost never observed nowadays. Thus, for evident reasons of realism, we will work within the framework of a monetary economy.[2] Money is the medium of exchange. It is also the numéraire and a reserve of value. There are ℓ nonmonetary goods indexed by $h = 1, \ldots, \ell$ in addition to money. The money price of good h is p_h. An agent i may make a purchase d_{ih}, for which he pays $p_h d_{ih}$ units of money, or a sale s_{ih}, for which he receives $p_h s_{ih}$ units of money.

1 Since there is no such thing as the market for a good against itself, the boxes along the diagonal have been eliminated.

2 The theories presented here have been developed in nonmonetary exchange structures as well, but the formalization is both less realistic and more complex. See, in particular, Bénassy (1975b, 1982).

1.3 Transactions in Nonclearing Markets

A most important element of the theory is obviously to show how transactions can occur in a nonclearing market and how quantity signals are generated in the decentralized trading process.

1.3.1 Demands and Transactions

In nonmarket-clearing models we must make an important distinction that by nature is not made in market-clearing models: that between demands and supplies, on the one hand, and the resulting transactions, on the other. We distinguish them by different notations. Demands and supplies, denoted $\tilde{d}_{ih}$ and $\tilde{s}_{ih}$, are signals sent by each agent to the market (i.e., to the other agents) before an exchange takes place. They represent, as a first approximation, the exchanges agents wish to carry, and so do not necessarily match on the market. Transactions, namely purchases and sales of goods, are denoted d_{ih}^* and s_{ih}^*. They are the actual exchanges carried on markets, and as such they are subject to all traditional accounting identities. In particular, in each market h, aggregate purchases must be equal to aggregate sales. With n agents in the economy, this is written

$$D_h^* = \sum_{i=1}^n d_{ih}^* = \sum_{i=1}^n s_{ih}^* = S_h^* \tag{4}$$

As we indicated earlier, no such equality holds a priori for demands and supplies. We will study in this section the functioning of a market for a particular good h, where a price p_h has already been quoted. Since everything pertains to the same market, we can suppress the index h for the rest of this chapter.

1.3.2 Rationing Schemes

Since the price is not necessarily a market-clearing one, we may have

$$\tilde{D} = \sum_{i=1}^n \tilde{d}_i \neq \sum_{i=1}^n \tilde{s}_i = \tilde{S} \tag{5}$$

From any such set of possibly inconsistent demands and supplies, the exchange process must generate consistent transactions satisfying equation (4). Evidently, as soon as $\tilde{D} \neq \tilde{S}$, some demands and supplies cannot be satisfied in the exchange process and some agents must be rationed. In real life this is done

through a variety of procedures, such as uniform rationing, queueing, proportional rationing, and priority systems, depending on the particular organization of each market. We will call rationing scheme the mathematical representation of the exchange process in the market being considered. This rationing scheme gives the transactions of each agent as a function of the demands and supplies of all agents present in that market (a general formalization appears in chapter 3). Before describing the various properties that rationing schemes may have, we give a simple example.

1.3.3   Example: A Queue

In a queueing system the demanders (or the suppliers) are ranked in a predetermined order and served according to that order. Let there be $n - 1$ demanders ranked in the order $i = 1, \ldots, n - 1$, each having a demand $\tilde{d}_i$, and a supplier, indexed by n, who supplies $\tilde{s}_n$. When the turn of demander i comes, the maximum quantity he can obtain is what demanders before him (i.e., agents $j < i$) have not taken, namely

$$\tilde{s}_n - \sum_{j < i} d_j^* = \max\left(0, \tilde{s}_n - \sum_{j < i} \tilde{d}_j\right) \tag{6}$$

The level of his purchase is simply the minimum of this quantity and his demand:

$$d_i^* = \min\left[\tilde{d}_i, \max\left(0, \tilde{s}_n - \sum_{j < i} \tilde{d}_j\right)\right] \tag{7}$$

As for the supplier, he sells the minimum of his supply and of total demand:

$$s_n^* = \min\left(\tilde{s}_n, \sum_{i=1}^{n-1} \tilde{d}_i\right) \tag{8}$$

It is easy to verify that whatever the demands and supplies, aggregate purchases and sales always match.

   We now turn to study successively a number of properties of rationing schemes.

1.3.4   Voluntary Exchange and Market Efficiency

The first property we consider is a very natural one in a free market economy, namely that of *voluntary exchange,* according to which no agent can be forced

to purchase more than he demands, or to sell more than he supplies. This will
be expressed by

$$d_i^* \leq \tilde{d}_i \tag{9}$$

$$s_i^* \leq \tilde{s}_i \tag{10}$$

Such a condition is quite natural and actually verified in most markets, ex-
cept maybe for some labor markets regulated by more complex contractual
arrangements. It is clearly satisfied by the queueing example above.

Under voluntary exchange agents fall into two categories: rationed agents, for
which $d_i^* < \tilde{d}_i$ or $s_i^* < \tilde{s}_i$, and nonrationed ones, for which $d_i^* = \tilde{d}_i$ or $s_i^* = \tilde{s}_i$.
We say that a rationing scheme on a market is *efficient* or *frictionless* if there are
not both rationed demanders and rationed suppliers in that market. The intuitive
idea behind this is that in an efficiently organized market, a rationed buyer and
a rationed seller should be able to meet and exchange until one of the two is
not rationed anymore. Together with the voluntary exchange assumption, the
efficiency assumption implies the "short-side" rule, according to which agents
on the short side of the market can realize their desired transactions

$$\tilde{D} \geq \tilde{S} \Rightarrow s_i^* = \tilde{s}_i \qquad \text{for all } i \tag{11}$$

$$\tilde{S} \geq \tilde{D} \Rightarrow d_i^* = \tilde{d}_i \qquad \text{for all } i \tag{12}$$

This also yields the "rule of the minimum," which says that the aggregate level
of transactions is equal to the minimum of aggregate demand and supply:

$$D^* = S^* = \min(\tilde{D}, \tilde{S}) \tag{13}$$

The market efficiency assumption is quite acceptable when one considers a
small decentralized market where each demander meets each supplier (as in
the queue of section 1.3.3). Market efficiency becomes a less fitting assumption
when we consider a wide and decentralized market where some buyers and
sellers might not meet pairwise. One may note in particular that the efficiency
property is usually lost through the aggregation of submarkets. As a result the
global level of transactions may be smaller than both total demand and supply.
Figure 1.2 shows indeed how the aggregation of two frictionless submarkets
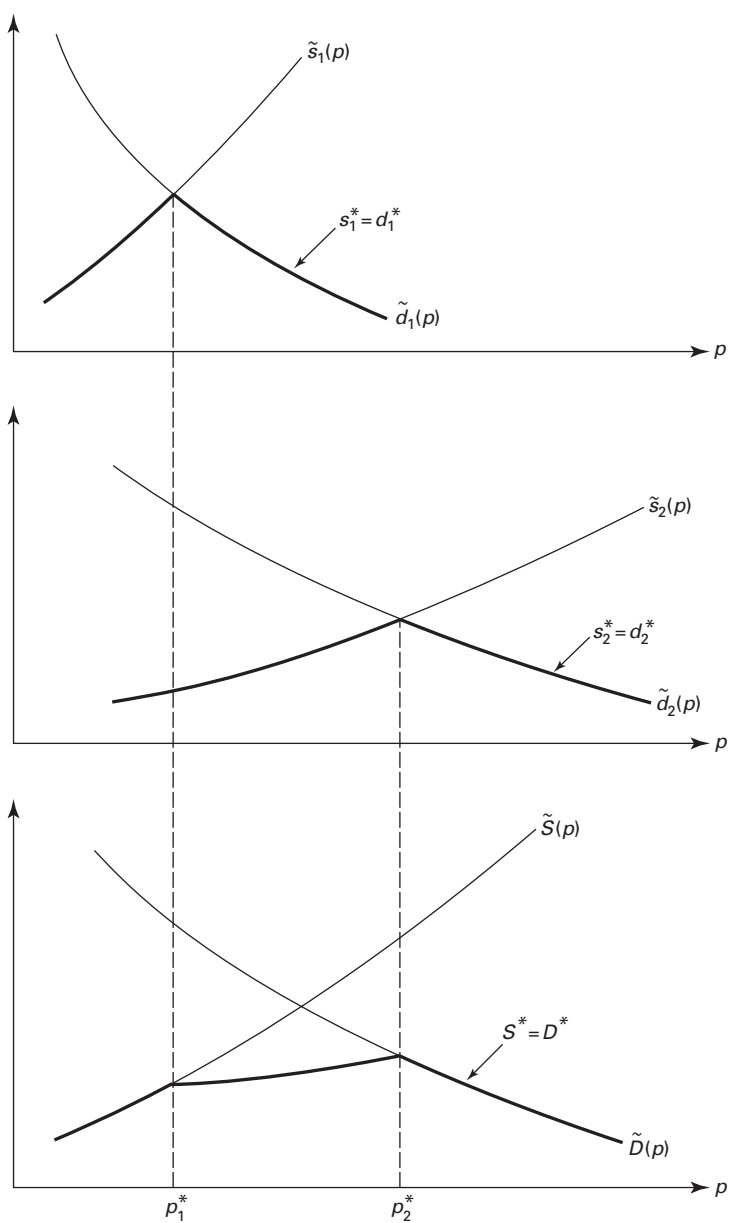yields an inefficient aggregate market, at least in some price range.

Figure 1.2   Aggregation and inefficiency

In the macroeconomic examples of the subsequent chapters we will assume frictionless markets, but we should note that the concepts that follow do not actually depend on this assumption.

1.4 Quantity Signals

Now it is quite clear that, since they cannot trade what they want, at least the rationed agents must perceive some quantity signals in addition to the price signals. Let us look at an example of how this occurs.

1.4.1 An Example

In order to see how quantity signals are formed in the transaction process, we begin with the simplest possible example, where only two agents are present in the market considered. Agent 1 demands $\tilde{d}_1$, and agent 2 supplies $\tilde{s}_2$. In such a simple market the "rule of the minimum" applies:

$$d_1^* = s_2^* = \min(\tilde{d}_1, \tilde{s}_2) \tag{14}$$

Now, as transactions take place, quantity signals are sent across the market: faced with a supply $\tilde{s}_2$, and under voluntary exchange, demander 1 knows that he will not be able to purchase more than $\tilde{s}_2$. Symmetrically supplier 2 knows that he cannot sell more than $\tilde{d}_1$. Each agent thus receives from the other a "quantity signal," respectively denoted $\bar{d}_1$ and $\bar{s}_2$, that tells him the maximum quantity he can respectively buy and sell. In this example, we have

$$\bar{d}_1 = \tilde{s}_2, \quad \bar{s}_2 = \tilde{d}_1 \tag{15}$$

so the rationing scheme (14) can be alternatively be expressed as

$$d_1^* = \min(\tilde{d}_1, \bar{d}_1) \tag{16}$$

$$s_2^* = \min(\tilde{s}_2, \bar{s}_2) \tag{17}$$

1.4.2 Quantity Signals

It turns out that many rationing schemes, and actually those we shall study in that follows, share the simple representation given by equations (16) and (17). Every agent i receives in the market a quantity signal, respectively $\bar{d}_i$ or $\bar{s}_i$ on
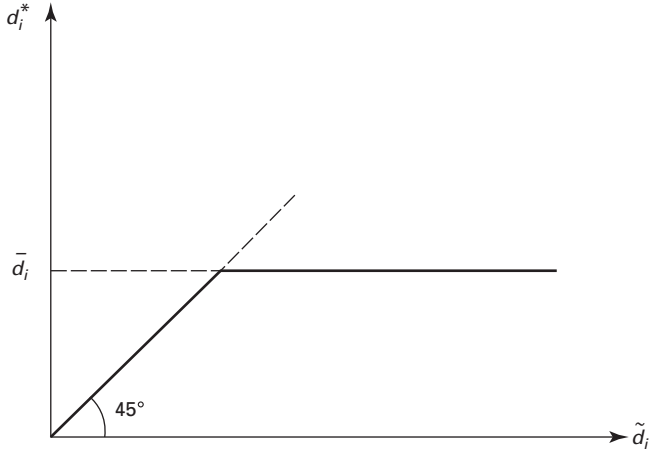
Figure 1.3   Nonmanipulable rationing scheme

the demand or supply side, which tells him the maximum quantity he can buy or sell. So the rationing scheme is simply written

$$d_i^* = \min(\tilde{d}_i, \bar{d}_i) \tag{18}$$

$$s_i^* = \min(\tilde{s}_i, \bar{s}_i) \tag{19}$$

where the quantity signals are functions of the demands and supplies of the *other* agents in the market. The relation between the demand $\tilde{d}_i$ and purchase d_i^* would appear as in figure 1.3.

As an example, for the queueing scheme of section 1.3.3 (equations 7 and 8), the quantity signals are given by

$$\bar{d}_i = \max\left(0, \tilde{s}_n - \sum_{j<i} \tilde{d}_j\right), \qquad i = 1, \dots, n-1 \tag{20}$$

$$\bar{s}_n = \sum_{j=1}^{n-1} \tilde{d}_j \tag{21}$$

We may note that under the representation given by (18) and (19), the rationing scheme displays obviously voluntary exchange, but also another important property, that of nonmanipulability. A scheme is called *nonmanipulable* if, once rationed, an agent cannot increase his transaction by increasing the

size of his demand. This property is quite evidently present in figure 1.3: once the transaction level $\bar{d}_i$ is attained, no increase of demand can yield a greater transaction. This assumption of nonmanipulability will be maintained in what follows.[3]

It is further clear that the quantity signals perceived by the agents should have an effect on demand, supply, and price formation. This is the relationship we explore next.

1.5   Effective Demand and Supply

As indicated above, demands and supplies are "signals" that agents send to the market in order to obtain the best transactions according to their own criteria. The traditional Walrasian demands and supplies are constructed on the assumption (which is verified ex post in Walrasian equilibrium) that every agent can buy and sell as much as he wants in the marketplace. Demands and supplies are thus functions of price signals only. We must now look more closely at how demands and supplies are formed when markets do not clear, and for that purpose we develop a theory of *effective demands and supplies,* which are functions of both price and quantity signals.

1.5.1   A Definition

When formulating his effective demands and supplies, agent i knows that his transactions will be related to them by equalities like (18) and (19) above, that is,

$$d_i^* = \min(\tilde{d}_i, \bar{d}_i) \qquad (22)$$

$$s_i^* = \min(\tilde{s}_i, \bar{s}_i) \qquad (23)$$

Maximizing the expected utility of these resulting transactions may lead to complex calculations (especially if constraints are stochastic). In the case of deterministic constraints, which is what we will consider here, there exists a simple

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3 Of course, there exist some rationing schemes, like the proportional rationing scheme, that are *manipulable* in the sense that an agent can, even when rationed, continue to increase the level of his transaction by overstating his demand. Such schemes are studied in the appendix to this chapter where it is shown that they typically lead, in a nonclearing market, to divergent demands or supplies and possibly no equilibrium.

and workable definition that generalizes Clower's (1965) seminal insight: effective demand (or supply) of a particular good is the trade that maximizes the agent's criterion subject to the usual constraints *and* to the quantity constraints on the *other* markets. A more formal definition is given in chapter 3, but before we get to it, we will study a well-known example.

1.5.2   The Employment Function

A good illustrative example of our definition of effective demand and supply is the employment function due to Patinkin (1956) and Barro and Grossman (1971). We take a firm with a diminishing returns to scale production function $Y = F(N)$ faced with a price P and wage W. The Walrasian labor demand is equal to $F'^{-1}(W/P)$. Assume now that the firm faces a constraint $\bar{Y}$ on its sales of output (in a complete model, such as will be developed in chapter 2, $\bar{Y}$ is equal to total demand from the other agents). By the definition of section 1.5.1, the effective demand for labor $\tilde{N}^d$ is the solution in N of the program

$$\max PY - WN \quad \text{s.t.}$$

$$Y = F(N)$$

$$Y \leq \bar{Y}$$

This yields

$$\tilde{N}^d = \min\left\{ F'^{-1}\left(\frac{W}{P}\right), F^{-1}(\bar{Y}) \right\} \tag{24}$$

We see that the effective demand for labor may actually have two forms: the Walrasian one $F'^{-1}(W/P)$ if the sales constraint is not binding, or, if this constraint is binding, a more "Keynesian" form equal to the quantity of labor just necessary to produce the output demand $F^{-1}(\bar{Y})$. We immediately see in this example that effective demand may take various functional forms, which intuitively explains why non-Walrasian models often have multiple regimes, as we will discover in the next chapter.

We also see that the definition of effective demand above naturally includes the well-known *spillover effects:* we say indeed that there is a spillover effect when an agent who is constrained to exchange less than he wants in a market because of rationing modifies his demands or supplies in the other markets. Here insufficient sales in the goods market "spill" into the labor market, resulting in a

reduction of labor demand. We shall see in the next chapter how the combination of such spillover effects can lead to the famous "multiplier" effects.

1.6   The Formation of Prices

We are now ready to address the problem of price setting by agents internal to the system, and we will see that as in demand and supply theory, quantity signals play a fundamental role. The general idea relating the concepts of this section to those of the preceding ones is that price setters change their prices so as to "manipulate" the quantity constraints they face, that is, so as to increase or decrease their possible sales or purchases.

As a result of this introduction of quantity signals into the price-setting process, the theory bears, at least formally, some strong resemblance to the traditional theories of imperfect competition. This will be so even if the market is highly competitive. As was pointed out by Arrow (1959), the absence of quantity signals is characteristic only of an auctioneer-engineered market, and not of the more or less competitive market structure.

1.6.1   The Institutional Framework

Various price-setting scenarios integrating the above ideas can actually be envisioned. We will focus here on a particular (and realistic for many markets) pricing process where agents on one side of the market (usually the sellers) quote prices and agents on the other side are price takers.[4]

Consider thus, to fix ideas, the case where sellers set the price (the exposition would be symmetrical if demanders were setting the price). In order to have a single price per market, as was the case in all we said before, we will characterize a good not only by its physical and temporal attributes, but also by the agent who sets its price (this way two goods sold by different sellers are considered as different goods, which is a fairly usual assumption in microeconomic theory since these goods differ at least by location, quality, etc.). With markets so defined, each price setter is alone on his side of the market, and thus appears formally as a monopolist. Note, however, that this does not imply anything about to his actual monopoly power because there may be competitors' markets where other agents sell goods that are very close substitutes.

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4 An alternative is for prices to be bargained between the two sides of the market. A model with such bargaining is developed in chapter 5.

1.6.2   Perceived Demand and Supply Curves

Consider thus a seller i who sets the price P in a certain market.[5] As we saw above, once he has posted his price, demands are expressed, transactions occur, and this seller faces a constraint $\bar{s}_i$ which is equal to the sum of all other agents' demands:

$$\bar{s}_i = \sum_{j \neq i} \tilde{d}_j = \tilde{D} \tag{25}$$

Now, if we consider the market *before* seller i set price P, we see that he does not, contrarily to a price taker, consider his quantity constraint $\bar{s}_i$ as parametric. Rather, and this is how the price-setting theory developed here relates to what we saw previously, he will use the price P to "manipulate" the quantity constraint $\bar{s}_i$ he faces, that is, so as to increase or decrease the demand addressed to him. The relation between the maximum quantity seller i expects to sell and the price he sets is called the perceived demand curve. If expectations are deterministic (which we assume here) the perceived demand curve will be denoted[6]

$$\bar{S}_i(P) \tag{26}$$

In view of equation (25), this perceived demand is what the price setter expects the aggregate demand of the other agents to be, conditional on the price P that he sets. Now, depending on what the price setter knows about the economy, two main forms of perceived demand curves can be considered, objective or subjective:

• In the "objective demand curve" approach, which we will be using in most of this book, it is assumed that the price setter knows the *exact* form of the other agents' demand functions, so that

$$\bar{S}_i(P) = \tilde{D}(P) \tag{27}$$

where $\tilde{D}(P)$ is the exact aggregate effective demand of agents facing the price

5 Although the price P is set by agent i, we do not index it by i because it is the (unique) market price.

6 Although it may seem odd that the perceived *demand* curve is denoted $\bar{S}_i(P)$, this is fully logical since it is a constraint on the *supply* of the price setter. The connection with the actual demand is expressed in formula (27) below.

setter. Although the construction of such an objective demand curve in a partial equilibrium framework is a trivial matter, things become much more complicated in a multimarket situation, as this requires a sophisticated general equilibrium argument. We will see in chapter 3 how to use the concepts developed here to rigorously construct an objective demand curve in a full general equilibrium system. Simple macroeconomic applications will be developed explicitly in chapters 4 and 5.

· In the "subjective demand curve" approach, it is assumed that the price setter does not have full information about the form of the demand curve facing him, so his perceived demand curve $\bar{S}_i(P)$ is partly "subjective." Most often isoelastic subjective demand curves are used, with the following form:

$$\bar{S}_i(P) = \xi_i P^{-\eta} \tag{28}$$

It should be noted that although the elasticity η is somewhat arbitrary, the position parameter ξ_i is not, since each realization $(P, \bar{s}_i)$ is a point on the "true" demand curve, and the subjective demand curve must pass through this point (Bushaw and Clower 1957). For example, with the isoelastic curves (28), if the price setter faces a quantity constraint $\bar{s}_i$ after setting a price $\bar{P}$, the parameter ξ_i must be such that

$$\bar{s}_i = \xi_i \bar{P}^{-\eta} \tag{29}$$

The functional form and the elasticity may be wrong, but at least the position must be right, as shown in figure 1.4. An example of equilibria using the subjective demand curve approach is found in chapter 2.

The subjective and objective demand curves approaches are not actually antagonistic, at least not in our theory. Knowing the exact objective demand curve requires very high amounts of information and computational ability, and the subjective demand curve should be thought of as what the price setter expects the "true" curve to be in an ongoing learning process. Whether this learning will lead to a good approximation of the "true" objective demand curve is still an unresolved problem.

1.6.3 Price Setting

Once the parameters of the perceived demand curve are known, price setting proceeds along lines that are traditional in imperfect competition theories: the price setter maximizes his objective function subject to the constraint that his
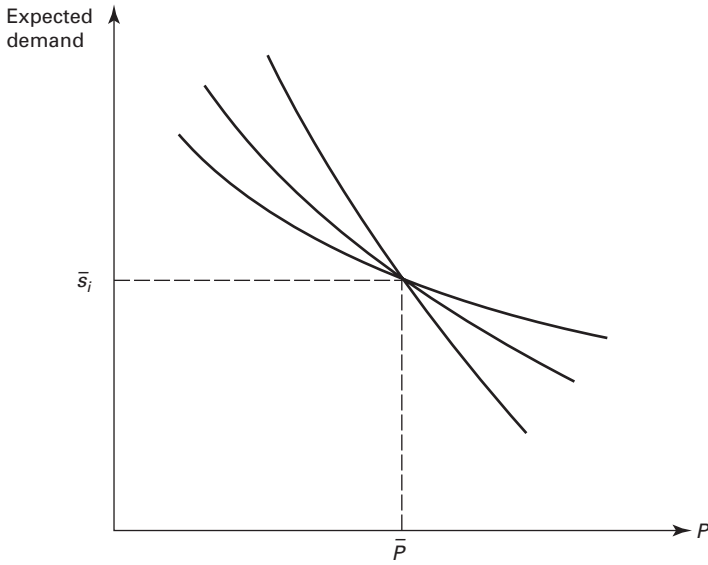
Figure 1.4   Subjective demand curves

sales can be no greater than the amount given by the perceived demand curve on the markets he controls (in addition to the usual constraints).

For example, take a firm with a cost function $\Gamma(Y)$, and assume that it faces an *objective* demand curve $\tilde{D}(P)$. The program giving the optimal price of the firm is thus written:

$$\max PY - \Gamma(Y) \quad \text{s.t.}$$

$$Y \leq \tilde{D}(P)$$

To solve this, we first note that the price setter will always choose a combination of P and Y such that he is "on" the demand curve, meaning such that $Y = \tilde{D}(P)$. If he were not, he could increase price P without modifying Y, thus increasing his profits. The solution is thus first characterized by

$$Y = \tilde{D}(P) \tag{30}$$

Now inserting (30) into the expression of profits and maximizing, we obtain

the first-order condition

$$\Gamma'(Y) = \frac{\eta(P) - 1}{\eta(P)} P \tag{31}$$

where

$$\eta(P) = -\frac{\partial \log \tilde{D}(P)}{\partial \log P} > 0 \tag{32}$$

Equation (31) is the well-known "marginal cost equals marginal revenue" condition, in which we see that the firm will choose a price high enough so that it will not only want to serve the actual demand, but would even be willing to serve more demand at the price it has chosen. In fact the firm would be content to meet demand up to $\Gamma'^{-1}(P) > Y$. There is thus in our sense an "excess supply" of the good, although this excess supply is fully voluntary on the part of the price setter.

The imperfectly competitive price and production are determined by equations (30) and (31). They are drawn together in figure 1.5, where
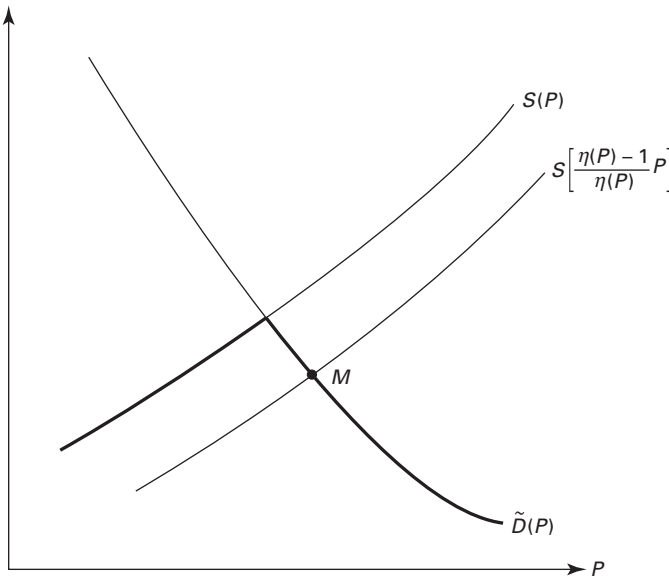

Figure 1.5   Imperfect competition equilibrium

$S(P) = \Gamma'^{-1}(P)$ is the "competitive" supply of the firm. The resulting equilibrium corresponds to point M.

Figure 1.5 also shows the "fixprice allocations" given by the minimum of supply and demand, that is,

$$Y = \min[\tilde{D}(P), S(P)] \qquad (33)$$

In the figure we see that the imperfectly competitive solution corresponds to one of the "fixprice" points, and one that is clearly in the excess supply zone.

1.7   Conclusions

We reviewed in this chapter the most basic elements for a rigorous theory of nonclearing markets. We saw how quantity signals are naturally formed in such markets, and how demands and supplies are responsive to these quantity signals. We also considered the issue of rational price setting, and we saw clearly that there is a natural relation between the functioning of nonclearing markets and the theory of price setting in circumstances of imperfect competition. This relation between nonclearing markets and imperfect competition will appear throughout the book.

For the simplicity of exposition we described all the above in a somewhat partial equilibrium framework. Obviously the next step is to move to a general equilibrium framework. Chapter 2 will start with the simplest macroeconomic model embedding the features of chapter 1, and show the rich variety of results that can be obtained. Chapter 3 will integrate the same concepts in a full-fledged multimarket general equilibrium setting. Finally the subsequent chapters will introduce successively time and uncertainty, moving to dynamic general equilibrium macromodels.

1.8   References

This chapter is based on Bénassy (1976c, 1982, 1993).

The starting point of many works on nonclearing markets is found in the article by Clower (1965), where he showed how to reinterpret the Keynesian consumption function through some type of effective demand, and in the book by Leijonhufvud (1968). Early elements of theory in the same direction are found in Hansen (1951), who introduced the ideas of active demand, close in spirit to that of effective demand, in Patinkin (1956), who studied the employment function of firms unable to sell their Walrasian output, and Hahn and

Negishi (1962), who studied nontâtonnement processes where trade takes place outside Walrasian equilibrium.

The representation of rationing schemes and quantity signals in this chapter are taken from Bénassy (1975a, 1977b, 1982). The voluntary exchange and market efficiency properties had been discussed under various forms in Clower (1960, 1965), Hahn and Negishi (1962), Barro and Grossman (1971), Grossman (1971), and Howitt (1974). The problem of manipulability was studied in Bénassy (1977b). The theory of effective demand originates in Clower (1965), and the more general definition is found in Bénassy (1975a, 1977b).

As we noted, the model of price setting is quite reminiscent of the imperfect competition line of thought (Chamberlin 1933; Robinson 1933; Triffin 1940; Bushaw and Clower 1957; Arrow 1959) and more particularly of the theories of general equilibrium with monopolistic competition, as developed notably by Negishi (1961, 1972). Their relation to the above non-Walrasian theories was developed in Bénassy (1976a, 1977a).

Appendix 1.1: Manipulable Rationing Schemes

Throughout this chapter we implicitly studied nonmanipulable rationing schemes. These are rationing schemes where an agent, once rationed, cannot increase his transaction by overstating his demand. In this appendix we will look at the opposite case of *manipulable* rationing schemes where a rationed agent can increase his transaction by overstating his demand, hence the name "manipulable." This is represented in figure 1.6, which represents the relation between the demand $\tilde{d}_i$ of a demander i and the transaction $d_i^* = \phi_i(\tilde{d}_i)$ he will obtain from the market.

An Example

We begin with a well-known example of a manipulable rationing scheme, that of proportional rationing. In a proportional rationing scheme, total transactions are equal to the minimum of aggregate demand and aggregate supply. These trades are allocated among the various agents proportionately to their demand or supply. This is expressed mathematically as

$$d_i^* = \tilde{d}_i \times \min\left(1, \frac{\tilde{S}}{\tilde{D}}\right) \tag{34}$$

$$s_i^* = \tilde{s}_i \times \min\left(1, \frac{\tilde{D}}{\tilde{S}}\right) \tag{35}$$
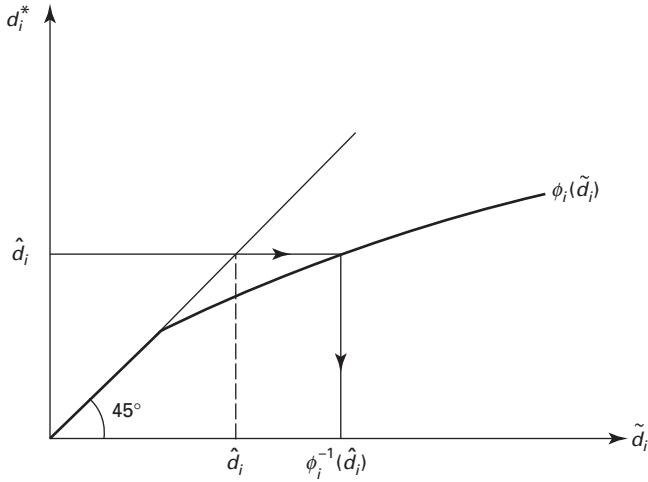
Figure 1.6   Manipulable rationing scheme

It is easy to verify that whatever the demands and supplies, the trades realized always match at the aggregate level.

Manipulation and Overbidding

We will now see that manipulable rationing schemes lead to a perverse phenomenon of overbidding, which may totally jeopardize the existence of an equilibrium in demands and supplies. The mechanism is easy to understand: consider an agent i who would like to obtain a transaction $\hat{d}_i$, but who would be rationed if he announces that level. As shown in figure 1.6, he will be naturally led to overstate his demand to the level $\phi_i^{-1}(\hat{d}_i)$ in order to reach $\hat{d}_i$. The problem is that all rationed demanders will do exactly the same thing, and as a result the perceived rationing schemes will move in time in such a way that the same demand yields an ever lower transaction. It is easy to see that because of this overbidding phenomenon, demands may grow without bound, so that no equilibrium with finite demands and supplies exists, as we will now observe in a simple example.

An Example

Let us consider the case of a supplier facing two demanders, indexed by 1 and 2. In each period t we have, respectively, a supply $\tilde{s}(t)$ and demands $\tilde{d}_1(t)$ and

$\tilde{d}_2(t)$. The rationing scheme is thus written as

$$d_i^*(t) = \tilde{d}_i(t) \times \min\left[1, \frac{\tilde{s}(t)}{\tilde{d}_1(t) + \tilde{d}_2(t)}\right], \qquad i = 1, 2 \tag{36}$$

$$s^*(t) = \min[\tilde{s}(t), \tilde{d}_1(t) + \tilde{d}_2(t)] \tag{37}$$

Let us assume that each trader knows the rationing rule and the demands and supplies of the others after they have been expressed. Moreover he expects these to remain the same from period $t - 1$ to period t. The perceived rationing scheme is thus for demander 1,

$$\phi_{1t}(\tilde{d}_1) = \tilde{d}_1 \times \min\left[1, \frac{\tilde{s}(t - 1)}{\tilde{d}_1 + \tilde{d}_2(t - 1)}\right] \tag{38}$$

and similarly for demander 2. Now let us assume that the agents have "target transactions" $\hat{d}_1, \hat{d}_2$, and $\hat{s}$ such that the supplier could serve each demander individually, but not both, that is,

$$\hat{d}_1 < \hat{s}, \quad \hat{d}_2 < \hat{s}, \quad \hat{s} < \hat{d}_1 + \hat{d}_2 \tag{39}$$

Under these conditions the supplier will never be rationed and will express his target transaction as effective supply:

$$\tilde{s}(t) = \hat{s} \tag{40}$$

The demanders, in the contrary, will be rationed, and they will in each period overstate their demands in a way such that they (wrongly) believe to reach their target transaction.[7] Their effective demands will thus be given by

$$\phi_{1t}[\tilde{d}_1(t)] = \hat{d}_1 \tag{41}$$

$$\phi_{2t}[\tilde{d}_2(t)] = \hat{d}_2 \tag{42}$$

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7 This belief is false simply because each demander computes his demand assuming that the other demander will not change his demand from the previous period, whereas, as we will soon see, these demands will generally increase over time.

In view of (38), and the corresponding formula for demander 2, these two equations yield

$$\tilde{d}_1(t) = \tilde{d}_2(t-1) \times \frac{\hat{d}_1}{\hat{s} - \hat{d}_1} \tag{43}$$

$$\tilde{d}_2(t) = \tilde{d}_1(t-1) \times \frac{\hat{d}_2}{\hat{s} - \hat{d}_2} \tag{44}$$

Combining (43) and (44), we obtain

$$\tilde{d}_1(t) = \tilde{d}_1(t-2) \times \frac{\hat{d}_1}{\hat{s} - \hat{d}_1} \times \frac{\hat{d}_2}{\hat{s} - \hat{d}_2} \tag{45}$$

which is an indefinitely divergent sequence since $\hat{s} < \hat{d}_1 + \hat{d}_2$.