1 THE LINEAR PROGRAMMING MODEL

The major problems of air pollution occur as a consequence of the clustering of polluting activities. Emissions from many sources intermix and accumulate within geographic spaces that we call airsheds. The problem is one of reducing emissions to achieve desired air quality.

Three themes appear in this book. They are (1) airshed planning, (2) simulation of pollution abatement, and (3) economic efficiency. The method of linear programming provides a powerful tool for air pollution control planning at the local level. The usefulness of this method is demonstrated in this book by a score of empirical applications, based on data for the St. Louis airshed. Many of these results are sufficiently general that they are applicable to other airsheds, as well as to pollution control efforts that are not necessarily least-cost. In effect, then, this research provides a simulation model for examining some important issues in air pollution control.

The Linear Programming Model (which is now capitalized to denote this particular application to air pollution control) is a submodel of a general equilibrium model developed in this chapter. It is useful to view an airshed as an independent economy in which the utility levels of the inhabitants are maximized. The tools of welfare economics can then be applied to determine conditions for an efficient allocation of resources into production and abatement activities. It follows that abatement strategy should be formulated at the airshed level and should comprise a least-cost set of pollution control activities.

The first sections of this chapter are introductory in nature. The three themes of airshed planning, simulation, and economic efficiency are discussed. Following this, the mathematical analysis is presented in which we progress from a general equilibrium model to the Linear Programming Model for Air Pollution Control.

Airshed Planning

It is frequently the case that individual airsheds are politically fragmented, and different regulations and policies are enforced in the separate jurisdictions of the same airshed. Because emissions intermix, it is crucial that planing should take place at the airshed level.

Several planning models are presented in this book. The simpler one is a version of the Linear Programming Model in which it is assumed that the ambient air concentration of a pollutant, over and above the background concentration, is proportional to total emissions of that pollutant. Although this assumption on total emissions has limitations, it is the basis of many federally sponsored strategies, such as Inspection/Maintenance (of automobiles) and the Offset Rule (for new sources in an airshed).

An alternative version of the Linear Programming Model is based on a diffusion formula in which the geographic locations of individual pollution sources are taken into account. In this version of the model, abatement strategy includes the selective location of new sources and even the relocation of existing sources. Some useful information, based on the empirical application of this model, is presented in chapters 4 and 7.

Both types of planning models have their place. The model based on total emission flows, however, is less complex and can be more easily implemented by regulatory officials. Because local agencies are required to maintain emission inventories, the data base is easily kept current. For these reasons, the simpler model is a recommended planning tool.

In general, there is no simple trade-off between money spent on abatement and the level of air quality. Air quality is based on a number of different pollutants, and there are trade-offs between them. The Linear Programming Model is essential when there are multiple requirements. This is illustrated in chapter 3 by the example of low sulfur coal. For many years, pollution control in St. Louis was hampered by legal controversy over regulations that specified the maximum sulfur content of coal. Other pollutants in addition to sulfur dioxide were affected, and the controversy was complicated by side issues such as the scarcity of natural gas, which might be substituted for coal. With the Linear Programming Model the various trade-offs were put into proper perspective.

The solution of the Linear Programming Model shows how an entire set of air quality standards can be achieved at the least total cost of abatement. Furthermore, the sensitivity of the solution to each of the control method costs can be readily determined. This allows greater flexibility in using the model. For example, the enforcement of a particular abatement activity might appear inequitable to control officials; if a more acceptable control activity were inefficient by a small percents of its unit cost, some trade-off of efficiency for equity might be justified. The power of linear programming is greatly enhanced by various capabilities for sensitivity analysis.

The emphasis here is on a model to assist in planning air quality strategy. New problems inevitably confront policy makers. In one year it makes sense to promote the conversion of coal furnaces to natural gas. In a later year the prevailing sentiment may be for conserving natural gas and converting back to coal. Whether the control agency should alter its earlier policy could depend on the magnitude of the increased cost of pollution abatement per cubic foot of gas to be replaced by coal. This cost estimate is easily obtained with the Linear Programming Model. Similarly, the emphasis that should be placed on reducing automotive emissions by Inspection/Maintenance or by car pooling can be given a dollar value in saved abatement costs. Given the many conflicting objectives in air pollution control, regulatory officials could use this planning tool to expand their awareness and to reinforce or modify their own intuitive judgments.

Simulation of Pollution Abatement

The Linear Programming Model is implemented with data for the St. Louis airshed. The data presented and discussed in chapter 2 underlie a series of models that are described in this book. These are listed in table 1.1. Although the table includes only thirteen models, some of them have several subversions, so that there are in fact more than twenty models.

In total this represents an extensive simulation of the economics of air pollution control. In many cases, issues are resolved that would have continued to trouble policy makers. Some of the questions examined in this book and identified by the corresponding model number in the table are as follows:

Efficiency savings. Aside from regulatory and enforcement costs, how great is the saving in total abatement cost for the efficient solution as compared to the current regulatory solution in the St. Louis airshed? (I)

Joint-wastes. Is it necessary to include all forms of wastes in an environmental model? Is there a danger that a planning model for air pollution alone will yield a solution that would seriously augment the flows of solid, liquid, and thermal wastes (which occur as a consequence of the cessation of open burning, increased use of scrubbers, generation of electricity to operate control equipment, etc.)? (II)

Cost of confidence. The relationship between emissions and ambient air concentrations is affected by stochastic meteorological variables such as wind velocity. What is the risk in using average values for these stochastic

Table 1.1
Major versions of the Linear Programming Model

Model Number	Chapter	Description
I	2, 3	The basic model with fixed pollution source levels and maximum allowable total annual emission flows.
II	3	This model is identical to Model I except that it includes output coefficients for liquid, thermal, and solid wastes as well as air pollutants.
III	4	This model is the same as Model I except that it incorporates the Larsen formula relating total emissions and annual average concentrations. Accordingly, maximum pollutant concentrations can be entered directly into the model.
IV	4	This is an elaboration of Model III, in which the linear relationship between total emissions and pollutant concentrations is stochastic. A specific probability that the desired air quality goals will be achieved is a parameter of this model.
V	4	A diffusion formula relates emissions of each source to pollutant concentrations at the CAMP Station. Composite sources are disaggregated according to their location in the airshed. With Model V, the location of any point source can be varied and the consequent effect on pollutant concentrations and abatement costs thereby determined.
VI	4	An alternative version of Model I in which source magnitudes are projected for 1985. The total allowable pollutant flows for 1985 are the same as those in Model I.
VII	5	This is a benefit-cost version of Model III. In place of maximum pollutant concentrations, there is an objective function equal to the dot product of pollutant concentrations (now variables) and their respective shadow prices from Model III, plus the total cost of abatement. The inclusion of unit capital coefficients for the abatement activity variables allows for alternative rates of interest.
VIII	5	A benefit-effectiveness version of Model III in which a pollution index is minimized subject to a constraint on the total cost of abatement. In the two-pollutant case, the concentration of particulates is minimized for a range of sulfur dioxide concentrations, with the concentrations of the other pollutants and the total cost of abatement held constant.
IX	6	A version of Model I incorporating the feedback of abatement activities on pollution source magnitudes. As a consequence of abatement, the selling prices of pollution related goods are higher, and the quantities demanded are accordingly reduced.

Model Number	Chapter	Description
X	6	An extension of Model IX, allowing for voluntary substitutions of natural gas to avoid costly restrictions on coal burning.
XI	6	In this version of Model I, the feedback of abatement activity on pollution source magnitudes occurs through the production of inputs for abatement.
XII	6	An extension of Model XI that includes a matrix of input-output multipliers and accounts for both the direct and the indirect demand for inputs for abatement.
XIII	7	An extension of Model IX in which job displacement, caused by reductions in output, is measured. In addition, there is a direct loss of jobs as a consequence of abatement technology. In an equity version of this model, job displacement is minimized subject to a constraint on the total cost of abatement.

variables, and how great is the cost of insuring against this risk by more intensive permanent abatement? Is this cost so high as to justify greater reliance on episode control measures? (IV)

Locational selectivity. How important is the selective location of new sources as a strategy for pollution control? (V)

Abatement and industrial growth. If total emissions are held to fixed allowable levels, is this likely to seriously restrain economic growth in an airshed? (VI)

Benefit-effectiveness. Are air quality standards in the St. Louis airshed benefit-effective, or should the standards for certain pollutants be made more stringent? (VIII)

Substitution effects. The costs of abatement are likely to increase prices of pollution-related goods and induce some substitutions of other goods. Will these substitutions significantly improve air quality and should they be reflected in regulatory planning? (IX)

Derived demands. The abatement of pollution requires inputs whose production is itself polluting. To what extent, therefore, must an abatement effort be augmented to offset this feedback effect? (XI)

Employment impact. Should regulatory agencies adjust their control strategies to preserve existing jobs? (XIII)

These are some of the issues that are examined empirically in this book. In addition, the Linear Programming Model is used to simulate the revenue potential of a program of pollution fees and the capability of such a program for internalizing the pollution costs of land use. The results of the various simulation models are interpreted in this book. This aspect of the research should be useful to economic theorists as well as regulatory policy makers. Furthermore, it demonstrates the wide versatility of the Linear Programming Model.

Economic Efficiency

In the economic literature on pollution control in an airshed, there are two major theoretical approaches. One is typified by the *general equilibrium model*, in which prices and outputs of individual goods are variable. Such models are useful in defining optimal levels of environmental quality and quantities of private goods. The second approach is *partial equilibrium analysis*, in which prices are essentially fixed and pollution control is accomplished without any changes in the quantities of goods and services associated with polluting activities. While this approach facilitates empirical research, the underlying assumption is faulty; pollution abatement, through price, income, and derived demand effects, does alter the levels of pollution-related outputs.

There has been some effort by economists to reconcile the two approaches to environmental analysis. Dick (1974, p. 125) constructs a partial equilibrium model in which the pollution from a single industry is being controlled, while the outputs and pollution flows from all other industries are assumed to be optimal to begin with. A similar assumption is implicit in the partial equilibrium models of Dorfman (1972, pp. xvi–xviii) and of Meade (1973, p. 58). It is more characteristic of the real world, however, that an entire spectrum of polluting activities is not properly controlled to begin with, and a strategy for simultaneous abatement by all sources is desired.

In the mathematical section of this chapter, we develop a general equilibrium model that has the useful properties of a partial equilibrium model. This is achieved with a simplifying assumption that air pollution originates exclusively in the production of intermediate goods, for which quantities demanded are proportional to some resource input. Assuming that resources are fixed, this model is one in which prices and outputs of the final goods are variable, but the magnitude of each pollution source is fixed by virtue of the assumption of technology. Furthermore, we assume away the relative price effects of abatement, as well as the real income effects, which would otherwise alter pollution source levels.

The Linear Programming Model of air pollution control is a component of the aforementioned model. Although the simplifying assumptions on pollution and intermediate activities are artificial, they permit us to relate the results of the Linear Programming Model to some important conditions for economic efficiency. For example, the shadow prices of the individual pollutant standards must be equal to (or less than) the marginal benefits of abatement for the respective pollutants. It follows that there is an optimal concentration for each pollutant. This analysis is pursued in chapter 5, where the economic efficiency of the pollutant concentrations is empirically tested.

In welfare economic theory, efficiency and equity are separate policy considerations. The level of air quality that is optimal under one distribution of income may not be optimal under a different distribution of income. It is appropriate, therefore, that regulatory agencies be cognizant of equity as well as cost-effectiveness. In chapter 7 alternative governmental programs for pollution control are evaluated with respect to efficiency and equity criteria, and it is found that there is an overlapping of the two objectives. For this evaluation of governmental programs, the empirical data from the Linear Programming Model are useful.

Although the basic planning model is based on the assumption that pollution source levels are constant, this assumption is dropped in chapter 6. In that chapter the Linear Programming Model is adapted to the case in which abatement increases the market prices of goods and, as a consequence of estimated reductions in quantities demanded, decreases pollution source levels. The effect of this feedback is to reduce the total cost of achieving a given set of air quality standards. Alternatively, pollution control requires inputs for abatement. The derived demand for these inputs increases pollution source levels, and the effect of this feedback is to increase the total cost of achieving a given set of standards. The models in chapter 6 more closely approximate the economic interactions that are characteristic of general equilibrium analysis.

Mathematical Analysis

In the remainder of this chapter mathematical models are developed, that lead ultimately to the Linear Programming Model for Air Pollution Control. The reader who plans to follow the mathematical derivations is urged, at the outset, to become acquainted with the Glossary of Mathematical Symbols that follows the Appendix.

The sequence of models that complete this chapter are as follows. We first examine a general equilibrium model in which there are two goods and one pollutant. Here pollution originates during the production of one of the goods and is undesirable because it diminishes the utility of both households. The thrust of the model is that an excessive level of pollution will be reduced by (1) adoption of alternative processes of production that are less polluting per unit of output, (2) shifts in consumption away from the good that is polluting in production, and (3) a contraction in units of total output.

Next we develop a pure abatement model in which economic efficiency is achieved solely by abatement, not by shifts in the composition of final outputs nor by a contraction of units of output. Such a model is more closely related to conventional linear programming models of air pollution control, in which the outputs of polluting activities are assumed fixed. This model is extended to include more than one air pollutant and is reformulated as a linear programming problem. The chapter concludes with an appendix that illustrates the equivalence of emission standards and pollution fees in the case of the pure abatement model.

The General Equilibrium Model

Consider a simple economy consisting of two households, household-one and household-two. We shall assume that the labor supplied by these households is the sole factor of production and that the quantity supplied is perfectly inelastic. In this economy two goods are manufactured, good-one and good-two, and their quantities are expressed by the variables y_1 and y_2. The allocations to the households are the sets (y_{11}, y_{21}) and (y_{12}, y_{22}), where the second subscript denotes the consuming household. The quantities produced are entirely consumed:

$$\begin{aligned} y_{11} + y_{12} &= y_1, \\ y_{21} + y_{22} &= y_2. \end{aligned} \tag{1.1}$$

We shall assume that the production of good-one, but not of good-two, is polluting. The activity level of the least-cost process for making good-one is the variable x_{1a}, and the corresponding rate of emissions is e_{1a}. The technology of abatement is formulated in terms of alternative processes, which are less polluting but more costly. Assuming that there are four processes la, 1b, 1c, and 1d for making good-one and measuring their activity levels x_{1a}, x_{1b}, x_{1c}, and x_{1d}, in units of good-one produced it follows that

$$x_{1a} + x_{1b} + x_{1c} + x_{1d} = y_1. \quad (1.2)$$

The alphanumeric ordering of subscripts is such that

$$\begin{aligned} e_{1a} &> e_{1b} > e_{1c} > e_{1d}, \\ c_{1a} &< c_{1b} < c_{1c} < c_{1d}, \end{aligned} \quad (1.3)$$

where e_j is the emission rate, expressed in parts per million (or micrograms per cubic meter) per unit of activity-j, and c_j is the cost coefficient measured in labor units. If the activity of the nonpolluting process for making good-two is y_2 and the unit cost is c_2, it follows that

$$\begin{aligned} c_{1a}x_{1a} + c_{1b}x_{1b} + c_{1c}x_{1c} + c_{1d}x_{1d} + c_2 y_2 &= R, \\ e_{1a}x_{1a} + e_{1b}x_{1b} + e_{1c}x_{1c} + e_{1d}x_{1d} &= q, \end{aligned} \quad (1.4)$$

where R is the fixed supply of labor and q is the level of air pollution.

According to welfare economic theory, a vector of outputs, $(y_{11}^*, y_{12}^*, y_{21}^*, y_{22}^*, q^*)$, is optimal if there is no alternative attainable set of outputs that would make one of the households better off without making the other household worse off. Better or worse off for a household is measured according to a utility function for that household. Thus

$$U^i(\bar{y}_{11}, \bar{y}_{21}, \bar{q}) > U^i(\bar{\bar{y}}_{11}, \bar{\bar{y}}_{21}, \bar{\bar{q}}) \quad (1.5)$$

is equivalent to the statement that the ith household is better off[1] with the combination $(\bar{y}_{11}, \bar{y}_{21}, \bar{q})$ than with $(\bar{\bar{y}}_{11}, \bar{\bar{y}}_{21}, \bar{\bar{q}})$. We shall make the conventional assumptions that utility increases at a decreasing rate as the quantity of a private good increases, and decreases at an increasing rate as the level of air pollution increases.

The welfare economic conditions for an efficient allocation of outputs, for example one in which $U^2 \geqslant \bar{U}^2$ and U^1 is maximized, are derived from a Lagrangian expression incorporating (1.1), (1.2), and (1.4):

$$\begin{aligned} \mathscr{L} = {} & U^1(y_{11}, y_{21}, q) + \lambda_u[U^2(y_{12}, y_{22}, q) - \bar{U}^2] \\ & + \lambda_r[R - c_{1a}x_{1a} - c_{1b}x_{1b} - c_{1c}x_{1c} - c_{1d}x_{1d} - c_2(y_{21} + y_{22})] \\ & + \lambda_1(y_{11} + y_{12} - x_{1a} - x_{1b} - x_{1c} - x_{1d}), \end{aligned} \quad (1.6)$$

where $q = e_{1a}x_{1a} + e_{1b}x_{1b} + e_{1c}x_{1c} + e_{1d}x_{1d}$. Observe that q, without a subscript, enters both utility functions, implying that the two households are exposed to the same level of pollution. (The assumption of equal exposure can be dropped without altering the essential results. See Kohn, 1975b, pp. 26–28). Because exposure by one household does not change the level to which the other household is exposed, the level of air pollution is analogous to a pure public good as conceived by Samuelson (1954).

We shall examine the case in which the solution of (1.6) is one in which each household consumes both goods and a combination of processes 1b and 1c is optimal.

From the Kuhn-Tucker conditions for optimality,

$$\begin{aligned} y_{ik}(\partial L/\partial y_{ik}) &= 0, \\ x_j(\partial L/\partial x_j) &= 0, \end{aligned} \tag{1.7}$$

it follows that

$$-\left(U_q^1/U_2^1 + U_q^1/U_2^2\right) = \frac{\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)}{c_2}, \tag{1.8}$$

$$U_1^1/U_2^1 = U_1^2/U_2^2 = \frac{c_{1b} + e_{1b}\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)}{c_2}, \tag{1.9}$$

where U_j^i/U_k^i is the ith household's marginal rate of substitution of good k for good j (or air quality q). Equation (1.8) states that the quantities of good-two which the two households together would exchange for a unit decrease in air pollution must equal the opportunity cost to the producing sector, as measured in units of good-two, of abating one unit of pollution, holding the output of good-one constant. The right-hand-side value of (1.8) is the ratio of the marginal cost of abatement, $(c_{1c} - c_{1b})/(e_{1b} - e_{1c})$, to the marginal cost of good-two.[2] For convenience we shall henceforth assume that the wage rate is one dollar so that all costs can be expressed in dollar values. If (1.8) is multiplied through by c_2, the left-hand side becomes the marginal benefit of abatement, measured in dollars, and the right-hand side the marginal cost of abatement. The equality of marginal benefits and costs is a well-known condition for an optimal supply of a public good.

Equation (1.9) states that the marginal rate of substitution in consumption between good-one and good-two is the same for both households and is equal to the rate of transformation in production, holding the pollution level constant. The rate of transformation between the two goods is the ratio of their marginal costs. Whereas the marginal cost of good-two is simply the direct cost c_2, the marginal cost of good-one, holding the level of air pollution constant, is $c_{1b} + e_{1b}\ [(c_{1c} - c_{1b})/(e_{1b} - e_{1c})]$. This is the direct cost of labor, c_{1b}, plus the cost of eliminating the incremental pollution, e_{1b}. An equivalent expression for the marginal cost of good-one is $c_{1c} + e_{1c}\ [(c_{1c} - c_{1b})/(e_{1b} - e_{1c})]$.

An allocation that fulfills the above conditions is illustrated graphically by the point P in figure 1.1. This point is on th facet bce of the simplex abcde. Observe that vertices a, b, c, and d are on the q-y_1 plane and vertex e is on the q-y_2 plane. For expositional convenience the q-axis begins at q_0, which is the maximum level of pollution for this economy. Thus more desirable levels of all three outputs are outward from the origin. Points on the facet abe represent sets of q, y_1, and y_2 obtainable with combinations of processes 1a, 1b, and 2. Points on the facet bec are obtained with combinations of processes 1b, 1c, and 2. The final facet cde is generated by combinations of processes 1c, 1d, and 2. The frontier in figure 1.1 is convex because all four processes for making good-one are technically efficient. This is the case because

$$\frac{c_{1b} - c_{1a}}{e_{1a} - e_{1b}} < \frac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}} < \frac{c_{1d} - c_{1c}}{e_{1c} - e_{1d}}. \qquad (1.10)$$

Whereas P denotes the vector (y_1^*, y_2^*, q^*), points A^1 and A^2, which are inside the simplex (see upper right-hand side inset of figure 1.1), denote the sets $(y_{11}^*, y_{21}^*, q^*)$ and $(y_{12}^*, y_{22}^*, q^*)$, respectively. Geometrically, A^1 and A^2 are vertices of a parallelogram $A^1PA^2q^*$ that lies on a plane parallel to the y_1-y_2 axis through q^*. This plane contains the coordinate axes in the right-hand inset of figure 1.1. The attained indifference curve of each household is superimposed on the same coordinate system, with consumption by each household measured independently along the corresponding axes. The slopes of the indifference curves $\partial y_{21}/\partial y_{11}$ at A^1 and $\partial y_{22}/\partial y_{12}$ at A^2 are equal to each other and to the slope $\partial y_2/\partial y_1$ of the facet through P, thereby satisfying condition (1.9). Condition (1.8) will be graphically illustrated in a comparable model later in this chapter.

If we assume that the two goods in this economy are sold in perfectly competitive markets, that households do not consider the pollution-generating consequences of their consumption decisions, and there is no government program for pollution control, producers of good-one would use the least-cost

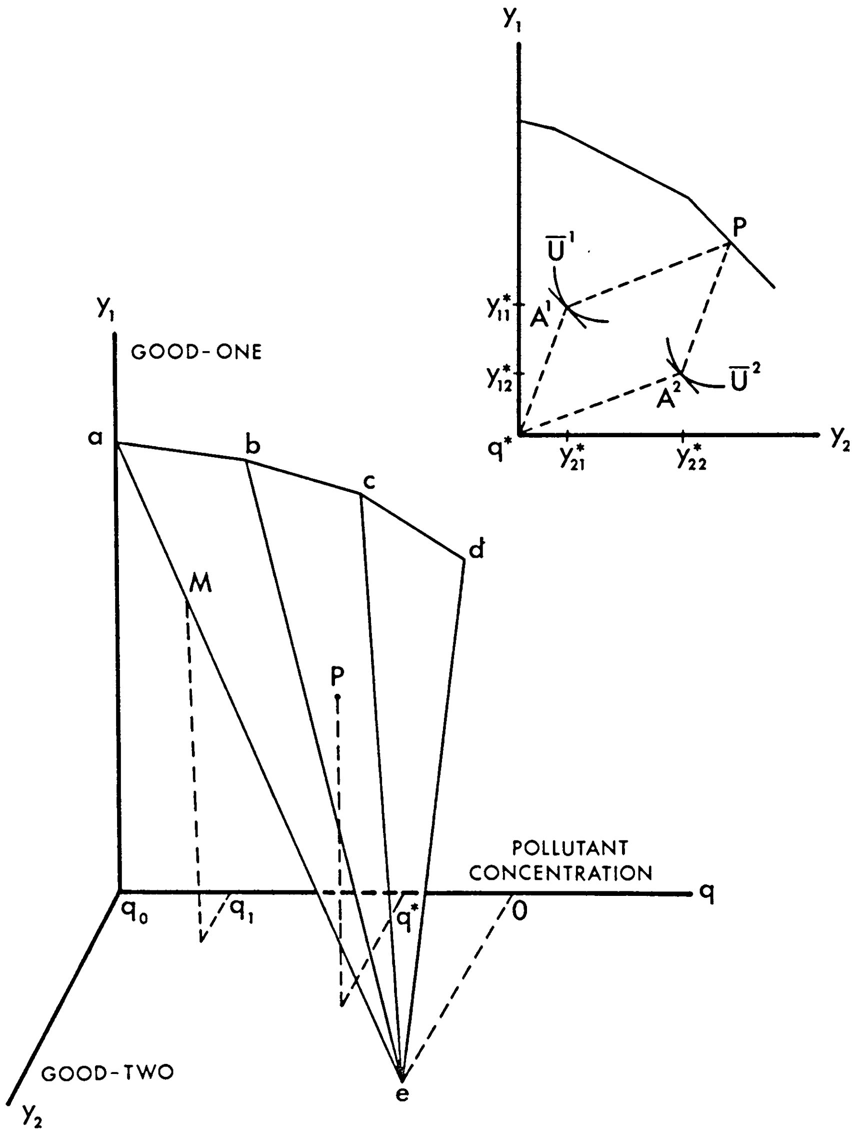

Figure 1.1
The production-possibility frontier and an optimal allocation

process, and the selling prices of the two goods would be c_{1a} and c_2, respectively. The competitive market allocation would correspond to a point such as M in figure 1.1, and the level of pollution would be q_1.

To reduce the pollution level from q_1 to q^*, three things generally happen: (1) technological abatement, (2) a shift in the ratio of goods consumed, and (3) a contraction in total units of output. Condition (1.8) implies that a combination of processes 1b and 1c should be used for making good-one. Because emission rates for this combination of processes are less than e_{1a} there would be a decrease in the level of pollution. Condition (1.9) implies that consumption decisions should be based on a price for good-one that is higher than the initial price, c_{1a}. In general, a higher relative price for good-one will result in some substitution of good-two, which is nonpolluting in production, for good-one. In addition, the increased cost of production (c_{1b} exceeds c_{1a}) reduces the quantity of goods that can be produced with the fixed supply of labor. This further decreases the output of good-one and hence the total flow of emissions. The three effects combine to reduce pollution from q_1 to q^*.

In theory this result would be achived in a perfectly competitive economy by assessing a pollution fee equal to $(c_{1c} - c_{1b})/(e_{1b} - e_{1c})$ per unit of pollution emitted. Producers of good-one would minimize total costs of production (which includes pollution fee charges) by using a combination of process 1b and 1c. There is the problem of divisibility here (see Kohn, 1975b, pp. 81–86) in that *any* combination of these two processes will result in the same total cost of producing a given quantity of good-one. We shall simply assume that the combination of processes 1b and 1c is chosen such that the resulting equilibrium market allocation is one in which condition (1.8) is satisfied. The marginal cost of good-one will be $c_{1b} + e_{1b}\ [(c_{1c} - c_{1b})/(e_{1b} - e_{1c})]$, which is the sum of direct costs c_{1b} plus pollution fees per unit of output. The marginal cost of good-two will be c^2. Assuming perfect competition, goods will be priced at marginal cost and condition (1.9) satisfied as a consequence of utility maximization by consumers. For the economy to be at full employment, the government must transfer the fee revenue paid by producers, totalling $q^*[(c_{1c} - c_{1b})/(e_{1b} - e_{1c})]$, to the households.[3] These lump sum transfers can, in theory, be given in such ratios as to satisfy the constraint in (1.6) on the utility level of household-two.

If pollution control were accomplished by emission standards, conditions

(1.8) and (1.9) would not be satisfied. Even though the emission standards require producers of good-one to use the optimal combination of process 1b and 1c, the allowable emission flows would cause reductions in utility levels that would not be reflected in the relative price of good-one. In the next section we develop a model in which economic efficiency can be achieved by emission standards as well as by emission fees.

The Pure Abatement Model

The construction of economic models of air pollution control is inevitably complex. There are numerous sources of pollution in an airshed, and it is a formidable task to identify all of these sources and the corresponding alternative production-abatement processes. To further account for the effect of abatement costs on individual output levels would be enormously complicating. It is therefore common for model builders to assume that pollution-related output levels are fixed and independent of abatement. Thus, for example, it is generally assumed in such models that households will demand a specific quantity of electricity, regardless of the cost of abatement at the power plant; or if the substitution of natural gas for coal is an abatement alternative for specific industrial furnaces, that the same total heat will be required with either fuel.

Accordingly, we revise our general equilibrium model so that polluting activity levels are in fact independent of abatement costs. This result is obtained by assuming that the good that is polluting in production is an intermediate good, used by firms in a fixed proportion to their labor input. Four alternative processes may contribute to the quantity of this intermediate good s_1:

$$x_{1a} + x_{1b} + x_{1c} + x_{1d} = s_1. \tag{1.11}$$

There are two final goods, y_2 and y_3, produced by nonpolluting processes, y_2 and y_3. The total resource requirements in this economy are

$$c_{1a}x_{1a} + c_{1b}x_{1b} + c_{1c}x_{1c} + c_{1d}x_{1d} + c_2y_2 + c_3y_3 = R. \tag{1.12}$$

All firms, including those which manufacture the intermediate good, require a quantity of the intermediate good proportional to their labor requirements. This proportion α is the same for all firms.[4] Thus

$$\alpha c_{1a}x_{1a} + \alpha c_{1b}x_{1b} + \alpha c_{1c}x_{1c} + \alpha c_{1d}x_{1d} + \alpha c_2y_2 + \alpha c_3y_3 = s_1. \tag{1.13}$$

It follows from (1.12) and (1.13) that

$$\alpha R = s_1 \tag{1.14}$$

and that the quantity of the polluting output s_1 is indeed fixed. That abatement activity itself should require inputs that give rise to pollution was originally suggested by Leontief (1970).

The welfare economic conditions for an efficient allocation of inputs and outputs are derived from the following Lagrangian expression, which incorporates (1.11), (1.12), and (1.13):

$$\begin{aligned}\mathscr{L} = {} & U^1(y_{21}, y_{31}, q) + \lambda_u[U^2(y_{22}, y_{32}, q) - \bar{U}^2] \\ & + \lambda_r[R - c_{1a}x_{1a} - c_{1b}x_{1b} - c_{1c}x_{1c} - c_{1d}x_{1d} - c_2(y_{21} + y_{22}) \\ & - c_3(y_{31} + y_{32})] + \lambda_1[\alpha c_2(y_{21} + y_{22}) + \alpha c_3(y_{31} + y_{32}) \\ & - (1 - \alpha c_{1a})x_{1a} - (1 - \alpha c_{1b})x_{1b} - (1 - \alpha c_{1c})x_{1c} - (1 - \alpha c_{1d})x_{1d}],\end{aligned} \tag{1.15}$$

where $q = e_{1a}x_{1a} + e_{1b}x_{1b} + e_{1c}x_{1c} + e_{1d}x_{1d}$. For the case in which the optimal solution to (1.15) is one in which some of each good is consumed by both households and the intermediate good is produced by a combination of processes 1b and 1c, the conditions for economic efficiency include the following:

$$-(U_q^1/U_2^1 + U_q^2/U_2^2) = \frac{\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)}{c_2}, \tag{1.16}$$

$$U_2^1/U_3^1 = U_2^2/U_3^2 = c_2/c_3. \tag{1.17}$$

Condition (1.16) is identical to (1.8). Condition (1.17), which differs from (1.9), indicates that the optimal marginal rate of substitution is independent of the level of pollution or of abatement. Although both final goods are indirectly polluting, the pollution is proportional to their production costs, and their relative prices do not change. There is a real income effect of technological abatement, causing consumption of final goods to decrease.

In this model, pollution control is accomplished entirely by technological abatement. The output of the intermediate good, which is the source of air pollution, is constant. Shifts in consumption between the final goods and contractions in their total output have no additional impact on the level of pollution. The properties of the Pure Abatement model are such that economic efficiency can be achieved either by emission standards or by emission fees.

Furthermore, economic efficiency can be interpreted in terms of a composite good.

Composite Good

The production-possibility frontier is illustrated in figure 1.2. Each facet, generated by a combination of four processes, is an equilateral trapezoid. The slope $\partial y_2/\partial y_3$ is equal to c_3/c_2 on each of the facets. This is a consequence of the Pure Abatement model in which relative prices of final goods are constant. Hicks (1965, p. 33) has noted that "A collection of physical things can always be treated as if they were divisible into units of a single commodity so long as their relative prices can be assumed to be unchanged in the particular problem at hand." Consequently, this single commodity can be treated as an argument in utility functions. Thus we may define a composite good that is equivalent in cost to, say, $1/c_2$ units of good-two and $1/c_3$ units of good-three, and whose total quantity is $Y = Y_1 + Y_2$.

With this simplification, the Lagrangian expression, becomes

$$\begin{aligned}\mathscr{L} = {} & U^1(Y_1, q) + \lambda_u[U^2(Y_2, q) - \bar{U}^2] \\ & + \lambda_r[R - c_{1a}x_{1a} - c_{1b}x_{1b} - c_{1c}x_{1c} - c_{1d}x_{1d} - Y_1 - Y_2] \\ & + \lambda_1[\alpha(Y_1 + Y_2) - (1 - \alpha c_{1a})x_{1a} - (1 - \alpha c_{1b})x_{1b} \\ & - (1 - \alpha c_{1c})x_{1c} - (1 - \alpha c_{1d})x_{1d}],\end{aligned} \tag{1.18}$$

where $q = e_{1a}x_{1a} + e_{1b}x_{1b} + e_{1c}x_{1c} + e_{1d}x_{1d}$. The condition of interest for the case in which x_{1b} and x_{1c} are nonzero and both households consume private goods is

$$-(U^1_q/U^1_Y + U^2_q/U^2_Y) = (c_{1c} - c_{1b})/(e_{1b} - e_{1c}). \tag{1.19}$$

This condition is illustrated in figure 1.3. The set of production possibilities between air pollution and the composite good is represented by the frontier abcd. The level of utility $\bar{U}^2$ allowed household-two is attainable by combinations of outputs along the indifference curve $\bar{U}^2$. The consumption possibilities left for household-one are the vertical distances between the frontier abcd and the indifference curve $\bar{U}^2$ (because both households have the same exposure). These distances are denoted by the MM′ curve in figure 1.3.

The utility of household-one is maximized by the combination of outputs at which the MM′ curve is tangent to an indifference curve of that household. In figure 1.3 there is a tangency at (q^*, Y_1^*). Because the slope of the

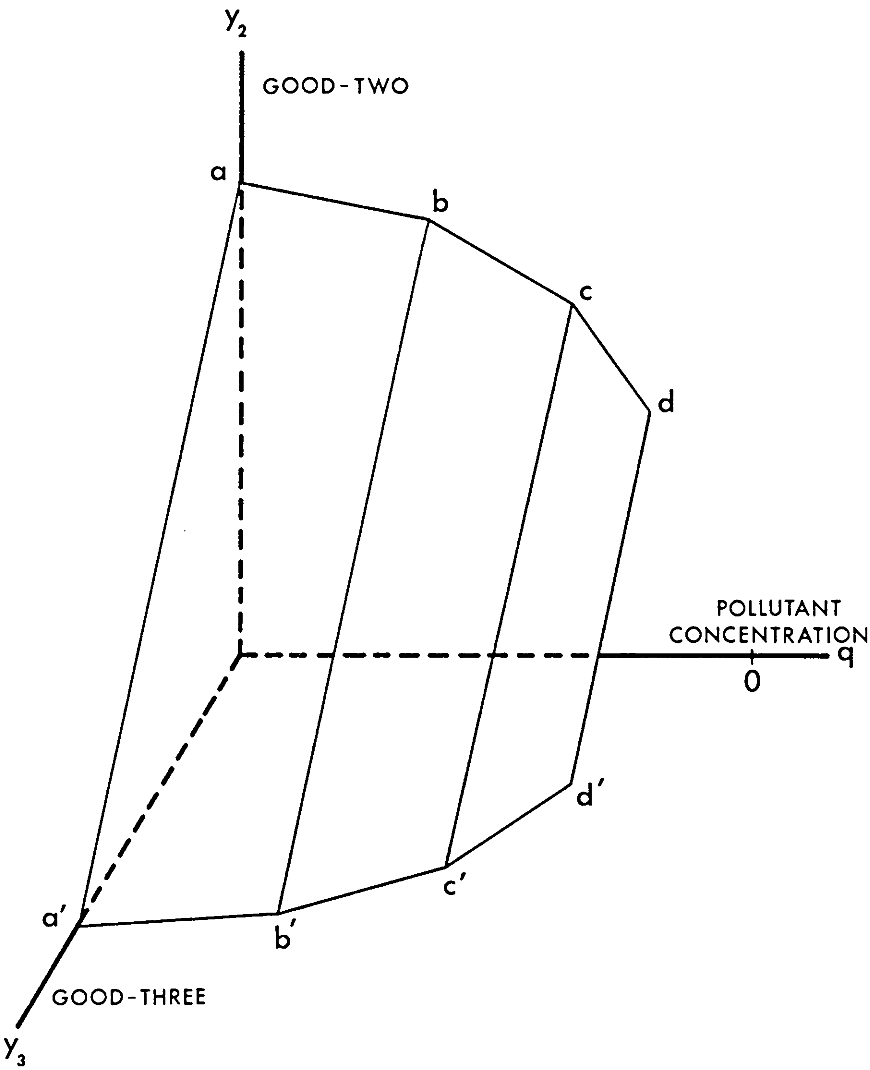

Figure 1.2
Production-possibility frontier for the case in which the polluting good in production is an intermediate good used by all firms in some common fixed proportion to their labor input

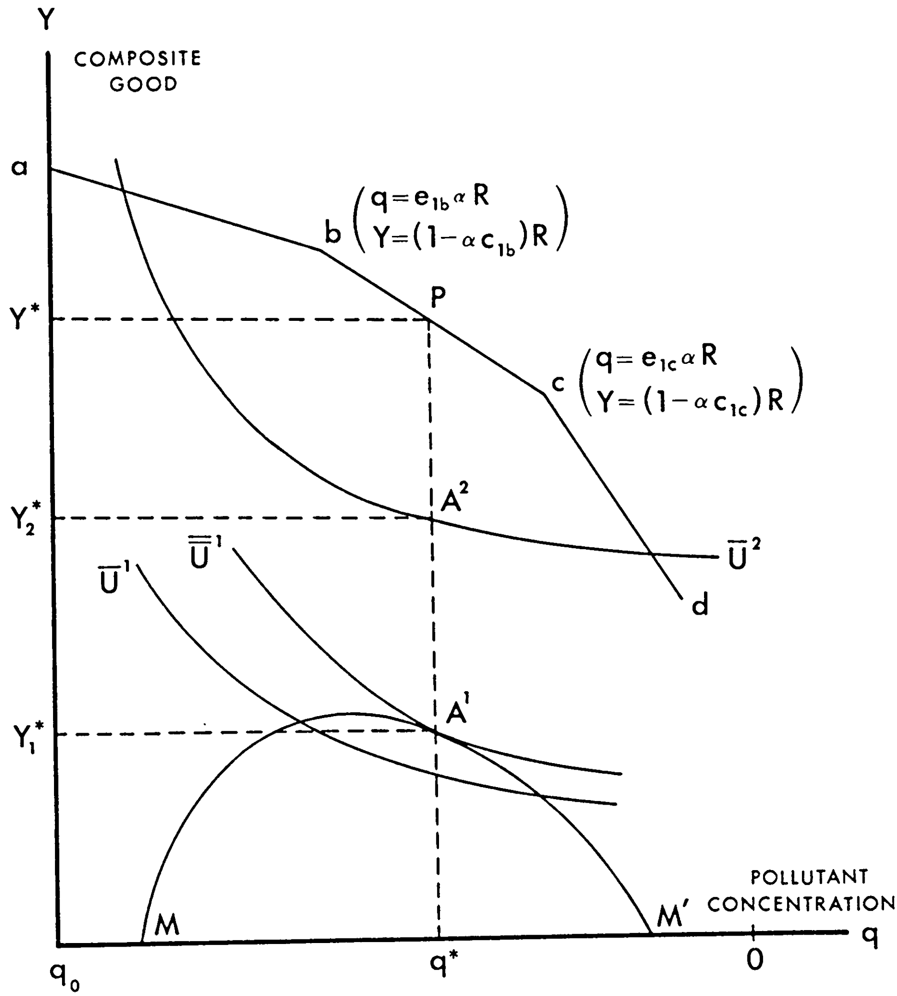

Figure 1.3
An optimal combination of a composite good and air pollution

MM′ curve is the difference between the slopes of the frontier abcd and the curve $\bar{U}^2$, it follows that the slopes of $\bar{U}^1$ at A^1 and $\bar{U}^2$ at A^2 sum to the slope of bc; and therefore condition (1.19) is satisfied. The reader may confirm algebraically that the slope of the line segment bc is indeed $(c_{1c} - c_{1b})/(e_{1b} - e_{1c})$.

It is a simple step to generalize the model so that the composite good is a combination of a great many private goods whose relative prices are fixed. The optimal level of air quality will be such that the summation of each household's trade-off of the composite good for air quality will equal the rate of transformation between the two.

The significance of the Pure Abatement model may be summarized as follows: it is based on the assumption that the intermediate good, which is polluting in production, is used by all producers in some fixed proportion to labor cost. The same result would obtain if s_i represented an intermediate production activity engaged in by producers of final goods. It follows that the level of the polluting activity s_i is fixed and the relative prices of final goods do not change because of abatement. The cost of abatement is reflected in reduced purchasing power for final goods. Given a competitive market economy with this technology, economic efficiency can be achieved either by an appropriate set of emission standards or by Pigouvian fees.[5] Furthermore, the Pure Abatement model permits us to represent an entire set of consumer goods by a single composite good.

The Multipollutant Case

The multipollutant case is characteristic of the real world in which emissions of carbon monoxide, hydrocarbons, nitrogen oxides, sulfur dioxide, particulates, benzo(a)pyrene, and other air pollutants may all be associated with a single production process. Here we extend the model of the preceding section to encompass two pollutants whose respective concentrations are q^1 and q^2. The processes for producing the intermediate good generate pollution as follows:

$$\begin{aligned} e^1_{1a}x_{1a} + e^1_{1b}x_{1b} + e^1_{1c}x_{1c} + e^1_{1d}x_{1d} &= q^1, \\ e^2_{1a}x_{1a} + e^2_{1b}x_{1b} + e^2_{1c}x_{1c} + e^2_{1d}x_{1d} &= q^2. \end{aligned} \tag{1.20}$$

For a production-abatement process to be considered, it must be the case that at least one of its emission coefficients is less than the emission coefficients (for that same pollutant) for each less costly process. This requirement is less stringent than (1.3) in the one-pollutant case.

The first-order conditions for an optimal combination of the composite good and the two-pollutant concentrations are obtained from the Lagrangian,

$$\begin{aligned}\mathscr{L} = {} & U^1(Y_1, q^1, q^2) + \lambda_u[U^2(Y_2, q^1, q^2) - \bar{U}^2] \\ & + \lambda_r[R - c_{1a}x_{1a} - c_{1b}x_{1b} - c_{1c}x_{1c} - c_{1d}x_{1d} - Y_1 - Y_2] \\ & + \lambda_1[\alpha(Y_1 + Y_2) - (1 - \alpha c_{1a})x_{1a} - (1 - \alpha c_{1b})x_{1b} \\ & - (1 - \alpha c_{1c})x_{1c} - (1 - c_{1d})x_{1d}],\end{aligned} \tag{1.21}$$

with pollution levels given by (1.20) above. For the case in which both households consume the composite good and x_{1a}, x_{1b}, and x_{1c} are positive, the Kuhn-Tucker conditions yield

$$(U^1_{q^1}/U^1_Y + U^2_{q^1}/U^2_Y) = \frac{(c_{1c} - c_{1b})(e^2_{1a} - e^2_{1b}) - (c_{1b} - c_{1a})(e^2_{1b} - e^2_{1c})}{(e^1_{1a} - e^1_{1b})(e^2_{1b} - e^2_{1c}) - (e^1_{1b} - e^1_{1c})(e^2_{1a} - e^2_{1b})} \tag{1.22}$$

$$(U^1_{q^2}/U^1_Y + U^2_{q^2}/U^2_Y) = \frac{-(c_{1c} - c_{1b})(e^1_{1a} - e^1_{1b}) + (c_{1b} - c_{1a})(e^1_{1b} - e^1_{1c})}{(e^1_{1a} - e^1_{1b})(e^2_{1b} - e^2_{1c}) - (e^1_{1b} - e^1_{1c})(e^2_{1a} - e^2_{1b})}. \tag{1.23}$$

These conditions are illustrated in figure 1.4 for the allocations A^1 and A^2. A plane cutting through q^{1*}, parallel to the q^2Y plane, would intersect the convex possibility frontier abcd along two facets. The allocations A^1 and A^2 would lie on a vertical line at q^{2*}, and, as in figure 1.3, the slope of the outward flaring indifference surfaces at A^1 and A^2, that is $U^1_{q^2}/U^1_Y$ and $U^2_{q^2}/U^2_Y$, would sum to the slope $\partial Y/\partial q^2$ of the facet abc. This slope is given by the ratio on the right-hand side of equation (1.23). Likewise, the condition for the optimal allocation of the composite good and pollutant-one could be illustrated on a plane through q^{2*}, parallel to the Yq^1 plane.

The three-dimensional production-possibility frontier in figure 1.4 for two concentrations and a composite good may be compared to the frontier in figure 1.1 for a single concentration and two goods. In the latter the vertices of each facet lie on the $y_1 = 0$ plane and $y_2 = 0$ plane, whereas in the former there are vertices in the interior space. The simplex in figure 1.4 contains three facets, abc, abd, and bcd, whose outer directed normals point in the direction of more Y and lower concentrations of q^1 and q^2. The fourth facet of the tetrahedron, acd, lies under the other three facets and contains technically inefficient combinations of outputs.

There are a number of empirical models in which a single pollutant is

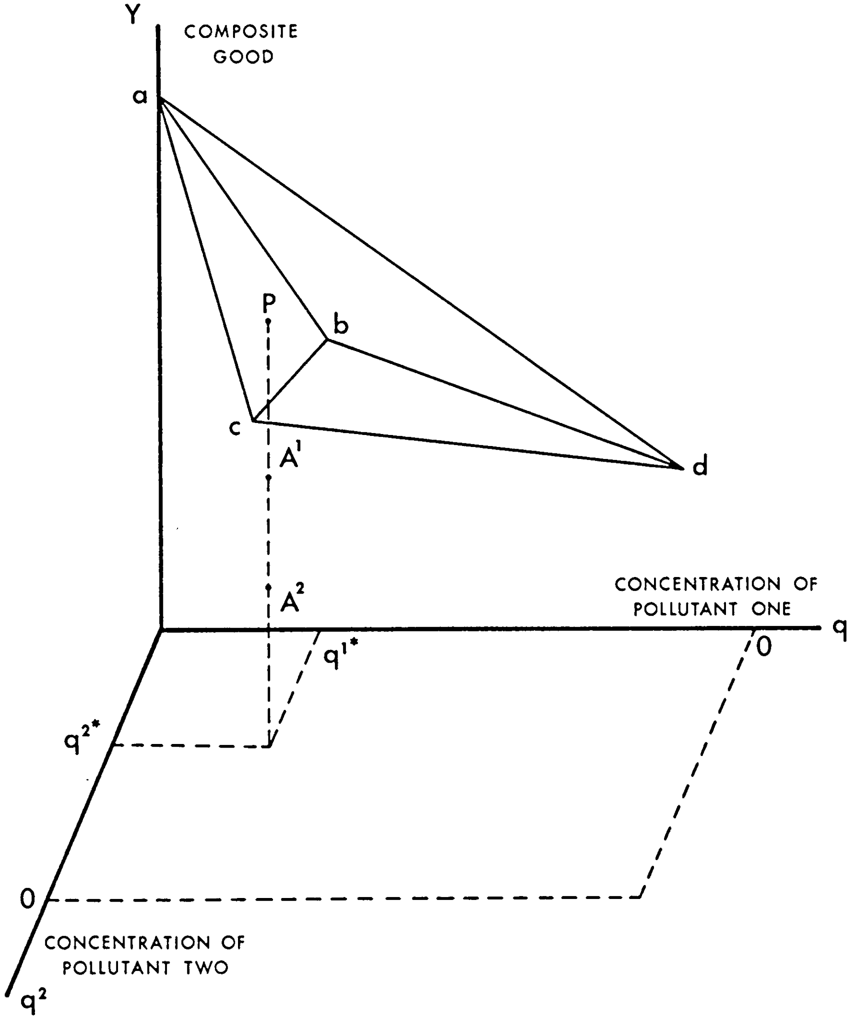

Figure 1.4
Production-possibility frontier for a composite good and two pollutant levels

controlled (see, for example, Kohn, 1968, Atkinson and Lewis, 1974, Shepard, 1970, Guldmann, 1973, and Wilson and Minnotte, 1969). In general, such models overstate the cost of controlling that particular pollutant because they fail to give credit for joint reductions of other pollutants. Although there are control methods that increase the flow of some pollutants while decreasing others, it is generally the case that the emission difference terms, such as those in (1.22) and (1.23), are positive. Therefore, the level of air quality that is optimal in a one-pollutant model is likely to be less stringent than the corresponding level in a multipollutant model.

The Linear Programming Model

Let us assume that an optimal pair of air quality levels, q^1 and q^2, like those in the solution of (1.21), are known in advance. The efficient production abatement technology would be the solution of the following linear programming model:

$$\text{Maximize } Y = R - c_{1a}x_{1a} - c_{1b}x_{1b} - c_{1c}x_{1c} - d_{1d}x_{1d}$$

subject to

$$\begin{aligned} e^1_{1a}x_{1a} + e^1_{1b}x_{1b} + e^1_{1c}x_{1c} + e^1_{1d}x_{1d} &\leqslant q^1, \\ e^2_{1a}x_{1a} + e^2_{1b}x_{1b} + e^2_{1c}x_{1c} + e^2_{1d}x_{1d} &\leqslant q^2, \\ x_{1a} + x_{1b} + x_{1c} + x_{1d} &= \alpha R, \\ x_{1a}, x_{1b}, x_{1c}, x_{1d} &\geqslant 0. \end{aligned} \tag{1.24}$$

In this model R is a constant; the sum of production activities for making the intermediate good is fixed at αR because of the technological assumption underlying the Pure Abatement model.

Each unit cost coefficient c_j consists of production cost and abatement cost. The latter will be called C_j. It follows that the difference

$$c_{1a} - C_{1a} = c_{1b} - C_{1b} = c_{1c} - C_{1c} = c_{1d} - C_{1d} = \sigma_1 \tag{1.25}$$

represents production cost alone. The objective function in (1.24) is equivalent to

$$\begin{aligned} \text{Maximize } Y = R &- (\sigma_1 + C_{1a})x_{1a} - (\sigma_1 + C_{1b})x_{1b} \\ &- (\sigma_1 + C_{1c})x_{1c} - (\sigma_1 + C_{1d})x_{1d}, \end{aligned} \tag{1.26}$$

which in turn is equal to

$$\text{Maximize } Y = R - \sigma_1(\alpha R) - C_{1a}x_{1a} - C_{1b}x_{1b} - C_{1c}x_{1c} - C_{1d}x_{1d}. \quad (1.27)$$

Because R and $\sigma_1(\alpha R)$ are constants, maximization of Y implies minimization of the negative quantities. Accordingly, we may substitute a new objective function

$$\text{Minimize } Z = C_{1a}x_{1a} + C_{1b}x_{1b} + C_{1c}x_{1c} + C_{1d}x_{1d}, \quad (1.28)$$

where Z is the total cost of pollution abatement.

The larger the outlay for abatement Z, the less the production of the composite good Y, which is a residual. The true cost of cleaner air is the opportunity cost of foregone consumption of the composite good. The value of the foregone comsumption is actually greater than Z. This is illustrated in figure 1.5, which depicts the demand curve (a rectangular hyperbola) of the ith household for the composite good at a given level of income. Prior to the establishment of, say, emission standards, the price of the composite good is $\hat{p}$. After abatement the price is $\bar{p}$. The cost of abatement borne by the ith household is Z_i, which is the area of either rectangle in figure 1.5. The reduced consumption $\hat{Y}_i - \bar{Y}_i$ is undervalued at $\hat{p}$ and overvalued at $\bar{p}$. The correct value of the foregone consumption, if we may borrow the partial equilibrium concept of consumer's surplus, is Z_i plus the shaded area under the demand curve.

In the Linear Programming Model, the optimal pollutant concentrations are achieved by technological abatement alone. There are no contractions in units of output, nor substitutions in consumption, that would diminish the level of the polluting activity itself. Thus, our linear programming model has the special properties of the Pure Abatement model.

Shadow Prices and Convexity

The revised linear programming model is

$$\text{Minimize } Z = C_{1a}x_{1a} + C_{1b}x_{1b} + C_{1c}x_{1c} + C_{1d}x_{1d}$$

subject to

$$\begin{aligned} e^1_{1a}x_{1a} + e^1_{1b}x_{1b} + e^1_{1c}x_{1c} + e^1_{1d}x_{1d} &\leqslant q^1, \\ e^2_{1a}x_{1a} + e^2_{1b}x_{1b} + e^2_{1c}x_{1c} + e^2_{1d}x_{1d} &\leqslant q^2, \\ x_{1a} + x_{1b} + x_{1c} + x_{1d} &= \alpha R, \\ x_{1a}, x_{1b}, x_{1c}, x_{1d} &\geqslant 0. \end{aligned} \quad (1.29)$$

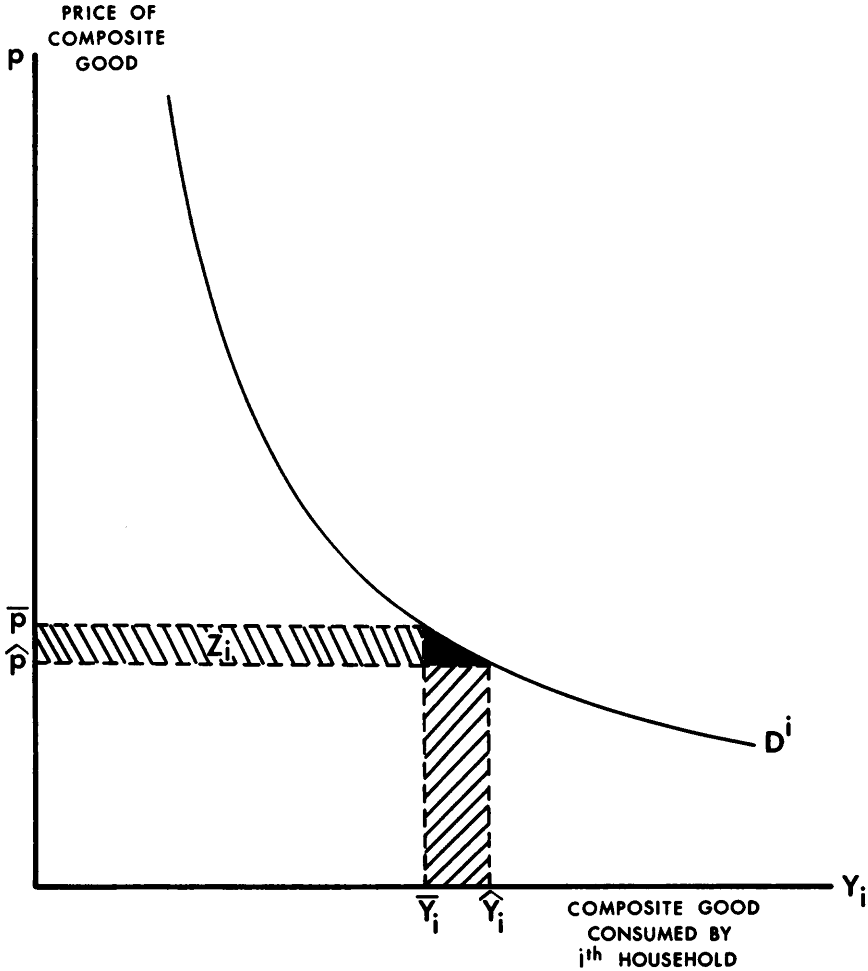

Figure 1.5
A household's demand curve for the composite good

Associated with the solution of (1.29) is a set of shadow prices $\{\pi^1, \pi^2, \pi_1\}$, each of which represents the change in Z when the corresponding constraint (on q^1, q^2, or αR, respectively) is increased by one unit, holding the other two constraints constant. For a solution in which, say, process 1a, 1b, and 1c are active, the vector of shadow prices $[\pi^1, \pi^2, \pi_1]$ is equal to the cost vector times the inverse of the basis matrix (see Hadley, 1962, p. 230), that is,

$$[C_{1a}\ C_{1b}\ C_{1c}]\begin{bmatrix} e^1_{1a} & e^1_{1b} & e^1_{1c} \\ e^2_{1a} & e^2_{1b} & e^2_{1c} \\ 1 & 1 & 1 \end{bmatrix}^{-1}. \tag{1.30}$$

There is a method for calculating shadow prices that is based on Cramer's Rule and is particularly useful when more than two pollutants are involved. In this method (see Dwyer, 1951, p. 138), the ith shadow price (that is, the ith entry in the row vector of shadow prices) is equal to the following ratio of determinants. The denominator of the ratio is the determinant of the basis matrix and the numerator is that same determinant with the ith row replaced by the row vector of abatement costs. For example, π^2 in (1.30) is

$$\pi^2 = \frac{\begin{vmatrix} e^1_{1a} & e^1_{1b} & e^1_{1c} \\ C_{1a} & C_{1b} & C_{1c} \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} e^1_{1a} & e^1_{1b} & {}^1_{1c} \\ e^2_{1a} & e^2_{1b} & e^2_{1c} \\ 1 & 1 & 1 \end{vmatrix}}. \tag{1.31}$$

Using (1.25) to express the results in terms of the c_j, it follows that

$$\pi^2 = \frac{-(c_{1c} - c_{1b})(e^1_{1a} - c^1_{1b}) + (c_{1b} - c_{1a})(e^1_{1b} - e^1_{1c})}{(e^1_{1a} - e^1_{1b})(e^2_{1b} - e^2_{1c}) - (e^1_{1b} - e^1_{1c})(e^2_{1a} - e^2_{1b})}. \tag{1.32}$$

By similar calculations,

$$\pi^1 = \frac{(c_{1c} - c_{1b})(e^2_{1a} - e^2_{1b}) - (c_{1b} - c_{1a})(e^2_{1b} - e^2_{1c})}{(e^1_{1a} - e^1_{1b})(e^2_{1b} - e^2_{1c}) - (e^1_{1b} - e^1_{1c})(e^2_{1a} - e^2_{1b})} \tag{1.33}$$

and

$$\pi_1 = c_{1a} - e^1_{1a}\pi^1 - e^2_{1a}\pi^2. \tag{1.34}$$

The meaning of π_1 is as follows. If the output of the intermediate good $s_1 = \alpha R$ is increased by one unit and the allowable flows of the two pollutants are held constant, there will be an increase in resource cost equal to c_{1a}, which is the direct labor cost of producing good-one with process 1a, plus the costs of eliminating the incremental emissions associated with one unit of activity of process 1a. These costs are the pollutant shadow prices times the incremental emissions e_{1a}^1 and e_{1a}^2. Because three processes are used, the shadow price of good-one can also be expressed in terms of the coefficients for process 1b or 1c; that is,

$$\pi^1 = c_{1b} - e_{1b}^1\pi^1 - e_{1b}^2\pi^2 = c_{1c} - e_{1c}^1\pi^1 - e_{1c}^2\pi^2. \tag{1.35}$$

One of the conditions for a convex production-possibility frontier between Y, q^1, and q^2 is that the right-hand-side values in (1.32) and (1.33) be negative.[6] This imposes certain conditions on the values of the coefficients. Observe that the denominators in (1.32) and (1.33) are identical and can be either positive or negative. For both right-hand-side values to be negative, the following conditions must hold. Let us first assume that the six terms that comprise these ratios are all positive. If the common denominator is less than zero, it must be the case that

$$\frac{c_{1b} - c_{1a}}{e_{1a}^2 - e_{1b}^2} < \frac{c_{1c} - c_{1b}}{e_{1b}^2 - e_{1c}^2}, \tag{1.36}$$

$$\frac{c_{1b} - c_{1a}}{e_{1a}^1 - e_{1b}^1} > \frac{c_{1c} - c_{1b}}{e_{1b}^1 - e_{1c}^1}.$$

If the common denominator is greater than zero, the inequalities in (1.36) are reversed.

If one of the six difference terms, say $(e_{1a}^2 - e_{1b}^2)$, is less than or equal to zero, it is only necessary that

$$\frac{c_{1b} - c_{1a}}{e_{1a}^1 - e_{1b}^1} < \frac{c_{1c} - c_{1b}}{e_{1b}^1 - e_{1c}^1}. \tag{1.37}$$

If $(e_{1b}^2 - e_{1c}^2)$ alone is less than or equal to zero, it is only necessary that

$$\frac{c_{1b} - c_{1a}}{e_{1a}^1 - e_{1b}^1} < \frac{c_{1c} - c_{1b}}{e_{1b}^1 - e_{1c}^1}. \tag{1.38}$$

If both $(e_{1a}^2 - e_{1b}^2)$ and $(e_{1b}^2 - e_{1c}^2)$ are negative, the same argument that led to condition (1.36) applies. The reader may determine the remaining conditions on the signs of the emission difference terms such that both right-hand-side ratios in (1.32) and (1.33) are negative. It is of interest that these conditions allow for production-abatement processes that increase the flow of some pollutants while decreasing that of others.

Multiple Sources of Pollution

We have assumed that all of the air pollution occurs in the production of a single intermediate good. We extend the model by assuming that there are many such intermediate goods, each of which is required by firms in some proportion α_j to their labor input. In the case of two intermediate outputs s_1 and s_2, which may be produced by a combination of two and three processes, respectively, we have

$$\begin{aligned} \alpha_1(c_{1a}x_{1a} + c_{1b}x_{1b} + c_{2a}x_{2a} + c_{2b}x_{2b} + c_{2c}x_{2c} + Y) &= x_{1a} + x_{1b} = s_1, \\ \alpha_2(c_{1a}x_{1a} + c_{1b}x_{1b} + c_{2a}x_{2a} + c_{2b}x_{22} + c_{2c}x_{2c} + Y) &= x_{2a} + x_{2b} + x_{2c} = s_2. \end{aligned} \tag{1.39}$$

It is equivalent to view the s_i not as intermediate goods but as production activity levels that take place in the factories where final goods are manufactured. In this model there are no price-induced shifts in consumption by households that would alter the polluting activity levels s_i. This together with the assumption that the single resource is inelastically supplied fixes the levels of the outputs that are polluting in production. Furthermore, each household can be presumed to have a utility function (in terms of the pollutant levels and a composite good) that does not shift with the level of abatement outlay Z.

These assumptions underlie the empirical model presented in chapters 2, 3, 4, and 5. However, they are not crucial to empirical analysis, and in the final chapters the model is revised to allow for changes in the quantity of resources and for shifts in relative prices with consequent feedbacks on the level of polluting activities.

APPENDIX: EQUIVALENCE OF EMISSION STANDARDS AND POLLUTION FEES AND A NUMERICAL EXAMPLE

In this appendix we demonstrate that in the Pure Abatement model, economic efficiency can be achieved by either emission standards or pollution fees. This is illustrated with a numerical example.

Conditions (1.16) and (1.17) may be satisfied in a competitive market economy by either emission standards or emission fees. If the government could predetermine the optimal level of q^* and the corresponding abatement activities such that

$$e_{1b}x_{1b}^* + e_{1c}x_{1c}^* = q^*, \tag{1.40}$$

it could establish an emission standard of $[\gamma e_{1b} + (1 - \gamma)e_{1c}]$ units of pollution per unit of output of the intermediate good, where

$$\gamma = x_{1b}^*/(x_{1b}^* + x_{1c}^*)$$

and (1.41)

$$x_{1b}^* + x_{1c}^* = s_1 = \alpha R.$$

The price of the intermediate good in this market economy would be equal to its marginal cost,

$$\begin{aligned} p_1 &= [\gamma c_{1b} + (1 - \gamma)c_{1c}] + p_1\alpha[\gamma c_{1b} + (1 - \gamma)c_{1c}] \\ &= [\gamma c_{1b} + (1 - \gamma)c_{1c}]/[1 - \alpha(\gamma c_{1b} + (1 - \gamma)c_{1c})]. \end{aligned} \tag{1.42}$$

The prices of the final goods would be

$$\begin{aligned} p_2 &= c_2 + p_1\alpha c_2 = c_2(1 + \alpha p_1), \\ p_3 &= c_3 + p_1\alpha c_3 = c_3(1 + \alpha p_1). \end{aligned} \tag{1.43}$$

The reader may confirm, using (1.12) and (1.41), that

$$p_2 y_2 + p_3 y_3 = R. \tag{1.44}$$

Letting one unit of R represent one dollar, it follows from (1.44) that the value of final goods will be equal to consumer income. This is essential if households are to have sufficient income to purchase the entire output of goods.

The ratio, p_1/p_2, of selling prices in (1.43) is equal to c_2/c_3, and if households maximize their utility, condition (1.17) will be satisfied. To confirm that the level of pollution is optimal, the government can ascertain that the marginal benefits of abatement, which equal $-p_2(U_q^1/U_2^1 + U_q^2/U_2^2)$, must equal the marginal cost of abatement. Allowing for purchases of the intermediate good, the latter is $[(c_{1c} - c_{1b})/(e_{1b} - e_{1c})][1 + \alpha p_1]$. This equivalence satisfies condition (1.16).

If the government controls pollution by means of emission fees, the optimal fee ϕ corresponding to (1.40) would be

$$\phi = \left(\frac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)(1 + \alpha p_1); \tag{1.45}$$

and the equilibrium price of the intermediate good would be

$$p_1 = c_{1b} + \alpha c_{1b} p_1 + e_{1b}\phi. \tag{1.46}$$

It follows by simultaneous solution that in equilibrium

$$\phi = \frac{\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)}{1 - \alpha\left[c_{1b} + e_{1b}\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)\right]} \tag{1.47}$$

and

$$p_1 = \frac{c_{1b} + e_{1b}\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)}{1 - \alpha\left[c_{1b} + e_{1b}\left(\dfrac{c_{1c} - c_{1b}}{e_{1b} - e_{1c}}\right)\right]}. \tag{1.48}$$

Denoting the common denominator in (1.47) and (1.48) by Ω, it can be determined that $p_2 = c_2/\Omega$ and $p_3 = c_3/\Omega$. Assuming that producers of good-one voluntarily adopted the divisible solution $\{x^*_{1b}, x^*_{1c}\}$, conditions (1.16) and (1.17) would be satisfied in the competitive market equilibrium based on emission fees.

Although absolute prices differ if pollution is controlled by emission fees rather than by emission standards, relative prices are the same and both programs are efficient. In the absence of government control of pollution, producers of the intermediate good would use process 1a and the level of pollution would be

$$q = e_{1a}\alpha R. \tag{1.49}$$

With government control, whether by standards or fees, the level of pollution would be

$$q = [\gamma e_{1b} + (1 - \gamma)e_{1c}]\alpha R. \tag{1.50}$$

The decline in pollution is a consequence of technological abatement alone. There is no income effect on output of the polluting intermediate good, nor are there any shifts in consumption that would alter the level of pollution.

This case is illustrated with the following numerical example. Consider a simple economy consisting of two households whose utility functions are

$$\begin{aligned} U^1 &= 1200 \ln Y_1 + 13 \ln (5000 - q^1) + 50 \ln (6000 - q^2), \\ U^2 &= 348 \ln Y_2 + \ln (5000 - q^1) + 10 \ln (6000 - q^2), \end{aligned} \tag{1.51}$$

where Υ_i is the quantity of output (represented by a composite good) consumed by the ith household and q^j is the level of the jth pollutant.

Pollution in this economy originates in the production of an intermediate good which is used by all firms including producers of the intermediate good, in quantities equal to 20 percent of their labor input. The total quantity of labor in this economy is 2500 units. Pollution can be reduced by using alternative processes for making the intermediate good. The production technology is described by the following equations:

$$\begin{aligned} 1.1x_{1a} + 1.2x_{1b} + 1.4x_{1c} + 1.45x_{1d} + 1.8x_{1e} + \Upsilon &= 2500, \\ 10x_{1a} + 8x_{1b} + 7x_{1c} + 7x_{1d} + 3x_{1e} &= q^1, \\ 12x_{1a} + 10x_{1b} + 5x_{1c} + 3x_{1d} + 4x_{1e} &= q^2, \\ -.78x_{1a} - .76x_{1b} - .72x_{1c} - .71x_{1d} - .64x_{1e} + .2\Upsilon &= 0. \end{aligned} \tag{1.52}$$

The first equation is the resource constraint, while the final equation is equivalent to

$$\begin{aligned} &.2(1.1x_{1a} + 1.2x_{1b} + 1.4x_{1c} + 1.45x_{1d} + 1.8x_{1c} + \Upsilon) \\ &\quad = x_{1a} + x_{1b} + x_{1c} + x_{1d} + x_{1e}. \end{aligned} \tag{1.53}$$

There are an infinite number of Pareto optimal allocations; one of these is

$$\begin{aligned} &\Upsilon = 1870,\ \Upsilon_1 = 1000,\ \Upsilon_2 = 870, \\ &q^1 = 4200,\ q^2 = 4000, \\ &x_{1a} = 200,\ x_{1b} = 100,\ x_{1d} = 200. \end{aligned} \tag{1.54}$$

It can be shown that this allocation satisfies conditions analogous to (1.19). When these three processes are nonzero, the rates of transformation in production are

$$\partial\Upsilon/\partial q^1 = -1/60 \quad \text{and} \quad \partial\Upsilon/\partial q^2 = -1/30.$$

The allocation satisfies the conditions for a Pareto optimum because

$$\begin{aligned} U^1_{q^1}/U^1_Y + U^2_{q^1}/U^2_Y &= \partial\Upsilon/\partial q^1, \\ \frac{-13\Upsilon_1}{(5000 - q_1)\,1200} + \frac{-\Upsilon_2}{(5000 - q^1)\,348} &= -\frac{1}{60}, \end{aligned} \tag{1.55}$$

and

$$\begin{aligned} U^1_{q^2}/U^1_Y + U^2_{q^2}/U^2_Y &= \partial\Upsilon/\partial q^2, \\ \frac{-50\Upsilon_1}{(6000 - q^2)(1200)} - \frac{10\Upsilon_2}{(6000 - q^2)(348)} &= -\frac{1}{30}. \end{aligned} \tag{1.56}$$

This solution could be achieved by either of two governmental programs, emissions standards or emission fees. If the allowable rates of emissions for pollutants one and two were set at 8.4 and 8.0, respectively, producers of the intermediate good would meet the standard at least-cost by using 40 percent of process 1a, 20 percent of 1b, and 40 percent of 1d. The prices of the intermediate and composite goods would be 315/187 and 250/187, respectively (see equations 1.42 and 1.43 above); and the maximum producible output of the latter would be 2500/(250/187), or 1870 units.

Alternatively, emission fees of 1/40 per unit of pollutant-one and 1/20 per unit of pollutant-two would result in market prices of 2.5 and 1.5 for the intermediate and composite goods respectively.[7] These fees would prompt producers of the intermediate good to use some combination of processes 1a, 1b, and 1d. Assuming, for convenience, that the 40 percent, 20 percent, 40 percent combination were chosen, the pollution levels would be $q^1 = 4200$ and $q^2 = 4000$. The fee revenue to the government, which would equal 305, could be transferred to the households so that disposable income would be 2805. The equilibrium output of the composite good would therefore be 2805/1.5, or 1870.

A simplified version of the model is one in which the Pareto optimal levels of pollution are given in advance and a least-cost set of abatement processes determined. Assuming that the pure production cost of the intermediate good is 1.0, the optimal combination of processes is the solution of the following linear programming problem:

Minimize

$$Z = .1x_{1a} + .2x_{1b} + .4x_{1c} + .45x_{1d} + .8x_{1e}$$

subject to

$$\begin{aligned} 10x_{1a} + 8x_{1b} + 7x_{1c} + 7x_{1d} + 3x_{1e} &\leqslant 4200, \\ 12x_{1a} + 10x_{1b} + 5x_{1c} + 3x_{1d} + 4x_{1e} &\leqslant 4000, \\ x_{1a} + x_{1b} + x_{1c} + x_{1d} + x_{1e} &= 500, \\ x_{1a}, x_{1b}, x_{1c}, x_{1d}, x_{1e} &\geqslant 0. \end{aligned} \tag{1.57}$$

The objective function of (1.57) is the total cost of abatement, while the equality constraint is derived from (1.52).

The five processes in (1.57) define a production-possibility frontier between q^1, q^2, and Z. This frontier is illustrated in figure 1.6. The facets of

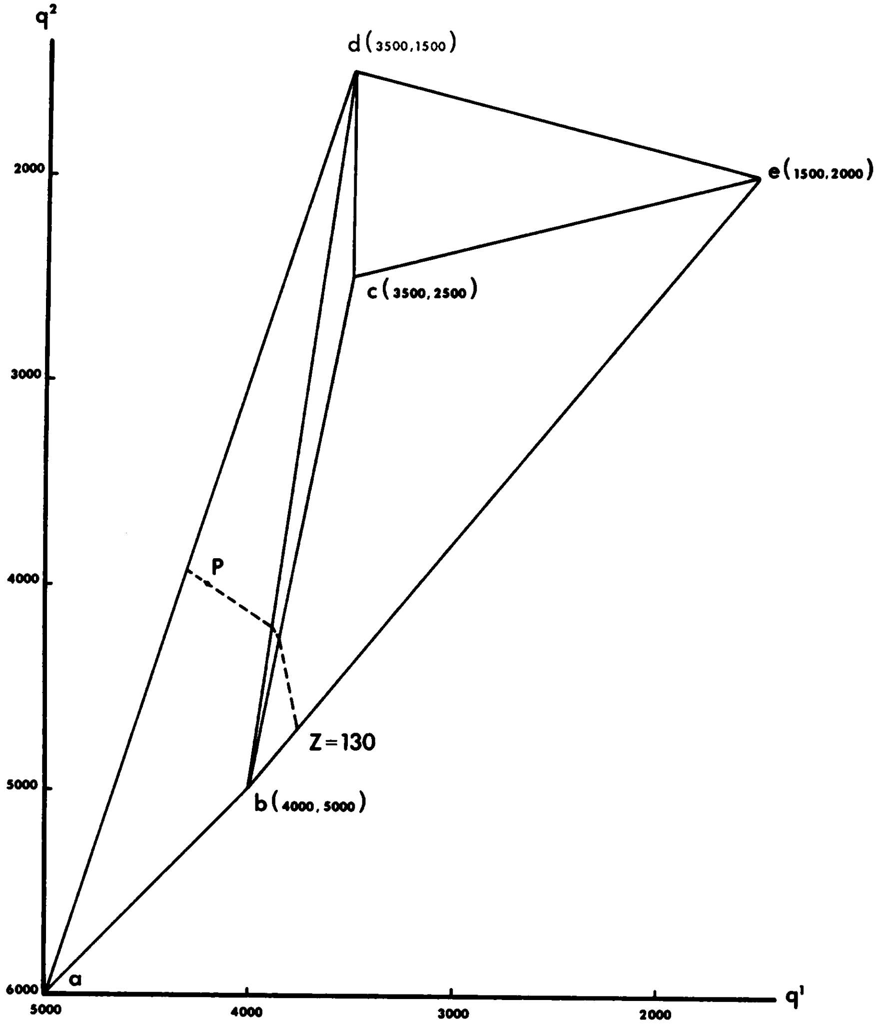

Figure 1.6
Production-possibility frontier for the numerical example

this frontier are projected on to the q^1q^2 plane. Because there are five processes, there can be no more than four facets containing technically efficient combinations of outputs. There are three other combinations of processes, (1a, 1b, 1c), (1a, 1d, 1e), and (1b, 1d, 1e), that generate facets with negative pollutant shadow prices; however, these facets lie behind the ones shown in figure 1.6. The pollutant shadow prices for the respective facets are as follows:

$$\begin{aligned} &\text{facet abd}: \pi^1 = -.01667, \ \pi^2 = -.03333; \\ &\text{facet bcd}: \pi^1 = -.075, \quad \pi^2 = -.025; \\ &\text{facet cde}: \pi^1 = -.09375, \ \pi^2 = -.025; \\ &\text{facet bce}: \pi^1 = -.09474, \ \pi^2 = -.02105. \end{aligned} \tag{1.58}$$

The optimal solution to (1.58) is denoted in figure 1.6 by the point P on the dotted frontier $Z = 130$. Frontiers for higher values of Z are northeast of the one illustrated.

Note that in moving along an isoquant such as $Z = 130$, the shadow price of one pollutant decreases while that of the other increases (or, in one case, stays the same). This is a condition of convexity of the production-possibility frontier.

This numerical example illustrates that in the case of the Pure Abatement model, both emission standards and pollution fees can promote economic efficiency.