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Twin Careers: Public Servant and Mathematician

Jean Baptiste Joseph Fourier was born at Auxerre in the province of the Yonne on 21 March 1768. The name "Joseph" was that of his father, a tailor of the town: the name "Fourier" is a variant on the word *fourrier*, which means in a military sense a quartermaster and in a figurative connotation a precursor or anticipator of ideas. More often than not during his life Fourier was to have his name spelled in that way; and inaccurate though it might have been, it admirably denoted the activity of a genius whose versatility encompassed both a distinguished administrative and diplomatic career and also scientific achievements of a truly revolutionary character. And these successes did not follow from a life of serene security: on the contrary, Fourier had to fight many social and political obstacles in the course of his career.[1]

The first of his difficulties arose while he was still a child. Before his ninth year both parents (of whom he was the nineteenth and not the last child) were dead, and only the support of a certain Madame Moitton in the town saved him from a life of apprenticeship and servitude. She recommended him to the Bishop of Auxerre, who in his turn sent him to the town's military school run by the Benedictines. Here his prospects were much brighter: if all went well, there was a chance that one day he might be able to pursue his special fields of interest at the Benedictine Collège de Montaigu in Paris. Soon he was showing the greatest promise, for around the age of thirteen he discovered mathematics and gave all his spare time to its study; but his main ambition at this time was a military rather than an intellectual career. He hoped very much to join either the artillery or the engineers (which were then open to everybody), and the school inspectors—one of whom was the mathematician Adrien Marie Legendre (1752–1833)—supported these ambitions. But for some reason he was turned down,[2]

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[1]The general biographical details in this chapter are drawn mainly from: D. F. J. Arago *1838a*, lxxii–ci; *Oeuvres*, 1, 298–330. J. J. Champollion-Figeac *1844a*, 1–43. V. Cousin *1831b*, 1–34. E. Duché *1871a*, 217–236. G. G. Mauger *1837a*, 270–275. V. Parisot *1856a*, 525–529. We mention these works (written by men who, apart from Duché, knew Fourier personally) only in connection with specific points of detail or controversy; but we must also record that they contradict each other at times.

[2]See V. Cousin *1831b*, 2 (especially the footnote).

and instead he entered the Benedictine abbey of St. Benoît-sur-Loire as a novice in 1787. He was put in charge of the teaching program there and worked sufficiently well on some results that he had found in the theory of equations to be able to send a paper to the Académie des Sciences in Paris in 1789. Legendre was one of the examiners; another was Gaspard Monge (1746–1818), also a leading mathematician of the time whose own life had had a modest beginning similar to Fourier's and who later was to exercise great influence on Fourier's career.[3] But that influence was not to begin to make itself felt in 1789, for that was the year of the French Revolution and Fourier's paper was one of its casualties. Fourier himself took the opportunity to relinquish his novitiate, and he returned to Auxerre to take up a teaching post in his old school. He was called on to teach not only mathematics but also rhetoric, philosophy, and history; and he seems to have carried out all these duties with remarkable success, building up a fine reputation as a lecturer. He also began to show his political and social sense, becoming a member both of the local Société Populaire associated with the revolutionary Jacobin party, and also of the Comité de Surveillance which enforced governmental decrees in the region. He tried his best to defend the people both from tyrannical leadership and from themselves: a forceful orator, he spoke so well at a meeting of the Société Populaire at Auxerre that the quota of men required from the area for the defense of the country was drawn from genuine volunteers, while on the other hand he worked hard for the protection of victims of tyranny.[4] His outspoken criticism of corruption among his fellow officials led to the issue of a decree in 1794 demanding his arrest and summary guillotining.[5] This was Fourier's baptism into real politics. He

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[3]The third examiner was the analyst Jacques Antoine Cousin (1739–1800). BN MFF 22511/76–78 are the last four pages of what appears to be a copy of this paper.

For letters from Fourier at St. Benoît-sur-Loire during 1788–1789 to his former teacher Bonard in Auxerre, see A. Challe *1858a*, 106–112, especially the letter of September 1789 on p. 109 where Fourier confided his hopes for the publication of his paper.

[4]V. Cousin recounts a fine example of Fourier's conduct in *1831b*, 4–5.

[5]Fourier later wrote to Bonard on 28 October 1794 from Montargis in the Loiret department, asking him to find out details of the affair [see A. Challe *1858a*, 113–114]. Previously he had thought of being made librarian of Auxerre when public libraries were being set up [see E. Duché *1871a*, 223].

went in person to Robespierre in Paris to plead his case; but he was unsuccessful, for on his return to Auxerre the Comité de Surveillance issued an order for his arrest. The public outcry in the town against the order secured its repeal, but it was reissued eight days later and Fourier was arrested and imprisoned. This time a deputation went on his behalf to St. Just, who grudgingly agreed to his release and was about to frame the order when Robespierre was arrested and executed on 28 July 1794, and Fourier was released anyway on a general amnesty. He returned to his school in Auxerre, but was then affected by an event of great significance: the opening of the Ecole Normale.

The Ecole Normale was a national college set up in Paris by a decree of the Convention to help repair the virtual breakdown in the system of higher education. It was to provide training in all branches of contemporary learning, as opposed to the specialization inherent in the structure of previous schools. There were to be 1500 students, each one chosen and financed by a district of the Republic. By the time of Fourier's release from prison Auxerre had already chosen its representative, but the neighboring district of St. Florentin invited him to go under their sponsorship. The professors were drawn from among the foremost men in the land: for example, Joseph Louis Lagrange (1736–1813) and Pierre Simon Laplace (1749–1827) taught mathematics, and Monge descriptive geometry. The school opened on 20 January 1795, but lasted only a few months. The failure was partly due to the severity of the winter weather, which caused the already impoverished students to live under conditions of great physical hardship; but there were other difficulties, caused by the poor organization of the teaching. Fourier himself described some of them in a letter: the lecture hall in the Jardin des Plantes was not big enough to hold all the students, often causing many of them to be locked out, while inside the method of instruction was closer to the theater than to education and can hardly have encouraged learning. The students, packed into their banked seats, witnessed—and frequently applauded—a continuous performance from eleven o'clock onwards of one-hour sessions given by the staff seated in three armchairs before them. Both Lagrange and Laplace were quiet speakers, with Lagrange outstanding for his Italian accent and Laplace for the rapidity of his teaching. The chemist Claude Louis Berthollet (1748–1822) was even more reticent, often repeating himself many times over; by contrast Monge caught Fourier's imagination, speaking with a loud voice and showing great skill in both theoretical and practical subjects.[6]

So the system was doomed to failure; in particular, the seminars—which had been intended as the backbone of the system—broke down because so few students had learned enough to be able to contribute. But it was at these seminars that Fourier made his mark, and when the school closed he transferred to the Ecole Polytechnique.

This school, initially called the Ecole Centrale des Travaux Publics, had been instituted under the general directorship of Monge by a decree of the Convention of 11 March 1794 to give a three-year training in science, engineering, and applied arts, together with the relevant mathematics. As opposed to the general aims of the Ecole Normale, the Ecole Polytechnique was designed as a military academy, for its graduates were intended mainly to provide the military elite of the Empire; and it took only four hundred students annually. It opened in November 1794 with a "trial run" of fifty of the most able students, and was converted into the full organization in the following May when the Ecole Normale closed. Most of the professors went over to the new establishment, but Fourier did not become a student there. He was over the age limit of twenty years which had been laid down, and in any case he had acquitted himself so well at the Ecole Normale that under Monge's especial support he was appointed to an assistant teaching post, with the title of *administrateur de police*.[7] But

[6]Undated letter to Bonard, in A. Challe *1858a*, 115–120; also in E. Duché *1871a*, 257–261.

The teaching at the Ecole Normale was taken down by stenographers and published in 1797 in 6 volumes of lectures and 2 volumes of *débats* as the *Séances des Ecoles Normales*. A second edition in 10 + 3 volumes was published in 1800–1801.

For an account of some troubles endured by Fourier from his political enemies in Auxerre while he was at the Ecole Normale, see *Le centenaire de l'Ecole Normale (1795–1895)* (1895, Paris), 132–133.

[7]For information on the reconstruction of French education at this time, including the founding of the Ecole Normale and the Ecole Polytechnique, see A. Fourcy, *Histoire de l'Ecole Polytechnique* (1828, Paris), 1–131, 375–379 and 391–404; G. Pinet, *Histoire de l'Ecole Polytechnique* (1887, Paris), 3–33

his political honesty still dogged his career. The new post-Robespierrian regime, in achieving the excesses against which it was a reaction, promoted the arrest of Fourier as a Jacobin and abettor of the same Robespierre who had rejected his appeal against the previous order for his arrest! Thus Fourier received the compliment of being arrested by both sides; this time he was saved from execution by the intervention of his colleagues at the Ecole Polytechnique.

One of Fourier's main duties as *administrateur de police* was to help with the running of Monge's course in descriptive geometry. This course embodied Monge's favorite subjects and dealt largely with the use of science and mathematics in military contexts, the art of attack and defense, and the organization of simulated battle situations: because of this, and perhaps also because of Monge's influence and activity at the school, it claimed more time on the timetable than any other course.[8] Otherwise, Fourier's academic teaching included courses in Lagrange's curricula in analysis, and there he taught the new results of equations which he found in his youth. The two main surviving collections of his lecture notes have several courses in common, showing that he made at least these two copies in his own hand for the benefit of his students. No wonder that once again he built up an enviable reputation for his teaching; as the table of these collections of notes (Table 1) shows, he must have been giving at times one lecture every day,

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and 351–401; *Le centenaire de l'Ecole Normale (1795–1895)* (1895, Paris), esp. pp. 72–192; R. Taton, *L'oeuvre scientifique de Monge* (1951, Paris), 37–43; L. Pearce Williams, "Science, education and the French Revolution," *Isis*, 44 (1953), 311–330, "Science, education and Napoleon I," *Isis*, 47 (1956), 369–382, and "The politics of science in the French Revolution," *Critical Problems in the History of Science* (1959, Madison), 293–308. The information in Fourier's biographies is mostly inaccurate; the reason for the strange title of what was certainly a teaching post for him may have been that no more funds were available for staff and so the authorities appointed "ushers" nominally to deal with discipline.

For an absorbing description of the situation in Parisian science at the end of the eighteenth century, see M. P. Crosland, ed., *Science in France in the Revolutionary Era described by Thomas Bugge, Danish Astronomer Royal and Member of the International Commission on the Metric System (1798–1799)* (1969, Cambridge, Mass.).

J. Fayet, *La révolution française et la science 1789–1795* (1960, Paris), gives a detailed account of the political consequences of the Revolution on the rejuvenation of French science.

[8] See the allocation of time for each course in A. Fourcy, *Histoire de l'Ecole Polytechnique* (1828, Paris), 376–377.

Table 1. Synopsis of the Surviving Collections of Fourier's Lecture Notes

The two collections are as follows:

1. A bound volume of 559 pages in the library of the Institut de France in Paris [Manuscript number 2044], and entitled *Calcul Différentiel et Intégral.*

2. A group of 10 unbound *cahiers* in the library of the Ecole Nationale des Ponts et Chaussées in Paris [Manuscript number 1852] with the covering title of *8 [sic] cahiers des Cours professés à l'Ecole Polytechnique par M. Fourier*, totaling 368 pages.

The table below outlines the content and the timetable of each course. In the column giving the number of pages for each course is also indicated the probable completeness (C)/incompleteness (I) of the corresponding set of notes; where the completeness question remains undecided, the letter (U) has been placed.

For the courses where there are sets of notes in both collections, those in 2 are less full, suggesting that they might have been written as detailed summaries of the corresponding set in 1 for the students' benefit. (In fact, two of the covers of the *cahiers* in 2 have written on them the names "Vincent" and "Daydé" of two first year students at the Ecole Polytechnique in 1796.) Each lecture is dated by day and month (usually in the Republic calendar of the time, rather than the orthodox calendar) but only occasionally is a year given as well. It has been possible to find years for courses 1–6 in the table (either directly or by cross-references), but in courses 7–9 the year-datings are conjectural; those suggested bring the lectures into the period December 1795–October 1796 in which most of the other courses definitely lie. From the table we see that in that period Fourier definitely had to give 122 lectures on 6 different subjects; and if the datings of courses 7–9 are correct these figures are raised to 153 lectures over 8 subjects. Without question they are underestimates; some sets of notes are definitely incomplete, and there may well have been still other courses for which notes no longer survive.

No.	Course	Dates	Frequency of delivery of lectures (days)	Number of lectures	Number of pages (completeness): Institut de France	Ecole Nat. des P. et C.	Remarks	
1	Analysis	25 Dec. 1795– 13 May 1796	5	29	116(C)	103(C)	Covers "Eulerian" analysis – roots of equations (including Rule of Signs), logarithms, complex numbers, etc. "Base course" for several others.	
2	Differential calculus	23 May 1796– 21 Aug. 1796	Mostly 5	18	68(C)	45(C)	Optimization and geometry of curves	Lectures clearly planned as a sequence; set in the Ecole Nationale des Ponts et Chaussées much abbreviated relative to the other set.
	Integral calculus	31 Aug. 1796– 10 Oct. 1796	Mostly 5	8	29(C)	27(C)	Foundations of integral calculus	

3	Differential calculus	31 Jan. 1796– 31 Mar. 1796	5	13	55(C)	50(C)	**Foundations; Taylor's theorem, etc, See part II below**	Lectures clearly planned as a sequence in this order. Basic course in the Euler-Lagrange approach to the calculus. Only direct dating in Integral calculus part II; other courses dated by cross-references within the sequence and to courses 1 and 2.
	Integral calculus I	5 Apr. 1796– 25 May 1796	5	11	See part II below			
	Calculus of variations	30 May 1796– 8 Aug. 1796	5	15	—	48(C)	Isoperimetrical problems, etc.	
	Integral calculus II	13 Aug. 1796– 12 Oct. 1796	5 (with gaps)	11	78(C)	78(C)	Applications of the Calculus (geometry etc.)	
4	Geometry	22 Dec. 1795– 21 Feb. 1796	Mostly 5	13	67(U)	—	Euclidean geometry, conic sections, etc. Notes may be complete at both the beginning and the end.	
5	Dynamics	23 Sep. 1796– 8 Oct. 1796	Mostly 5	4	—	10(I)	Newton's laws; notes probably incomplete at both the beginning and the end.	
6	Stereotomy	11 Mar. 1795– 7 May 1795	Irregular; average 2	27	40(U)	—	Three-dimensional geometry, part of Monge's "descriptive geometry." 17 pages of "model answers" at the end.	
7	Differential calculus	18 Mar. 1796?– 27 May 1796?	Irregular; average 6	11	23(C)	—	Very elementary treatment of principles. Might have been a preliminary course for course 8 below.	
8	Statics	26 May 1796?– 19 Aug. 1796?	Mostly 5	18	68(U)	—	Balances, pulleys, weights, traction, parallelogram of forces, etc. Dates correspond in the sets with one exception. Correspondence of material not sufficiently close to establish that the sets apply to the same course.	
		26 June 1796?– 9 Aug. 1796?	Mostly 5	9	—	25(I)		
9	Hydrostatics	24 Aug. 1796?– 29 Aug. 1796?	5	2	3(I)	—	Fluid flow. Course definitely incomplete at end. The course might have followed on from course 8.	

as well as all the seminar and tutorial work which was an important and novel feature of the organization of the Ecole Polytechnique. Yet he still found time for research, for he continued his investigation into the theory of equations and began work also on problems in applied mathematics. In 1798 he published his first paper, in the *Journal de l'Ecole Polytechnique* — a treatment of the principle of virtual work, a formulation of the foundations of rational mechanics which had been promoted by Lagrange and which was now explained by Fourier with great clarity.[9] At one point in the paper, Fourier announced his intention of publishing in the *Journal* a series of papers on the theory of equations,[10] for his results had become widely known in Paris since he had taught them at the Ecole Polytechnique.

Fourier's main theorems dated from his school days at Auxerre, where he had found a new proof of Descartes's rule of signs. This rule may be expressed as follows: let

$$f(x) = x^m + a_1 x^{m-1} + \cdots + a_m, \tag{1.1}$$

where the a_r are constants (some possibly zero). The signs of the coefficients are incorporated in the a_r of (1.1), but if in practice some of these coefficients take negative values and some positive then we have a sequence of signs associated with the sequence of descending powers of x. We shall call a pair of adjacent signs a *combination*: if it preserves sign ($++$ or $--$) it is a *permanence*,

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[9] J. B. J. Fourier *1798a*. Lagrange's main work on the principle of virtual work was his *Mechanique Analitique* (1st edition: 1788, Paris).

Fourier's paper was not the only one on the subject to appear in the *Journal de l'Ecole Polytechnique* at the time. Lagrange himself put one in the same volume ["Sur les principes des vitesses virtuelles," *cahier* 5, 2 (1798), 115–118; *Oeuvres*, 12, 315–321], and so did his colleague G. C. F. Riche de Prony (1755–1839) ["Sur le principe des vitesses virtuelles et la décomposition des mouvements circulaires," *cahier* 5, 2 (1798), 191–208]. According to de Prony, Fourier also had views on Lagrange's work on maxima and minima ["Suite des leçons d'analyse ..." *cahier* 4, 1 (1796), 459–569 (p. 565)].

Some early notes, which may well date from Fourier's time at the Ecole Polytechnique, show that he also tried to develop notations for representing the logic of propositions, the alphabet, and music [see BN MFF 22501/27–44 and 48–64]. He also prepared some notes on psychology [see 22501/13–24].

[10] J. B. J. Fourier *1798a*, 46; *Oeuvres*, 2, 506.

In a letter to Bonard of March 1795, Fourier reported that both Lagrange and Laplace encouraged him to publish his results [see A. Challe *1858a*, 121–125; also in E. Duché *1871a*, 254–257]. In later letters to Bonard he wrote of the difficulty of entering the Ecole Polytechnique and the bad selection system by the votes of a jury of professors [see A. Challe *1858a*, 125–128].

while otherwise (+ − or − +) it is a *variation*. Descartes's rule says that the number of positive roots of $f(x)$ is not more than the number of variations in the sequence, while the number of negative roots is not more than the number of permanences.

Descartes announced the result in 1637, but he seems to have obtained it from considering polynomials of low degree and did not offer a general proof.[11] He mentioned also the possibility of imaginary roots, and Newton gave examples of roots in complex conjugate pairs in his own consideration of the rule.[12] But while he indicated the limitations of the rule, he gave no general proof either, and the problem of supplying one (with or without imaginary roots) was not tackled successfully until the eighteenth century. As Fourier reported in an undated lecture at the Ecole Polytechnique:

"Segner est le premier qu'en donne une démonstration dans une lettre à Hamberges en 1728.[13] M. l'Abbé de Gua fit insérer deux belles démonstrations de cette même règle dans les mémoires de Paris, année 1741, dont l'une est purement algébrique et l'autre puisée dans la géométrie des courbes. . .[14] on en trouve dans le Calcul Différentiel d'Euler déduite des équations différentielles de la proposée.[15] Mais celle qui par sa simplicité mérite d'être préferée à toutes les autres est due encore à Segner. Elle se trouve dans les mémoires de Berlin."[16]

[11]R. Descartes, *Discours de la méthode pour bien conduire la raison, et chercher la vérité dans les sciences* (1637, Leyden): [Appendix] *La géométrie*, 373. There have been many reissues and translations of both the *Discours* and the *Géométrie*: see especially *Oeuvres de Descartes* (ed. C. Adam and P. Tannery), 6, 445; and D. E. Smith and M. L. Latham, *The Geometry of René Descartes* (1925, Chicago: reprinted 1954, New York), 160–161.

In his posthumous *Artis Analyticae Praxis* (1631, London), Thomas Harriot (1560–1621) showed himself to be aware of this result: see section 5 (numbered as pp. 72, 79–86).

[12]I. Newton, *Arithmetica universalis*, . . . (1707, London), esp. pp. 242–245 in "De natura radicum aequationis."

[13]J. A. von Segner, *Dissertatio epistolica qua regulam Harrioti*. . . (1728, Jena).

[14]J. de Gua de Malves, "Recherche du nombre des racines réelles ou imaginaires. . . ," *Mémoires de l'Académie Royale des Sciences*, (1741: publ. 1744), 435–494.

[15]L. Euler, *Institutiones calculi differentialis* (1755, St. Petersburg), part 2, chs. 12–13; *Opera Omnia*, (1) 10, 501–542.

[16]J. A. von Segner, "Démonstration de la règle de Descartes . . . ," *Mémoires de l'Académie Royale de Berlin*, 12 (1756: publ. 1758), 292–299.

The quotation is from p. 21 of another set of lectures in the library of the Ecole Nationale des Ponts et Chaussées [manuscript number 668] entitled *Leçons d'Analyse de l'Ecole Polytechnique*, written out by a copyist or perhaps a student. The course of 19 lectures in this set is similar in content to course 1 of the table, but definitely not identical with it.

Fourier's tribute to Segner was characteristically self-effacing, for the achievement of his youth had been to produce a proof of Descartes's rule that so far surpasses all its predecessors that it has remained standard in the literature ever since. As this work was important not only for Fourier's mathematical career in general but also for certain problems that were to arise in heat diffusion in the course of his preparation of his 1807 manuscript, it is worth describing here in detail from some of the relevant surviving manuscripts. He demonstrated his proof of the rule in his lecture courses: on 14 January 1796, he used the example

(1.2) $$+ + + - - + - + - + + + - + - + + +,$$ [17]

while in the undated lecture quoted above he used the following sequence of 5 permanences and 9 variations:

(1.3) $$+ + - + + + - + - + - - + + -.$$

Of the two demonstrations we shall follow the latter, which is rather more clear. Fourier's method was to multiply $f(x)$ by $(x+p)$ to obtain a new sequence of signs for $(x+p)f(x)$, which would be one sign longer than that for $f(x)$ and therefore contain one more combination. His proof amounted to a demonstration of the fact that if the new root was negative (positive) the number of variations (permanences) would not be increased, and hence that there would be at least one more permanence (variation) than previously. We shall describe the derivation of this result in the case of a new negative root first, and then summarize the positive root case.

The negative root is given by $p > 0$, and we have

(1.4) $$\begin{aligned}(x+p)f(x) = {} & x^{m+1} + a_1 x^m + a_2 x^{m-1} + \cdots + a_m x \\ & + p x^m + p a_1 x^{m-1} + \cdots + p a_{m-1} x + p a_m,\end{aligned}$$

and so the new sign sequence is

(1.5) $$+ + - + + + - + - + - - + + - \qquad \text{from } xf(x),$$

and

(1.6) $$\quad + + - + + + - + - + - - + + - \quad \text{from } pf(x).$$

[17] See the lecture for *Nivôse* 24, *an* 4 (= 14 January 1796) in the analysis notes in both collections of lectures given in the table (course 1).

When we add by signs, terms with the same sign will yield a sum of that sign, but terms with a different sign add to a sum of undecided sign. Hence, from (1.5) and (1.6) we obtain the sequence for $(x+p)f(x)$

(1.7) $$+\ +\ ?\ ?\ +\ +\ ?\ ?\ ?\ ?\ ?\ -\ ?\ +\ ?\ -.$$

To show that the number of variations is not increased, we must arrange the signs for the question marks so that the number of variations is maximized. There may be more than one way of achieving the maximum; in the case of (1.7), for example, Fourier gave the two possibilities

(1.8) $$+\ +\ \ominus\oplus\ +\ +\ \ominus\oplus\ominus\oplus\ominus\ -\ \oplus\ +\ \ominus\ -$$

and

(1.9) $$+\ +\ \oplus\ominus\ +\ +\ \oplus\ominus\oplus\ominus\oplus\ -\ \ominus\ +\ \oplus\ -$$

(where the ringed signs are his insertions), but in no case is it possible for the number of variations to exceed 8, which is the number in the original sequence (1.3). Therefore, neither (1.8) nor (1.9) can have more variations than the original (1.2). Meanwhile the former must possess one more combination than the latter, for it has one extra sign. Therefore this combination must of necessity be a permanence, which proves the rule.

A similar argument applies in the case of an extra positive root. The sign sequence for $(x+p)f(x), p < 0$, is

(1.10) $$+\ +\ -\ +\ +\ +\ -\ +\ -\ +\ -\ -\ +\ +\ - \qquad \text{from } xf(x)$$

and

(1.11) $$-\ -\ +\ -\ -\ -\ +\ -\ +\ -\ +\ +\ -\ -\ + \qquad \text{from } pf(x),$$

which yields

(1.12) $$+\ ?\ \qquad ?\ ?\ -\ +\ -\ +\ -\ ?\ +\ ?$$

Permanences are now to be maximized, and this may be done by Fourier's suggestions:

(1.13) $$+\ \oplus\ -\ +\ \oplus\oplus\ -\ +\ -\ +\ -\ \ominus\ +\ \oplus\ -\ +$$

or

(1.14) $$+\ \ominus\ -\ +\ \ominus\ominus\ -\ +\ -\ +\ -\ \oplus\ +\ \ominus\ -\ +,$$

which imply that this time (1.12) cannot have more permanences than the original (1.3), and therefore that its extra root must be a variation.[18]

This is as far as he took the argument with the students, but other manuscripts show that he had a clear understanding of the generality of the reasoning. Let us suppose that within the sequence for $f(x)$ we have a subsequence of variations which, if it does not comprise the whole sequence, will be bounded by permanences. Then Fourier's example in his lectures demonstrates the scheme

(1.15)
$$\left.\begin{array}{ll} \cdots + + - \quad \cdot & \text{for } f(x) \\ \text{giving} & \\ \cdots \quad + \; ? \quad \cdot & \text{for } (x+p)f(x), \end{array}\right\}$$

as we can see by comparing (1.3) with (1.7). Over the whole subsequence this yields the pattern

(1.16)
$$\begin{array}{llllllll} \mathrm{P} & \mathrm{V} & \mathrm{V} & \qquad\qquad & \mathrm{V} & \mathrm{V} & \mathrm{P} & \text{for } f(x) \\ \text{giving} & & & & & & & \\ \mathrm{D} & ? & ? & & ? & ? & \mathrm{D} & \text{for } (x+p)f(x), \end{array}$$

where P represents a permanence and V a variation between signs, while D and ? stand for definite and indefinite signs. Now, however the undetermined signs in (1.16) are chosen, it is clearly impossible for there to be more resultant variations for $(x+p)f(x)$ than there are already in (1.15) for $f(x)$, for the subsequence for $f(x)$ is already maximized for variations. Clearly this argument applies to all subsequences of variations, and hence to all occurrences of variations in the sequence. The remaining parts of the sequence will necessarily be subsequences of permanences, and these cannot possess fewer variations. Therefore we can safely deduce that there will be no more variations in the sequence for $(x+p)f(x)$ than there were originally in the sequence for $f(x)$, and so the extra combination that it possesses must be a permanence. Thus Descartes's rule is established for negative roots; and the case of positive roots can be established by similar reasoning.[19]

[18] J. B. J. Fourier (n. 16, p. 9), 18–19.
[19] For general discussion of the theorem, see especially the manuscript in BN MFF 22510/74–75, whose unusually large handwriting suggests that it might be a schoolboy text; and an early four-page note in the library of the Institut de France [*Anciens et nouveaux fonds*, volume 2038, folios 195–196].

But this was not the totality of Fourier's achievement. His main extension of Descartes's rule was the estimation of the number of real roots of an equation within a given range of values of the variable. His surviving lecture notes do not contain any mention of it, although he claimed later to have taught it also at the Ecole Polytechnique;[20] certainly as an achievement it surpasses his proof of Descartes's rule. Like that proof it is based on consideration of sequences of signs, but this time the sequence is created from $f(x)$ and its derivatives. If $f(x)$ has a simple root at $x = a$ — that is, if no derivative of $f(x)$ is also zero at $x = a$ — then $f(x)$ changes sign through $x = a$, and Taylor's theorem shows that the signs of the subsequence $f'(x), f''(x), \ldots$ are

$$
\begin{array}{lcc}
 & f'(x) & f(x) \\
x < a & \pm & \mp \\
x = a & \pm & 0 \\
x > a & \pm & \pm
\end{array}
\left.\vphantom{\begin{array}{c}x\\x\\x\\x\end{array}}\right\}
\begin{array}{l}
\text{where the choice of signs is} \\
\text{dictated} \\
\text{by the sign of } f'(x).
\end{array}
\tag{1.17}
$$

In both cases a variation is changed to a permanence as x increases its value, and so the whole sequence for $f^{(m)}(x)$, $f^{(m-1)}(x), \ldots, f'(x), f(x)$ gains a permanence from a variation through $x = a$. If a multiple root of $f(x)$ occurs at $x = a$, then there will arise a subsequence of zeros in the sequence equal to that root's multiplicity, and Taylor's theorem will suggest a corresponding loss of variations.

These deductions indicate the behavior of the sequence as x increases its value. When $x = -\infty$ the terms of the sequence are dominated respectively by the terms

$$
m!, \quad m!x, \qquad\qquad m(m-1)x^{m-2}, \quad mx^{m-1}, \quad x^m, \tag{1.18}
$$

and so the sequence of signs will be

$$
+ \quad - \quad + \qquad\qquad \pm \qquad\qquad \mp \qquad \pm; \tag{1.19}
$$

while at $x = +\infty$ it will be

$$
+ \quad + \quad + \quad + \qquad + \qquad\qquad + \qquad +. \tag{1.20}
$$

Hence as x passes gradually from $-\infty$ to $+\infty$ the sequence loses all of its m variations to become a sequence of m permanences

[20]J. B. J. Fourier *1820c*, 160; *Oeuvres*, 2, 295.

and one is lost every time a real root is crossed (counting multiple roots multiply). Therefore if the sequence loses k_a variations as x passes from $-\infty$ to a, then there will be at most k_a real roots less than a (assuming that $f(a) \neq 0$). Similarly there will be at most k_b roots between $x = -\infty$ and $x = b$, and therefore—because of the possible presence of complex roots for $[a, b]$, of course, rather than by simple subtraction—at most $(k_b - k_a)$ roots within $[a, b]$.[21] This was Fourier's generalized theorem, and he realized that the number was an upper estimate rather than the exact answer because of the presence of complex roots to the equation. So he was led to the problem of detecting complex roots in a polynomial by means of his sequence of signs, and also to problems of approximation to the value of a real root, to investigations of polynomials of special types, to . . . .

But all these plans of work were rudely interrupted. Napoleon Bonaparte (1769–1821) had been put in charge of the forthcoming French expedition to Egypt, and Berthollet and Monge were entrusted with the selection of its scientific members. Fourier was chosen in May 1798, and when the Institut d'Egypte was founded in Cairo in the following August he was appointed *secrétaire perpétuel.* The Institut d'Egypte was organized along the lines of the Institut de France into four sections—mathematics, physics, literature and arts, and political economy—with nominally twelve members in each section.[22] Only a year later Bonaparte suddenly left Egypt to return to France to become first consul of the French Empire. He took his chance for power; and he left behind him a venture which perhaps he knew was doomed to failure. For the challenges of English forces made the occupation difficult to maintain; and the more awkward the situation became, the greater were the burdens placed upon Fourier in order to resolve it. General Jean Baptiste Kléber (1754–1800) was put in command after Bonaparte's departure, but he was assassinated in Cairo in June 1800. His successor was Jean François Abdullah Menou (1750–1810), who held his post until the end of the occupation.

[21] Various manuscripts in the collection at the Bibliothèque Nationale are concerned with the generalization of Descartes's rule. Some at least could date from this early period; see especially BN MFF 22509/119–133 and 22510/12, 34–47.

[22] A thirteen-page manuscript by Fourier of the members and *procès-verbaux* of the Institut d'Egypte during 1798 and 1799 is now in the library of the Institut de France [*Anciens et nouveaux fonds,* volume 3818].

Under both of these leaders Fourier held many important administrative and judicial positions: he read the *éloges* of both Kléber and Louis Charles Antoine Desaix de Veygoux (1768–1800) (another general in Egypt, who was killed at the battle of Marengo in November 1800 while "on loan" to the Italian campaign) at their funeral services,[23] and under Menou was put virtually in control of all nonmilitary affairs.[24] One of his most important assignments was to conduct the French side of the negotiations with the Egyptian beys, who were represented by Sitty-Nefiçah, the beautiful and talented wife of the chief bey, Mourâd.[25] Yet he still found time for research; in fact, one of Bonaparte's last acts in Egypt was to make Fourier leader of one of the two scientific expeditions which spent about two months of the autumn of 1799 investigating the monuments and inscriptions in Upper Egypt.[26] At the Institut d'Egypte Fourier not only edited its proceedings in the journal *La Décade Egyptienne*[27] but also read papers on a wide variety of subjects besides the theory of equations: statistical researches in Egypt,

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[23]They were published in the occupation newspaper, the *Courier de l'Egypte*, which appeared in 116 four-page issues between 12 *Fructidor, an* 6 (= 29 August 1798) and 20 *Prairial, an* 9 (= 9 June 1801) at approximately half-*décade* (that is, 5-day) intervals. Fourier was quite heavily involved in its editorship: his *éloges* of Kléber and Desaix appeared in nos. 72 (9 *Messidor, an* 8 = 28 June 1800), 2–4; and 88 (24 *Brumaire, an* 9 = 15 November 1800), 2–4 respectively. They were republished in V. Cousin *1831b*, 21–28; and J. J. Champollion-Figeac *1844a*, 317–325.

[24]Most of these duties are described in V. Cousin *1831b*, 14–21.

For a detailed, if rather biased, study of the Egyptian campaign up to the time of Bonaparte's departure and a reproduction of our plate 1, see F. Charles-Roux, *Bonaparte, Gouverneur d'Egypte* (1936, Paris) [English translation as *Bonaparte, Governor of Egypt* (1937, London)].

[25]He also negotiated with her for the freedom of her house-slaves, especially those who interested the French generals! [See *Kléber et Menou en Egypte depuis le départ de Bonaparte (août 1799–septembre 1801)* (ed. M. F. Rousseau: 1900, Paris), 252.] For his treaty with Mourâd, see J. J. Champollion-Figeac *1844a*, 332–333.

[26]The expedition left Cairo in the middle of September 1799 [see *Courier de l'Egypte*, no. 37 (29 *Fructidor, an* 7 = 15 September 1799), 1] and returned in early November [*Courier de l'Egypte*, no. 44 (18 *Brumaire, an* 8 = 9 November 1799), 1].

One member of Fourier's party was the biologist Etienne Geoffroy-St.-Hilaire (1772–1844), who established a firm dislike for him while in Egypt. In a letter of December 1801, Geoffroy-St.-Hilaire wrote that Fourier had tried to show that his colleagues were fools and that his own (civil engineering) students were the only ones of real merit [see *Lettres écrites d'Egypte* (ed. E. J. Hamy: 1901, Paris), 215–217].

[27]The journal appeared in 3 volumes during 1799 and 1800, and contained military and civil decrees as well as research papers by members of the Institut d'Egypte.

mechanics, researches into oases, ancient monuments, and irrigation machines![28] But for the second time in his brief scientific career, his researches were abruptly terminated: under the terms of a general treaty signed between England and France, the French forces withdrew from Egypt in the autumn of 1801. Fourier organized much of the departure and returned to France in the middle of November 1801 on the English brig *Good Design*, taking the opportunity to discuss problems of equations with a colleague from the Institut d'Egypte.[29] He hoped to resume his teaching post at the Ecole Polytechnique and in fact was able to give a few lectures there,[30] but Bonaparte could not bear to let such a diplomatic talent be wasted on education and research. In February 1802 the prefect of the department of Isère, an area of France on the Italian border and centered at Grenoble, died in office, and Bonaparte appointed Fourier as his successor.[31]

It was as prefect of Isère that Fourier spent his middle years and presented his monograph on heat diffusion to the Institut de France in December 1807, so in order to indicate the circumstances of its preparation we shall describe the nature and range of his prefectural duties and the special events that were to demand his attention during his first six years in office. The post of prefect was an onerous one: the administrative orders of the government had to be implemented in the department, and at the same time the needs and requests of the area had to be put forward to the government for consideration and action. In Isère the responsibilities were especially heavy.

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[28]The reports of these papers are listed in V. Cousin *1831b*, 13–17; only *1799a* was actually published. Extracts from Fourier's notebook during the Egyptian campaign were published as *1904a*: they deal mainly with places visited and monuments examined during the 1799 expedition of which he had been the leader.

BN MFF 22520 is devoted to problems in mechanics; one of its manuscripts is dated in *an* 7 (1798–1799) and several others also seem to be from his Egyptian period [see esp. folios 41–45]. In addition, 22514/75–93 is an essay on problems in the theory of equations which is dated December 1800–January 1801.

[29]The colleague was Louis Alexandre Olivier de Corancez (1770–1832); see his letter in J. B. J. Fourier *1831a*, xxi.

On landing at Toulon, Fourier wrote to his friend Bonard in Auxerre on 20 November 1801, and told him that he had to prepare some of his researches on Egypt for publication [see A. Challe *1858a*, 129–131].

[30]According to A. Fourcy (n. 7, p. 4), 272.

[31]Bonaparte made the appointment in consultation with Berthollet and Monge; see J. J. Champollion-Figeac *1844a*, 22.

Fourier's predecessor, Ricard de Séalt, was the first prefect, and he had set up little administrative machinery during his two years of office; in addition, the area was both backward in development and particularly independent of spirit. Thus when Fourier arrived in Grenoble in the spring of 1802, he had to start almost afresh both with revitalizing the department and also making it amenable to the demands of the regime. Although he never wanted the job he carried it out with truly extraordinary vigor, and began at once with organizational problems. He appointed local men to the prefecture: four *conseillers de fonction* who would help with general administration; a *chef de cabinet* called Ambroise Auguste Lepasquier, who spent a year at the Ecole Polytechnique in 1806 and then returned and helped with the preparation of the 1807 manuscript; another administrative officer by the name of André Raynaud, who also wrote out scientific papers for him. Fourier also appointed three subprefects in small towns in his department: they changed with great regularity, some to become prefects elsewhere. All these people helped with day-to-day problems and left him free to carry out the larger-scale tasks neglected by Ricard.[32] One of these was to compile *carnets*, or notebooks, on prominent people in the area. Fourier began a round of visits, which on the whole seem to have been enjoyable: the retired General Joubert de la Salette, for example, who talked most of the time about music and urged him to set up a music school in Auxerre; and especially Jacques Joseph Champollion-Figeac (1778–1867) and his brother Jean François (1790–1832), who became — much under Fourier's influence — important archaeologists and Egyptologists, and his close friends.[33]

Fourier found an interesting collection of people in Grenoble, but he had to carry out other inquiries into the local

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[32]The details of these appointments are given in A. L. Champollion-Figeac *1880a*, 79–82.

[33]For details of these visits and compilation of the *carnets* over six years, see A. L. Champollion-Figeac *1880a*, 120–127, 147–149, 167–170, 175–189. In *1881a*, 173, Champollion-Figeac mentions that some of these *carnets* are missing.

Aimé Louis Champollion-Figeac (1813–1894) was the second son of Jacques Joseph, whose *1844a*, a semibiography of Fourier, we cite from time to time; we also refer to his own rambling histories *1880a* and *1881a* of the Grenoble area in Fourier's time.

At the village of La Côté St. André, near Grenoble, in December 1803 was born the composer Hector Berlioz (1803–1869).

people which must have offended his sense of honor. Bonaparte's regime soon began to develop into a form of police state, and Fourier had to execute orders concerning the opening of letters, the suppression of antigovernmental pamphlets and the restriction of religious sects.[34] He also assumed the editorial supervision of the *Annales du département de l'Isère, journal administratif, politiques et littéraires*, the official thrice-weekly newsheet, in order to keep both revolution and scandal from its columns: he was not always successful, and more than once thought of removing the editor from his post.[35] He organized local elections and referenda, and from his *carnets* he had to abstract information on local senatorial members and important local families for the acquaintance of the Ministry of the Interior.[36]

A particularly eventful year in Fourier's prefecture was 1804. In February Bonaparte visited Grenoble,[37] and in the following May crowned himself Emperor Napoleon. The local people were suitably pleased: Fourier wrote to Napoleon to tell him that 82,084 of the electorate supported the plebiscite on hereditary descent, with 12 dissenters,[38] and he spent more than three months in the capital for the celebrations.[39] At the turn of the year he had been appointed *chevalier* of the new order of the Légion d'Honneur: there was a ceremony in Grenoble on his return to celebrate the award,[40] and in October of 1804 a service was held in the cathedral for the local awards of the order, where he delivered one of the speeches.[41] Then he became involved with the visit to France of Pope Pius VII (1740–1823). The religious situation in France had become very delicate: in 1801 Napoleon had insisted on a concordat with the Pope in which he could appoint bishops and take an oath of allegiance

[34]For various of Fourier's decrees on these matters, see A. L. Champollion-Figeac *1881a*, 149–163.
[35]For quotations from the dubious articles, see A. L. Champollion-Figeac *1880a*, 91–97.
[36]For information on the elections, see A. L. Champollion-Figeac *1880a*, 200–202; and for details of the information sent to the ministry, *1881a*, 171–214.
[37]For extracts from the numerous letters concerning the visit, see A. L. Champollion-Figeac *1881a*, 141–147.
[38]See A. L. Champollion-Figeac *1881a*, 136–137.
[39]According to his own list of absences from Grenoble; see G. Letonnelier *1923a*, 137.
[40]See A. L. Champollion-Figeac *1880a*, 202–203, for a report of the occasion.
[41]See the reports in A. L. Champollion-Figeac *1881a*, 271–278.

from them, and then in 1802 he added articles to the effect that papal law could be implemented only by governmental permission. This seemed to the Pope to be an excessive imposition of authority, and he came to Paris in November 1804 to try to obtain concessions. His route passed through the Isère department, and Fourier had much correspondence to handle concerning the passage.[42]

Besides the multitude of political affairs Fourier managed to do much for the development and welfare of the department. When he arrived in Grenoble, he commented on the dilapidated state of the town, especially the Town Hall and the prefect's office in the Place St. André.[43] In addition, many of the institutions of the area were not functioning. So he began to reopen schools and colleges and revitalize the mining and crafts industries.[44] He also planned an important and spectacular road through the French Alps from Grenoble to Turin via Briançon and Pinerolo. The Ministry of the Interior was opposed to the scheme, but Fourier used all his powers of persuasion and his Paris contacts to support it, and finally secured approval from Napoleon himself by means of a succinct paper summarizing its advantages.[45] The road was not completed beyond Briançon in Fourier's time because of the uncertain political situation in France following the fall of Napoleon in 1815, but when it was finished it created the quickest and shortest route from Lyons to Turin, and later Fourier recorded that of all his prefectural assignments this one gave him the greatest satisfaction.[46]

The other large-scale project handled by Fourier was the drainage of the huge area of marshland around the village of Bourgoin midway between Lyons and Grenoble. The project had been planned and attempted many times over more than a century, but opposing petty interests among the river communi-

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[42]For quotations from it, see A. L. Champollion-Figeac *1880a*, 216–220.

[43]See G. Letonnelier *1923a*, 140. Grenoble then had a population of about twenty thousand inhabitants.

[44]See A. L. Champollion-Figeac *1880a*, 152–166 and 206–208; *1881a*, 164–168 and 337; and G. Letonnelier *1923a*, 142.

[45]For documents concerning the road, see A. L. Champollion-Figeac *1881a*, 169–170. See also *1880a*, 167–169.

[46]See G. Letonnelier *1923a*, 143–144. The completed road is now numbered *Route N91* in France and *Strada 23* in Italy.

ties of the area had always brought it to a halt. Fourier took charge of the complete negotiations, going personally to almost every household in the area to explain the advantages of the scheme. The patience and tact needed to handle narrow-minded and superstitious peasants is difficult to imagine, but by August 1807 the details of planning and compensation had been worked out to the satisfaction of the thirty-seven communities involved. Later there were disagreements between the owners of property in the area and the state organization in charge of the drainage. Fourier sided with the local people, and their position was confirmed by a decree of the Council of State in December 1812. This was also the year in which the draining of the marshes was completed, and an area of first-quality farming land created out of the poisonous marshes. While the drainage was in progress archaeological remains were found, and Fourier instructed Jacques Champollion-Figeac, then the librarian of Grenoble, to collect them and preserve them in the library.[47]

Fourier tried hard to meet the enormous demands made on him by the central government, but sometimes they were too much even for him. During 1804 especially, he received a flood of letters from the minister of the interior demanding statistical information about the department. To obtain help on this work, he made use of the Société des Sciences et des Arts in Grenoble. Soon after his arrival, he had declined the presidency of the society offered him by the musical General Joubert de la Salette, becoming only an ordinary member;[48] but now he set up a 600 francs statistical prize in the society with an additional 300 francs of his own. Some effort was put in by the members, but no adequate papers were offered for the prize and Fourier was forced to write up much of the work himself; but he still had to report delays to the ministry, and in the end it was never finished.[49] Perhaps in recompense, he did later send to the minister various publications of the society, including archaeological researches by Jacques Champollion-Figeac.[50]

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[47]See A. L. Champollion-Figeac *1880a*, 169–170.

[48]See A. L. Champollion-Figeac *1880a*, 126–127. For a general paper on science read to the society, see J. J. Champollion-Figeac *1844a*, 333–337.

[49]For extracts from Fourier's apologies to the ministry, see A. L. Champollion-Figeac *1881a*, 323–328. See also J. J. Champollion-Figeac *1844a*, 26–27.

[50]See A. L. Champollion-Figeac *1880a*, 280–289.

With all these activities it would appear that Fourier was enjoying his prefecture after all. Certainly he must have felt the pleasure of the achievement of social aims and warmed to the demonstrations of loyalty and affection which he received at ceremonies and celebrations (of which he was very fond) and visits to public places such as the theater.[51] But he always hoped to be able to resign from the job—made even more difficult by maladministration and apathy—and return to an intellectual life.[52] Monge and Berthollet tried to persuade Napoleon to release him, but Napoleon expressed annoyance at the suggestion, and to an extent which made Fourier think of going into exile.[53] But he stayed at his post, and tried to make his life as interesting as possible for himself as well as for others. He worked hard to promote cultural activities in the department, and even put on scientific experiments as entertainment.[54] He held soirées from time to time to which he invited his cultural friends, such as Jacques Champollion-Figeac, who kept notes of the things that he said.[55] He never tried to spread his image through the kingdom, and had prepared only one lithograph while at Grenoble;[56] but on the other hand, he was concerned about his financial position, and was not satisfied with his salary of 12,000 francs per annum[57]—not a substantial sum in its day, and one which would have had to cover his considerable prefectural expenses. He was also worried about his health. A small man of slight build, his constitution was not robust: the rapid change from the tropical climate of Egypt to the chilling winds of the French Alps soon led to bad attacks of rheumatism.[58] Despite

[51]See A. L. Champollion-Figeac *1880a*, 196–200; and G. Letonnelier *1923a*, 138–139.

[52]On this point, see A. L. Champollion-Figeac *1881a*, 282–283.

[53]See A. L. Champollion-Figeac *1880a*, 172.

[54]See A. L. Champollion-Figeac *1881a*, 339–345; and G. Letonnelier *1923a*, 138.

[55]See A. L. Champollion-Figeac *1881a*, 288 and 360–367.

[56]He also had his portrait painted after returning from Egypt [see A. L. Champollion-Figeac *1881a*, 408–409]. Neither the portrait nor the lithograph seem to have survived.

[57]J. J. Champollion-Figeac *1844a*, 255. As a result of a complaint by Fourier in 1810, Napoleon decreed that he should receive a salary of 30,000 francs per annum, including the payment of his assistants [see G. Letonnelier *1923a*, 139]. One drain on his resources was his relatives: little is known about them, and for the apparent reason that there is nothing worth knowing, but there are indications that from time to time throughout his life he did have to support them[see A. Challe *1858a*, 131; E. Duché *1871a*, 240].

[58]See G. Letonnelier *1923a*, 136–137.

these preoccupations and difficulties, he still managed to find time for research – and even then demands were made on him by the government.

Soon after the expeditions had returned from Upper Egypt at the end of 1799, Fourier had suggested that a record of the discoveries that had been made be written, and he had been assigned to organize its preparation.[59] The prefectural duties to which he was now committed prevented him from taking the general editorship which was entrusted to another ex-member of the Institut d'Egypte, Edmé François Jomard (1777–1862); but he was still consulted over its organization and delegated to write certain articles, especially an introductory paper which would describe the history of the ancient civilization and its renaissance under French patronage. In order to prepare this work he took himself away from Grenoble when he could, to the Chateau de Beauregard in the village of Pariset a few kilometers west of the town and accessible from the village only by boat; and even then he had to write to the Ministry of the Interior explaining the necessity for these absences and denying the rumors of illegal holidays![60] It seems more than likely that he also used part of these precious periods for his mathematical researches. It may seem unbelievable that he could accomplish anything scientific at all, but in fact he worked on several problems in addition to the masterpiece of 1807 on heat diffusion. He wrote up various papers on equations which he had mentioned in his paper of 1798 on the principle of virtual work in the *Journal de l'Ecole Polytechnique*.[61] He developed some of Monge's ideas on differential geometry in unpublished work on the curvature of surfaces, which were further extended in two papers by a colleague from Egypt, Michel Ange Lancret (1774–1807),[62] and

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[59]See the *Courier de l'Egypte*, no. 48 (19 *Frimaire, an* 8=10 December 1799), 1.

[60]See a letter to the ministry in September 1806, in A. L. Champollion-Figeac *1881a*, 375.

[61]The opening pages of an early note entitled "Première mémoire sur l'algèbre" survives, and it begins: "J'avais annoncé dans un écrit sur le statique le dessein d'insérer dans ce receuil différents mémoires d'algèbre. Je n'ai pu remplir cet engagement pendant la durée d'un voyage considérable entrepris pour l'interêt des lettres et par ordre du governement" [see BN MFF 22510/48–50, and the draft in 72–73].

[62]Fourier had mentioned the problems of defining surfaces and their curvature first at Monge's early meetings at the Ecole Normale. [See *1800a*. The early (?) manuscript BN MFF 22519/23–33 deals with the curvature of lines.]

mentioned later by Sylvestre François Lacroix (1765–1843), the chief mathematics textbook writer of the day.[63] He continued to look for problems in rational mechanics, especially related to the analysis of friction;[64] but at some stage during his early years at Grenoble (or even perhaps in Egypt), he turned his back on mechanics altogether and took up a completely new problem – the diffusion of heat.

Fourier never described how or when he came to be motivated to this problem, though it seems likely that he simply saw it as one of the unsolved problems of his time rather than that some significant event brought it to his attention. He apparently began to obtain results with his n-body analysis during the period 1802–1804, found difficulties which we describe later, and does not seem to have found his new approach to the problem until the latter half of 1804. Then, during the next three years he achieved in the brief intervals of research time available to him the main body of his contributions to mathematical physics; and between September 1807 and February 1808 he forsook his prefectural duties for what he described as "an occupation prescribed by the government."[65] Presumably he convinced them of the necessity for extended Egyptological research in the capital; certainly he used some of the time to write up his results on heat diffusion,[66] and present them to the

Lancret's papers are "Mémoire sur les courbes à double courbure," *Mémoires présentés à l'Institut de France par divers savans*, (1) 1 (1805), 416–454 [read April 1802; for reference to Fourier see p. 420] and "Sur les devéloppoïdes des courbes planes, des courbes à double courbures et des surfaces devéloppables," ibid., (1) 2 (1811), 1–79. For commentary see R. Taton, *L'oeuvre scientifique de Monge* (1951, Paris), 236–239.

[63] S. F. Lacroix, *Traité du calcul différentiel et du calcul intégral* (2nd edition), 1 (1810), 503–505 and 633.

[64] Many of the mechanics manuscripts in BN MFF 22520 which are not of Fourier's Egyptian period would appear, by the handwriting, to belong to Fourier's early years at Grenoble. On friction, see folios 56–104, and also the paper on elastic bodies dated January–February 1804 in folios 106–107.

[65] According to his list of absences: see G. Letonnelier *1923a*, 137–138. During his preparation Fourier corresponded with Pierre Prevost (1751–1839), who was also interested in heat, concerning the latest works on the subject [see A. L. Champollion-Figeac *1881a*, 288–289]. His manuscripts occasionally deal with Prevost's work: see, for example, BN MFF 22525/105.

[66] See A. L. Champollion-Figeac *1881a*, 375–376 for a letter from Fourier at Grenoble to the minister of the interior announcing the need to work on his introductory paper for the Egypt volumes, and also mentioning the intention of presenting *and publishing* a work on the theory of heat. The date given by Champollion-Figeac to this letter – 1 January 1808 – however, must be a mistake, as Fourier had certainly presented his paper by then

Institut de France on 21 December 1807.[67] The *secrétaire perpétuel* for the mathematical and physical sciences was the astronomer Jean Baptiste Joseph Delambre (1749–1822), and he asked Lagrange, Laplace, Lacroix, and Monge to examine the paper. Monge would surely have supported the work of his protégé, and we shall see later that Laplace and Lacroix were certainly in favor; but Lagrange was adamant in his rejection of several of its features (especially the "Fourier series") and thus of the paper altogether. So the public saw only an unenthusiastic five-page summary and review by Siméon Denis Poisson (1781–1840), one of the rising young scientists in Paris who was interested in heat diffusion himself.[68] Fourier sent a note to Lagrange on the convergence of his trigonometric series[69] and a supplementary paper on the same subject to the Institut de France;[70] in October 1809 he submitted to the Institut de France a short nonmathematical paper on the general features of the 1807 manuscript together with a set of extended footnotes on points raised by the examiners.[71] He may well have sent in additional supplementary papers too (although no others have survived), for at the back of the 1807 manuscript is a note written by Delambre which reads as follows:

"Notes jointés au mémoire sur la propagation de la chaleur remises à Mm. Lg. [Lagrange] et Lp. [Laplace] mars et Sep^bre^ 1808, 8^bre^ [Octobre] 1809. Convergence des séries, diffusion de la chaleur dans un prisme infine, émission des rayons à la surface et constructions remarquables, formes générales.

"Notes diverses: nature des équations déterminées, températures terrestres périodiques."

Topics such as surface emission and terrestrial temperatures are not covered by the surviving papers, and belong to Fourier's later interests in heat: they were also mentioned by Fourier himself in a note which he put at the end of the publica-

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and was still in Paris. [See, for example, the letter of 4 January 1808 by Jacques Champollion-Figeac to his brother Jean in Paris inquiring after Fourier, on pp. 396–397.]

[67]See the minutes of the meeting in *Procès-Verbaux*, 3, 632.

[68]J. B. J. Fourier *1808a*.

[69]The manuscript is now with Lagrange's papers in the library at the Institut de France in Paris [*Anciens et nouveaux fonds*, volume 906, folio 103].

[70]J. B. J. Fourier *Convergence*. The paper is now with the 1807 manuscript.

[71]J. B. J. Fourier *Extrait* and *Notes*. Both papers (of which only the first ten pages of the *Extrait* have survived) are now with the 1807 manuscript.

tion of his prize paper of 1811.[72] But they can have made little difference to the general situation. Fourier wrote to Delambre asking for the date of publication of the 1807 manuscript, and Delambre assured him that something would be done. . . .[73] But nothing happened: Lagrange was unrepentant.

[72]J. B. J. Fourier 1811 paper, part 2, 245–246; *Oeuvres*, 2, 93–94.
[73]See BN MFF 22529/121 for the exchange of notes between Fourier and Delambre.