Preface

$\mathcal{A}$UCTIONS INTEREST economists for a number of reasons, on several levels. First, while the role of the fictional Walrasian auctioneer in the *tâtonnement* process of equilibrium-price determination is recognized as important, the mechanics of how such prices obtain are typically left vague. Prices are very important to economists because, under certain conditions, they represent the opportunity cost of goods and, thus, can aid both consumers and producers in making decisions. Because auction models can often provide simple analytic formulae describing the mechanics of price formation, such models have provided useful insights into what such prices represent.

Second, theoretical models of auctions are one of the major success stories in the application of game-theoretic methods, in particular John C. Harsanyi's (1967/68) *Noncooperative Games of Incomplete Information.* In fact, Harsanyi, along with John F. Nash, Jr. and Reinhard Selten, won the 1994 Nobel Prize in economics for their research in game theory. Auction theory delivers very specific predictions concerning how purposeful bidders should behave in equilibrium and how a seller might use this information to construct optimal selling mechanisms. Thus, from a purely scientific perspective, empirical estimation of auction models appears particularly fruitful.

Third, from a practical perspective, using auction theory to understand what happens in the real world is particularly relevant because many commodities and goods are bought and sold at auction. These include antiques, art, cotton, fish, flowers, furs, houses, land, livestock, oil, securities, timber, and, of course, the ever-important Beanie Babies on eBay.

This book is concerned with the empirical analysis of field data from auctions. For us, it is the culmination of nearly two decades of field research in which we have personally attended or studied in detail the following: bankruptcy and liquidation auctions in Clinton, British Columbia, Canada as well as Iowa City, Iowa; confiscated Oriental carpet sales held by Revenue Canada in Vancouver, Canada; fish auctions near Kiel, Germany, in Tokyo, Japan, in Grenå, Denmark, in Arenys de

Mar, Spain, in Alacante, Spain, and in Sydney, Australia; standing timber and tree-planting procurement auctions in the Prince George region of British Columbia, Canada; timber-export permit sales in the Krasnoyarsk region of Siberia, Russia; milk quotas in Toronto, Canada; highway-repair contracts in New Jersey; cattle in Kalona, Iowa; flowers in Aalsmeer, Holland and Aarhus, Denmark; used-car auctions in Mount Vernon, Iowa; furs in Copenhagen, Denmark; PCS spectrum auctions in the United States; real estate in Sydney, Australia; and Receiver General cash-management auctions conducted by the Bank of Canada in Ottawa, Canada.

1 Focus of the Book

Auction theory, when viewed as an application of Harsanyi's theory concerning noncooperative games of incomplete information, is a well-developed subfield of economic theory. Vijay Krishna (2002) claims that the *EconLit* CD lists over one thousand papers with *auction* in their title, over half of them theoretical. The complementary portion of these papers is presumably applied, with many of the papers being empirical ones.

In this book, we provide an introduction to the econometrics concerning auctions. But, like Krisha's treatment of auction theory, which is presented in his elegant text *Auction Theory*, we too have been selective. In particular, we have considered only some of the recent developments in what we refer to as the *structural econometrics of auction data* (SEAD). We have limited ourselves to what many economists refer to as the *structural-econometric approach* (SEA) for two reasons. First, we believe that the SEA is an internally-consistent one in the sense that the economic-theoretic model is mapped virtually seamlessly to the econometric specification; this is not true for another contending approach to the empirical analysis of auction data, often referred to by economists as the *reduced-form approach*. Second, we also believe that some of the most exciting applications of theoretical econometric research in the past decade have been in the SEAD.

Because the literature associated with the SEAD is somewhat large and growing, we have constrained ourselves to a subset of this literature concerned with economic models of agents who live in an environment sometimes referred to by theorists as the *independent private-values paradigm* (IPVP); we shall discuss this paradigm in detail below.

We have selected the IPVP for pedagogical reasons: first, models within the IPVP are relatively easy to investigate with a minimum of mathematics. Second, many of the important empirical techniques can

be illustrated within this informational environment in a straightforward way. Of course, one problem with a strategy involving omission is that some researchers are going to feel slighted—they should not. Our decision was not personal, but pedagogical.

We have also written this book in a particular style. Specifically, we have explicitly chosen not to judge the relative merits of the various approaches, even though in some cases we have strong opinions. Instead, we have provided the reader with a variety of different empirical exercises. By solving these exercises, a diligent learner will be able to form her or his own opinion concerning the merits as well as the difficulties of the various approaches.

2 Who Should Read This Book?

This book will be of primary interest to graduate students in industrial organization because auctions represent explicit examples involving the economics of strategic behavior under asymmetric information. Auctions are just one example of this; Reiss and Wolak (2005) document the structural econometric analysis of others. Of course, professional economists in industrial organization will also find the book of interest. However, because auctions are used to sell agricultural commodities as well as natural resources in most developed and some developing countries, we think graduate students in such fields as agriculture and forestry will also find this book useful. In fact, based on anecdotal evidence from our refereeing for journals in agricultural and forestry economics, we believe that students in these fields will be increasingly more interested in the use of market mechanisms such as auctions. Because of a recent interest in payment systems in monetary economics (and their relationship to such fiscal institutions as treasury-bill auctions) as well as the interest in market microstructure in finance, graduate students and professional economists in these fields may find the book of use too. We also believe that graduate students of microeconometrics will find a good portion of this book relevant to their research.

We believe this book fills an important gap in the literature. We can think of over two dozen researchers who are actively working on problems concerned solely with the SEAD. Most of the papers are written in different notations; often, no attempt is made to relate these papers to one another. Also, the economic theory motivating these papers, while well developed, is often implicitly assumed by the authors. Thus, integrating and understanding this literature is difficult for both students and professionals. By developing one of the existing theoretical models in a common notation and then mapping it into econometric specifications

that can be implemented in practice, we believe the following material will be a useful resource. By reviewing many of the major contributions, within a consistent framework, we hope to provide readers with enough information to evaluate the relative merits of different strategies and, thus, to make informed choices on how to proceed in research. Consequently, we hope this book will make it easier for both students and professionals to enter this exciting field.

3 Material We Intend to Presume

The minimum amount of economics that a reader will need is a good course in microeconomic theory at the level of Hal Varian's (1999) *Intermediate Microeconomics: A Modern Approach* as well as familiarity with some notions from game theory at the level of that covered in Robert Gibbon's (1992) *Game Theory for Applied Economists.* Of course, we presume knowledge of differential and integral calculus as well as linear algebra. We also presume that the reader has had courses in probability, statistics, and econometrics at the level of the text *Econometric Theory and Methods* by Russell Davidson and James G. MacKinnon (2004). Beyond that, we develop enough auction theory at the level of the 1987 survey paper by R. Preston McAfee and John McMillan in the *Journal of Economic Literature.*

We presume no background in numerical analysis, although anyone serious about using the SEA in her or his research should probably own and read avidly Kenneth L. Judd's (1998) *Numerical Methods in Economics.* Nevertheless, because one of our goals is to provide a serious learner with the tools necessary to implement an empirical analysis of data from an actual auction, we have devoted considerable space to describing how the various methods are implemented on a computer.

Implementing the methods described in this book on a computer will typically require using either a programming language (such as C or FORTRAN), with or without the aid of a scientific subroutine library, or a high-level programming environment (such as GAUSS or MATLAB). We have chosen to use MATLAB to implement the solutions to those practice problems that involve computation because we believe that it represents a reasonable compromise between speed of computation and elegance in presentation; MATLAB is also easy to learn. Moreover, many scholars around the world have written books describing how to harness MATLAB's features in a variety of different fields, so supporting documentation is readily available. When we tested this book in the classroom, our students found one such useful text to be Hanselman and Littlefield (2005), but as MATLAB evolves others will surely appear.

Many graduate students in industrial organization will be familiar with or will have read Vijay Krishna's *Auction Theory*. We believe our book complements that text. However, our book is not as comprehensive as Krishna's because the structural econometrics of auctions is not as mature a literature as the literature concerning auction theory. The modern theoretical literature on auctions is well over four decades old, while the structural-econometric literature concerning auctions is less than two decades old, with the first published papers only appearing about fifteen years ago. In addition, because ours is an introduction, we have made an explicit pedagogical decision to restrict ourselves to a particular theoretical paradigm, and to work out quite extensively implications within that paradigm. While this limits the applicability of our results, we believe that a motivated learner will be able to understand, to appreciate, and to extend the existing literature.

4 What a Learner Will Know

Having read this book, we hope that a student or a professional will be able to analyze data from an actual auction and then to derive policy conclusions from this research. As mentioned above, one aim is to provide both students and professionals with the practical tools to structure their research. To this end, our book is accompanied by a CD on which sample data sets reside. At the end of each chapter, we have also provided practice problems in the form of mathematical questions or empirical exercises that one of us (Paarsch) has used previously when teaching courses on auctions. Solutions to the mathematical questions are provided at the end of the book, while examples of the MATLAB code which implement solutions to the empirical exercises are contained on the CD that accompanies the book. By working through the empirical exercises, a motivated learner will be able to evaluate the relative merits of the different empirical strategies presented in this book.

5 Notation

In this book, we have sought to adopt a logically-consistent notation, but as in virtually all attempts to create languages that have such a property, we have not been totally successful. Because the main focus in auction theory is asymmetric information, which economic theorists have chosen to represent as random variables, the bulk of our notation centers around a consistent way to describe random variables. Typically, we denote random variables by uppercase roman letters; for example, V,

W, X, Y, and Z. Realizations of random variables are then denoted by lowercase roman letters; for example, v is a realization of V.

Two lowercase Greek letters, β and σ, are used to denote equilibrium bid functions, while the lowercase Greek letter κ is used to denote a kernel-density function. The lowercase script Greek letter φ is used to denote a function and, sometimes, the lowercase Greek letter λ is either a Lagrange multiplier or a function. Typically, however, lowercase Greek letters are used to denote parameters. Uppercase Greek letters are then used to denote the spaces in which the corresponding lowercase Greek letters live; e.g., $\theta \in \Theta$.

The calligraphic letter $\mathcal{N}$ is special and used to denote the number of potential bidders at an auction, while the calligraphic letters $\mathcal{C}$, $\mathcal{E}$, and $\mathcal{V}$ denote the characteristic function as well as the expectation and variance operators, respectively; the calligraphic letter $\mathcal{L}$ denotes the logarithm of the likelihood function. Boldfaced versions of the calligraphic letters $\mathcal{E}$, $\mathcal{N}$, and $\mathcal{W}$ are used to denote the distribution of exponential, normal, and Weibull random variables, respectively. Thus, $\boldsymbol{\mathcal{E}}(\lambda)$ is an exponential random variable with hazard rate λ, $\boldsymbol{\mathcal{N}}(\mu, 1)$ is a normal random variable with mean μ and variance one, while $\boldsymbol{\mathcal{W}}(\lambda, \gamma)$ is a Weibull random variable with parameters λ and γ. Under this notation, $\boldsymbol{\mathcal{W}}(\lambda, 1)$ corresponds to an exponential random variable $\boldsymbol{\mathcal{E}}(\lambda)$.

Vectors are denoted by boldfaced letters. Thus, the (1×3) vector of realized random variables (v_1, v_2, v_3) is denoted by the boldfaced, lowercase, math italic letter $\boldsymbol{v}$. The superscript $^\top$ denotes the transpose operator, and an $(n \times 1)$ vector of constants $(c_1, c_2, \ldots, c_n)^\top$ is denoted by the boldfaced, lowercase roman letter $\mathbf{c}$. Uppercase, boldfaced, math italic letters, like $\boldsymbol{V}$, denote vectors of random variables, while matrices of constants are typically denoted by uppercase, boldfaced, roman letters. Thus, $\mathbf{H}$ would be a matrix, while $\boldsymbol{H}$ would be a vector of random variables. While boldfaced, lowercase Greek letters typically denote vectors of parameters, with boldfaced, uppercase, Greek letters denoting the spaces in which those parameters live (hence $\boldsymbol{\theta} \in \boldsymbol{\Theta}$), the notable exceptions are, of course, $\boldsymbol{\beta}$ and $\boldsymbol{\sigma}$, which denote vectors of equilibrium-bid functions. Uppercase, boldfaced, but not math font, Greek letters, such as $\boldsymbol{\Sigma}$, denote matrices of parameters; e.g., a variance-covariance matrix of constants. Because we use the boldfaced number 1 to denote an indicator function of an event (e.g., $\mathbf{1}(\mathsf{A})$ equals one when the event A occur and zero otherwise), we use the boldfaced, lowercase Greek letter $\boldsymbol{\iota}_K$ to denote a $(K \times 1)$ vector of ones where, in general, the trailing subscript denotes the dimension of the vector. The matrix $\mathbf{I}_n$ denotes the $(n \times n)$ identity matrix, while $\mathbf{i}$ denotes the imaginary number $\sqrt{-1}$.

Whenever possible the uppercase letter F is used to denote a cumulative distribution function, while the lowercase letter f is used to

denote a probability density function. On some occasions, we denote the survivor function by S and the hazard-rate function by h. The uppercase subscript following F, f, S, and h then indicates the corresponding random variable. Hence, $F_V(v)$ denotes the cumulative distribution function of the random variable V, while $f_W(w)$ denotes the probability density function of the random variable W, $S_X(x)$ denotes the survivor function of the random variable X, and $h_Y(y)$ denotes the hazard-rate function of the random variable Y. We denote the joint cumulative distribution of two random variables X and Y by $F_{XY}(x,y)$ and its corresponding joint probability density function by $f_{XY}(x,y)$. The conditional cumulative distribution and probability density functions of Y given X are denoted $F_{Y|X}(y|x)$ and $f_{Y|X}(y|x)$, respectively.

The i subscript is invariably used to index (potential or actual) bidders and typically goes from 1 to $\mathcal{N}$ or n, depending on the situation. Thus, an independent and identically distributed sample of size $\mathcal{N}$ from $F_V(v)$ might be denoted by $\{V_i\}_{i=1}^{\mathcal{N}}$. The j subscript is used to index different distributions and typically goes $j = 1, \ldots, J$. Thus, if there are J different distributions from which valuations can be drawn, these will be typically denoted $\{F_j(v)\}_{j=1}^{J}$. These might be collected in the $(J \times 1)$ vector $\boldsymbol{F}$. When transformations of the V_is from the distribution F_j result in additional cumulative distribution and probability density functions, we typically use G_j and g_j next, and then H_j and h_j. Sometimes, however, H_k doubles as a Hermite polynomial of order k, while h doubles as the hazard-rate function, and triples as a bandwidth parameter in nonparametric estimation. In these cases, the trailing subscript should make clear our intent.

In estimation problems, the subscript t indexes different observations, different auctions, and typically goes from $t = 1, \ldots, T$. The superscript "0" typically denotes the truth or population value, while a "$\,\hat{}\,$" atop a letter, particularly an F, an f, or a lowercase Greek letter, typically denotes an estimator (or estimate). Thus, for example, $\hat{F}_V(v)$ would represent an estimator (or estimate) of the true cumulative distribution function $F_V^0(v)$ at the point v, while $\hat{\theta}$ would denote an estimator (or estimate) of the true parameter value θ^0. Also, $\hat{\boldsymbol{\gamma}}$ would denote an estimator (or estimate) of the vector of true parameter values $\boldsymbol{\gamma}^0$.

Note, under our conventions, because an estimator is a random variable, it should be an uppercase letter, while an estimate (a realization of the random variable) should be a lowercase letter. But we do not adopt this extension for the obvious reason: we can't. Also, when more than one estimator (estimate) exists, we distinguish among alternatives by first using "$\,\tilde{}\,$", then "$\,\bar{}\,$", with the symbol "$\,\check{}\,$" being used rarely.

The k subscript is used to index observed covariate or other heterogeneity and typically goes $k = 1, \ldots, K$. An exception, however,

would be for auction t, where the vector of covariates, when a constant is included, $\boldsymbol{z}_t$ would be conformable to an unknown parameter vector $(\gamma_0, \gamma_1, \ldots, \gamma_{K-1})^\top$.

We have adopted a numbering convention for chapters, sections, and subsections as well as for assumptions, corollaries, lemmata, theorems, and equations within chapters. Thus, with regard to parts of the book, the first number refers to the chapter, the second number to the section, and the third number to the subsection. Hence, **2.4.3** would mean subsection 3 of section 4 of chapter 2, while **A.2.3** would mean section 3 of appendix 2. As a visual aid, we have made chapter headings in seventeen-point font, section headings in twelve-point font, and subsection headings in ten-point font. For assumptions, corollaries, lemmata, and theorems, the following **Assumption 4.3.1:** conveys the first assumption made in section 3 of chapter 4. Tables are numbered sequentially throughout a chapter—Table 1.1 denotes the first table in chapter 1. We have also numbered equations sequentially throughout each chapter, so (3.10) marks the tenth numbered equation in chapter 3, while (A.2.1) marks the first numbered equation in the second appendix.

6 Comments and Suggestions

While several pairs of eyes have proofread the camera-ready copy of this book, we recognize that some typographical errors will remain; we apologize to the reader for these. We shall appreciate reports via e-mail of the typographical errors so that they may be eliminated in future printings. Also, although we hope that no errors exist in this book, we are realistic. Thus, we shall also appreciate reports of any errors too. In addition, readers are encouraged to send us e-mail should they have questions or comments concerning any part of the book. We shall endeavor to answer the questions as quickly as we can. As people move around a lot, perhaps the easiest way to find a current e-mail address for any one of us is to Google his name.

The sample MATLAB code contained on the CD that accompanies this book was developed to be accessible to beginning- and intermediate-level users of MATLAB. In addition to any written solutions provided at the end of the book, we have also included comments in the code which make clear the coding steps. While coding, we sought clarity rather than elegance, sometimes at the expense of efficiency and parsimony. While we have endeavored to check this code very carefully, we realize that errors often creep into computer programs. We regret any errors that may exist in the code. No warranty is provided with this code, nor is any

implied. We do, however, welcome comments via e-mail from users of the code. If you have found an error, then documenting the conditions under which this error occurs will be most helpful. In fact, the better the documentation, the more likely we shall be able to find a solution.

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Paarsch is also grateful to his coauthors Bjarne Brendstrup, Stephen G. Donald, and Jacques Robert as well as the *Journal of Applied Econometrics*, the *Journal of Econometrics*, and the journal *Econometric Theory* for giving permission to borrow liberally from published work. Thus, some material in this book derives from the following papers:

> Paarsch, Harry J., "Deciding between the Common and Private Value Paradigms in Empirical Models of Auctions," *Journal of Econometrics*, 51 (1992), 191–215.

> Paarsch, Harry, J., "Deriving an Estimate of the Optimal Reserve Price: An Application to British Columbian Timber Sales," *Journal of Econometrics*, 78 (1997), 333–357.

> Donald Stephen G. and Harry J. Paarsch, "Superconsistent Estimation and Inference in Structural Econometric Models Using Extreme Order Statistics," *Journal of Econometrics*, 109 (2002), 305–340.

> Brendstrup, Bjarne and Harry J. Paarsch, "Identification and Estimation in Sequential, Asymmetric, English Auctions," *Journal of Econometrics*, (forthcoming).

and is reprinted with the permission of Elsevier Publishing. Also, some material in this book derives from the following paper:

> Donald, Stephen G. and Harry J. Paarsch, "Identification, Estimation, and Testing in Parametric Empirical Models of Auctions within the Independent Private Values Paradigm," *Econometric Theory*, 12 (1996), 517–567.

and is reprinted with the permission of Cambridge University Press.

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Finally, some material in this book derives from the following paper:

> Donald, Stephen G., Harry J. Paarsch, and Jacques Robert, "An Empirical Model of the Multi-Unit, Sequential, Clock Auction," *Journal of Applied Econometrics*, (forthcoming).

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Just before we began to put our manuscript together, we benefitted from several instructive conversations with A. Colin Cameron, James G. MacKinnon, and Martin Osborne on how to produce a book. James and Martin were particularly helpful with some of the subtle mechanics of typesetting a book in TEX, as was Robert Picard, who created a number of useful macros, which proved essential in creating the table of contents as well as the name and subject indexes. Martin also provided us with the names and e-mail addresses of publishers, which ultimately helped us find a home for this book.

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