Preface

The many branches of analysis—the infinitesimal calculus, the theories of measure and integration, differential and integral equations, the calculus of variations, and the theory of functions of a complex variable—form the most comprehensive part of mathematics. The present volume, however, is not an encyclopedia or textbook of these mathematical disciplines; it is addressed to readers who are making practical use of mathematics in their daily lives—in particular, young instructors in colleges—and who wish to orient themselves quickly in the foundations and recent developments of their science before they consult specialized works in a literature which is by now voluminous and often almost inaccessible.

Consequently we are concerned here with presenting the essential features of the subject in a reasonably ample form.

The first chapter deals with convergence. This subject is fundamental in all branches of analysis (in fact, analysis can be described as that branch of mathematics in which the concept of convergence is indispensable); at the same time it forms a bridge to topology. In recent times the concept of convergence has been made more profound through the introduction of Moore-Smith sequences, filters (introduced by Henri Cartan and Bourbaki), and uniform structures. Chapter 1 presents these concepts to the reader. Chapter 2 deals with the concept of a function. A real function is a mapping of a point set in the space R^n into the space R^m, and in particular the "classical" function is a mapping from R^1 into R^1 (in other words, a mapping of real numbers into real numbers). In this chapter we discuss the concepts of continuity and particularly of differentiability, which for a mapping of R^n into R^m amount, in coordinate-free terms, to linear approximation; and then by introducing coordinate axes we arrive at the concept of partial differentiation. The third chapter introduces the Riemann integral in R^n as a preparation for abstract measure theory. By specialization of abstract measures we arrive at the theory of the Lebesgue integral, which in modern times is more and more replacing the Riemann integral. In chapter 3*a* the results of the preceding chapter are utilized to present a modern theory of probability. Chapter 4 deals with the transformation of multiple integrals and thus leads directly to the theory of alternating differential forms. In chapter 5 we turn to complex analysis. Here we begin anew with elementary matters. The rules for operating with complex numbers are given, and complex differentiability and integrability are discussed. With the concept of holomorphy we pass to chapter 6, which presents the fundamental principles of the classical theory of functions of a complex variable. The identity theorems and the introduction of Riemann surfaces as the domains of definition of "many-valued" functions form the main part of this chapter. The starting point of the following chapter is the fact that in projective geometry the infinitely distant points are not introduced in the same way as in theory of functions of a complex variable. Are we then to conclude that these points at infinity have a type of reality somehow different from that of finite points? This chapter also closes with a look at present-day research (the theory of modifications).

Now follow those parts of analysis that are most important for the applications. Chapters 8 and 9, on real analysis, deal with ordinary and partial differential equations. Chapter 10, based on function-theoretic methods, deals with difference equations and definite integrals, in particular the Γ-function. Chapter 11 is an introduction to the far-reaching theory of Hilbert and Banach spaces, with applications to the theory of integral equations. Chapter 12, with the title "Real Functions," describes various connections between the properties of point sets and functions. Chapter 13 is an introduction to the productive and invariably surprising applications of analysis to questions in the theory of numbers. Chapter 14 has the title "The Changing Structure of Modern Mathematics." This chapter represents our summarizing remarks on the subject of pure mathematics. The rapidly increasing number of specialized results threatens to divide mathematics into many separate special disciplines. Opposing this tendency are the modern efforts to create a unified treatment of the whole of mathematics, depending chiefly on a theory of structure and based, of course, on elements that are common to many different branches of mathematics. These efforts have been incorporated, in a form that has by now become historic, in the work of N. Bourbaki, whose great and often surprising success will not be denied by anyone well acquainted with the situation. The last chapter of the present volume will discuss this modern tendency. We recommend that our readers peruse it carefully since it gives articulate expression to the purposes we have had in mind throughout.

It was difficult to coordinate the various parts of the present volume. From this point of view we are particularly grateful to Professor H. Tietz and Dr. H. Rau, as well as to many other colleagues, among them H. Suchlandt, of Munich, who has greatly assisted Dr. Rau in the preparation of the manuscript. We are also grateful to Dr. H. Liermann and Dr. Kuhlmann, who have worked, respectively, on chapter 13 (analysis and number theory) and chapter 12 (real functions). Special gratitude is due to the association of sponsors, in particular to Dr. Fritz Gummert, for their munificent financial support, without which the volume could never have been written. We also express our gratitude to the publishers, who have carried out our requests with great patience and have given the volume a magnificent format.

A staff of more than one hundred authors have been working on this set of volumes for more than four years. During that period we have been bereaved by the death of Messrs. Dreetz, Lohmeyer, Reimann, Süss, and Zühlke. To all these colleagues we remain enduringly grateful.

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Spring 1962

Münster (Westf.)