Chapter 1

INTRODUCTION

Recently there has been considerable interest in the problem of the collective interactions of charged-particle beams (or streams) with plasmas. Such interactions arise from a coupling of perturbations in the macroscopic density and current of the beam with those in the plasma through the associated electromagnetic field. Under certain conditions this coupling can lead to an increase in the coherent oscillations of the beam and plasma particles, and in the strength of the electromagnetic field, at the expense of the dc energy of the beam; that is, these perturbations can be unstable. These interactions, or instabilities, are of interest not only because of the important role that they play in some of the basic physical processes occurring in plasmas but also from their possible applications as a means of amplifying microwave power and as a means of heating a plasma.

The streaming of charged particles through a plasma arises usually from either an externally injected beam of electrons (or ions) or from currents induced in the plasma by external electromagnetic fields. The mathematical models adopted in this monograph are chosen largely with the first physical situation in mind, namely, the case of an <u>injected</u> beam of electrons passing through a plasma. In addition, a substantial portion of this investigation is aimed at the application to plasma heating by the beam-plasma interaction, and, more specifically, to the heating of the plasma ions. For this reason, considerable attention is given to the low-frequency interactions in beam-plasma systems with the plasma electrons assumed to be relatively hot.

This work is concerned only with the linearized description of the beam-plasma system, that is, with the "small-signal" perturbations on some unperturbed state. This approach allows one to determine the conditions for which an instability occurs, and also to say something about its initial rate of growth in time and space, but it clearly does not provide any information on the resulting large-signal behavior. In all cases, the instabilities are classified according to whether they are convective instabilities (amplifying waves) that grow in space, or whether they are non-convective instabilities (absolute instabilities) that grow in time, when the system is uniform in (at least) one spatial dimension. (These terms are defined more precisely in Chapter 2, where a general mathematical procedure for distinguishing between convective and nonconvective instabilities is presented.) Moreover, an attempt is made, whenever possible, to determine the exact conditions for which a finite (nonuniform) system will become unstable.

It is, by now, well known that there is a rather large variety of different modes of instabilities in beam-plasma systems. In this monograph, a number of different limiting cases are analyzed in the hope of obtaining a more complete picture of the interactions. The interactions are analyzed in a one-dimensional system with a steady magnetic field aligned along the direction of the beam flow and in systems of finite size in the direction transverse to the beam velocity. The presence of the plasma ions is accounted for, and the interactions at both high and low frequencies are investigated. In every case, however, since a number of simplifying assumptions are necessary in order to make the analysis tractable, this study is by no means an exhaustive coverage of the problem of stream-plasma interaction.

There is a very close analogy between the interaction of an electron beam with a plasma and the beam-circuit interaction occurring in microwave beam tube amplifiers and oscillators. This analogy has been stressed by Smullin and Chorney[1,2] and Gould and Trivelpiece.[3] The analysis presented here has been rather strongly influenced by this point of view. In fact, the heavy emphasis placed here on classifying "instabilities" as amplification processes or "true" growth in time arises largely from the beam tube concept that the "instability" in a traveling-wave tube is quite a bit different physically from the "instability" in a backward-wave oscillator. It will be shown that these beam tube analogies are very useful in the interpretation of the various instabilities, and, in fact, in some cases important parameters such as the critical length for oscillation can be obtained directly from the analogue.

1.1 Historical Review of the Problem

The concept of a macroscopic (collective) beam-plasma interaction was first proposed by Langmuir in 1925.[4] He proposed this interaction as a possible mechanism for generating the high frequency oscillations observed in his hot-cathode discharge. It took until the late 1940's, however, before widespread interest in the subject developed; this interest arose largely as a consequence of several important theoretical papers which were published during that time. Pierce,[5] in 1948, showed that a beam of electrons passing through a cold ion cloud should cause amplification of signals at frequencies just below the ion plasma frequency. He proposed this mechanism as a possible explanation for the spurious oscillations that were observed in some microwave tubes. Haeff,[6] in 1948, showed that amplification results when two electron beams move with different velocities, and he indicated that this could be a possible source of solar noise. Several authors, essentially simultaneously, proposed a two-stream microwave amplifier based on this principle.[7] Bailey[8] stated that the transverse as well as the longitudinal waves in beam-plasma systems in finite steady magnetic fields could be amplified; however, his interpretation of the transverse waves as "amplifying" was later criticized by Twiss.[9] Finally, Bohm and Gross[10] and Akhiezer and Fainberg[11] treated the electrostatic instabilities in the absence of a steady magnetic field, using the kinetic equations; the former authors gave a fairly complete description of the physical process of the energy transfer in the interaction.

It is, by now, well known that there are a number of different ways in which a beam of charged particles in the presence of a steady magnetic field can interact with a plasma, as will be discussed in more detail in Sections 1.1.1 and 1.1.2. Rather complete formulations of the unbounded beam-plasma system have been given which account for the velocity spreads in both the beam and the plasma, and some work has also been presented which accounts for the finite dimensions of the system transverse to the beam velocity. (Note that all of the earlier works[5-11] assumed an unbounded beam-plasma system.) The more recent references that are most pertinent to the work in this monograph are also briefly discussed. It should also be mentioned that rather extensive bibliographies to both the Russian and English literature have been given in two recent review articles.[12,13]

The following discussion is divided into sections dealing with theories for unbounded and bounded beam-plasma systems. The question of absolute instabilities and amplifying waves is briefly reviewed in the final section.

1.1.1 Unbounded Beam-Plasma Systems. The earliest work on beam-plasma interactions dealt only with the so-called electrostatic (or longitudinal space-charge-wave) instability in the absence of a steady magnetic field. In the simplest formulation of the problem, which neglects collisions and temperature, the spatial growth rate of this interaction is infinite at the electron plasma frequency, as was shown by Pierce[5] for the similar problem of a beam drifting through a cold ion cloud. This high-frequency instability has been investigated in some detail by a large number of authors; in particular, Sumi[14] and Boyd, Field, and Gould[15] showed that the amplification rate is bounded when the effects of collisions and temperature are included.

In a relatively cold plasma, the electrostatic instability is strongest near the electron plasma frequency. It has been shown by Rukhadze[16] and Kitsenko and Stepanov[17] that an electrostatic instability can occur at the ion plasma frequency when the unperturbed beam velocity is much less than the average thermal speed of the plasma electrons. This instability is clearly of great interest from the standpoint of heating the ions of a plasma by the interaction with an electron beam, and will be discussed at some length in this monograph.

Birdsall[18] has shown that the collisional damping in the plasma can itself be a mechanism for inducing an instability of the electrostatic wave. The idea that a lossy medium around an electron beam should result in amplification of the beam space-charge waves was first theoretically predicted by L. J. Chu[19] on the basis of his kinetic power theorem, and was later experimentally demonstrated by Birdsall, Brewer, and Haeff.[20]

It has been shown by a number of authors that the transverse waves in a magnetized plasma-beam system which propagate in the direction of the steady magnetic field can be unstable.[21-25] For an electron beam traversing a cold plasma, an instability of the transverse wave occurs near the ion cyclotron frequency. It has been pointed out by Stix[26] that the ion temperature of the plasma can be very important in this interaction. A detailed discussion of the effect of ion temperature on this interaction is given in Chapter 3.

1.1.2 Bounded Beam-Plasma Systems. Theories that account for the finite dimensions of the beam-plasma system in the direction transverse to the beam velocity usually deal with the cold, collisionless model of the plasma.

The space-charge-wave interaction of an electron beam with a cold, collisionless plasma in the presence of an infinite magnetic field in the direction of the beam velocity has been considered by Bogdanov, Kislov, and Tchernov[27] and Vlaardingerbroek, Weimer, and Nunnink.[28] The latter authors show that the amplification rate of this interaction is infinite only when the beam space-charge wavelength is (roughly) less than the transverse dimension of the plasma, for the case of both the beam and the plasma filling a waveguide.

The space-charge-wave interactions in the case of a finite axial magnetic field has been considered by several authors with the aid of the quasi-static approximation.[1,29-34] There have been, however, some difficulties in the interpretation of solutions of the dispersion equation in this case, as regards the meaning of the roots of complex wave numbers for real frequemcy in the vicinity of the negative dispersion wave in the plasma. This point is clarified in the present work by use of the amplification criteria developed in Chapter 2.

Smullin and Chorney[1,2,35] considered the interaction of an electron beam with an ion cloud within a quasi-static approximation and stressed the very close analogy of these results with the interactions in various types of microwave beam tubes. For the case of both the beam and the ion cloud filling a waveguide, they showed that cyclotron-wave interactions could result as well as the usual space-charge-wave interactions. Morse[36] and Getty[37] used a similar model to analyze the interaction of an electron beam with the plasma electrons and presented computations of the dispersion near the interaction of the space-charge waves and slow cyclotron wave of the beam with the negative-dispersion plasma wave.

Kino and Gerchberg[38] have recently pointed out that a very thin electron beam can have transverse as well as longitudinal modes of instability when passing through a plasma of infinite extent. These transverse modes of instability are obtained for zero external magnetic field as well as for finite magnetic fields.

Some work has also been started on analyzing the beam-plasma interactions in a cold, collisionless plasma without making the quasi-static approximation.[39,40] In addition, the effect of plasma temperature in the case of beam-plasma systems of finite transverse dimensions has been included in a quasi-static formulation by accounting only for the velocity spread in the direction parallel to the external magnetic field.[41,42]

1.1.3 Absolute Instabilities and Amplifying Waves. The criterion of stability used by most authors is whether or not the dispersion equation of the uniform (infinite) system admits complex values of the frequency for some real values of the wave number (with the imaginary part of the frequency corresponding to growth in time). It was first pointed out by Twiss,[43,44] and later by Landau and Liftshitz[45] and by Sturrock,[46] that two distinct types of instabilities can be identified physically. A spatial pulse on a uniform system can propagate along the system so that the disturbance decays with time at a fixed point in space (convective instability) or it can increase with time at every point in space (nonconvective or absolute instability).

A somewhat related problem has arisen in the interpretation of solutions of a dispersion equation for which complex values of the wave number are found for real values of the frequency. This problem has arisen, for example, in the description of the spatial amplification process in the interaction of electron beams with circuits in microwave beam tubes.[47] The solutions with complex wave numbers can represent amplifying waves which grow in space away from some source, or they can represent evanescent waves which decay away from the source. In simple cases, the concept of small-signal energy and power has helped to resolve this question.[19,48-50]

In the course of this investigation, difficulties were encountered in the interpretation of solutions of some of the dispersion equations by use of the existing mathematical criteria for distinguishing between absolute and convective instabilities, and between amplifying and evanescent waves.[46,51-53] New criteria that avoid these difficulties are presented in Chapter 2. A critical review of the previous criteria is presented at the end of that chapter.

1.2 Assumptions and Mathematical Models

As was mentioned before, the physical situation to which this theoretical investigation is intended to be most applicable is that of an externally generated electron beam that is injected into a plasma. For this reason, it will be assumed throughout that the beam is "cold," that is, that all beam electrons have the same unperturbed velocity. In many such experimental situations the beam density is usually several orders of magnitude less than the plasma density; for this reason it will often be useful to consider first the interactions in the mathematical limit of the beam density approaching zero. Interesting transitions can occur, however, for small but finite beam densities, and the majority of the analysis is by no means limited to the case of a beam of infinitesimal density.

The model of the plasma which is adopted is that of a fully ionized gas composed of particles which interact only through the large-scale, macroscopic electromagnetic fields. These particles can then be described within the framework of the collisionless Boltzmann-Vlasov equation. The effect of collisions on the beam-plasma interaction can safely be ignored if the frequency of the wave is much larger than any collision frequency, and means that the theory should be most applicable to hot plasmas of moderate or low density. It should also be mentioned that the effects of temperature are not included by means of a transport equation formalism, where only the first few moments of the Boltzmann equation are considered. Our analysis deals with the full distribution of velocities and includes the effects of the Landau and cyclotron damping.

The criteria for distinguishing between amplifying and evanescent waves and for determing absolute instabilities are presented in Chapter 2. These criteria are not restricted to the case of beam-plasma systems but are applicable to a wide class of uniform, time-invariant systems.

Chapter 3 considers the waves propagating along the steady magnetic field in an unbounded beam-plasma system. These waves consist of the electrostatic longitudinal wave, which is independent of the steady magnetic field, and the transverse circularly polarized waves in which the electromagnetic fields are perpendicular to the steady magnetic field. A detailed examination is made of the low-frequency longitudinal interactions for the case when the beam velocity is much less than the average thermal speed of the plasma electrons. The instabilities of the transverse waves occur at frequencies below the ion cyclotron frequency. The effects of the plasma temperature on these transverse instabilities are determined.

The analysis of the interaction in a cold plasma that is of finite extent in the plane transverse to the steady magnetic field is presented in Chapter 4. Various limiting cases are studied in order to obtain analytical results. In Section 4.1 the case of a very low density electron beam in a finite axial magnetic field is considered. It is assumed that both the beam and the plasma fill a cylindrical waveguide structure, and the quasi-static approximation is made. The strength of the various interactions is determined for both high and low frequencies. In Section 4.2 the interaction of a very thin solid beam with a plasma that fills the waveguide is analyzed, again within the quasi-static assumption. The interaction of solid and hollow electron beams with a plasma in the presence of an infinite axial magnetic field is analyzed in Section 4.3.

The low-frequency interaction with ions in a hot-electron plasma of finite transverse dimensions is considered in Chapter 5. It is assumed that a large axial magnetic field confines the electrons to motion only along the field lines, so that only the random velocities of the plasma electrons along the field lines are of importance. The beam and plasma are assumed to fill a cylindrical waveguide structure, and the quasi-static assumption is made. It is shown that the interaction in a hot-electron plasma of finite transverse dimensions differs both qualitatively as well as quantitatively from the one-dimensional case.