Preface

Uncertainty is everywhere. There is no field in economics in which risk is not a dimension of the decision-making environment. The theory of finance provides the most obvious example of uncertainty in decision making. Similarly most recent developments in macroeconomics have been made possible by recognizing the importance of risk in explaining individual decisions. Consumption patterns, investments, and labor decisions can only be understood completely if uncertainty is taken into account into the decision-making process. Environmental economics provides another illustration. Public opinion is now very sensitive to the presence of potentially catastrophic risks related, for example, to the greenhouse effect and genetic manipulations. Environmental economists have introduced probabilistic scenarios in their models to exhibit socially efficient levels of prevention efforts. Finally, the extraordinary contributions of asymmetric information to game theory have reinforced interest in uncertainty among economists.

We are lucky enough to have a well-accepted and unified framework to introduce uncertainty in economic modeling. John von Neumann and Oskar Morgenstern, in the mid-1940s, developed the expected utility theory, building on Daniel Bernoulli's idea that agents facing risk maximize the expected value of the utility of their wealth. Expected utility (EU) theory is now fifty years old. It is a ubiquitous instrument in economic modeling. Most economists recognize that the theory has been very useful for explaining the functioning of our economies. The aim of this book is to provide a detailed and unified analysis of the implications of the expected utility model to the economic theory.

One of the innovation of this book is its refusal to use convenient utility functions to solve complex problems of decision and market

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equilibrium under uncertainty. The benefit of this method is twofold. First, it allows us to better understand the relevant basic concepts that determine optimal behavior under uncertainty. Second, it reestablishes the richness and diversity of the EU framework, something that tends to be forgotten in the strongly reductionist limitations imposed on the set of utility functions usually used for convenience.

This book provides an overview of the most recent developments in expected utility theory. It is aimed at any audience that is interested in problems related to efficient, or optimal, public and private strategies for dealing with risk. The book heavily relies on concepts that are standard in the theory of finance. But most findings presented in this book are useful for our understanding of various economic problems ranging from macroeconomic fluctuations to global warming.

Organization of the Book

Part I of this book is devoted to the presentation of the EU model and its basic related concepts. Chapter 1 presents the axioms that underlie the EU approach. It proves the von Neumann–Morgenstern EU theorem and discusses its main limitations. Chapter 2 shows how to introduce risk aversion, and "more risk aversion," in the EU model. The notion of risk premium is introduced here, together with the familiar utility functions that are used in macroeconomics and finance. Chapter 3 is devoted to the dual question of how to define risk, and "more risk," in this model. It also presents some recent developments in the literature on stochastic dominance.

In part II we examine the standard problem of choice under uncertainty in which an agent must decide how much to invest in a portfolio of two assets, one of the two being risk free. Chapter 4 reviews the effect of a change in wealth, in the degree of risk aversion, or in the degree of risk on the optimal demand for the risky asset. This model is also helpful for understanding the behavior of a policyholder insuring a given risk at an unfair premium, or the behavior of a risk-averse entrepreneur who must determine her productions capacity under price uncertainty. Chapter 5 is devoted to an equilibrium version of this problem. It presents the simplest version of the equity premium puzzle.

Part III introduces some technical tools that are useful to solve various classes of decision problems under uncertainty. It shows how the linearity of the EU objective function with respect to probabilities is very helpful. Simple hyperplanes separation theorems can Preface xvii

be used in such an environment. The existence of such simple technical tool in the case of EU is central to our understanding of the success of the EU model in economics. Chapter 6 is devoted to the presentation of the basic hyperplane separation theorem—a tool richer than Jensen's inequality—and its simplest applications in the economics of uncertainty. Chapter 7 examines more sophisticated applications that rely on the notion of log-supermodular functions. The use of these tools will allow us to unify the wide range of results that has appeared in the literature during the last thirty years.

Part IV takes various decision problems to more than one risk. We take the standard portfolio model which was developed in the early 1980s and treat portfolio risk as a unique source of risk borne by the consumer. To this we introduce the real world, where people must manage and control several sources of risk simultaneously. Chapters 8 and 9 show how the presence of an exogenous risk in background wealth affects the demand for other independent risks. New considerations like properness, standardness, and risk vulnerability are introduced and discussed. Chapter 10 covers the problem of whether independent risks can be substitute. To illustrate, we characterize the conditions under which the opportunity to invest in one risky asset reduces the optimal demand for another independent risky asset. The remaining of this part deals with how to use dynamic programming methods to show how the time horizon length affects the optimal in behavior toward repeated risks. Chapter 11 does this for the simplest case where financial markets are frictionless; then chapter 12 explores the effects of various limitations in dynamic trading strategies on optimal dynamic portfolio management.

The canonical decision problem in the theory of finance, namely the Arrow-Debreu portfolio problem, is developed in part V. Chapter 13 shows that the investor can exchange contingent claim contracts in competitive markets. We determine the impact of risk aversion on the investors optimal portfolio. Several properties of the value function for the investor's wealth are derived in chapter 14.

Consumption and savings form the framework of part VI. We start with the consumption problem under certainty in chapter 15. There, we see how people evaluate their welfare when they consume their wealth over different periods. We learn that the structure of this problem is equivalent to the structure of the static Arrow-Debreu portfolio. We then explore the properties of the marginal propensity to consume and the concept of "time diversification." The assumption of a time-separable utility function makes the treatment of time

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very similar to the treatment of uncertainty in classical economics. In chapter 16 the important notion of prudence is introduced. Saving is examined as a way to build a precautionary reserve to forearm one-self against future exogenous risks. The equilibrium version of this model is examined in chapter 17, where the equilibrium risk-free rate is derived. In these three chapters we assume that financial markets are perfect; that is to say, there is a single risk-free interest rate at which consumers can borrow and lend. This is obviously not a realistic assumption. So a market imperfection is introduced in chapter 18 by the way of a liquidity constraint. The existence of a liquidity constraint provides another incentive to save. Related to this is the Merton-Samuelson problem of the joint decisions on consumption and risk-taking. This problem is discussed in chapter 19. Finally, chapter 20 covers a model that allows us to disentangle the notions of risk aversion and aversion to consumption over time.

In part VII several of the previous results are gathered in order to determine the equilibrium price of risk and time in an Arrow-Debreu economy. More generally, we look at an analysis of how risks are traded in our economies. Chapter 21 starts with the characterization of socially efficient risk-sharing arrangements. Chapter 22 covers how competition in financial markets can generate an efficient allocation of risks. It also discusses how to determine the equilibrium price of risk and time as a function of the characteristics of the particular economy. The discussion includes the properties of the capital asset pricing model. The chapter then provides an introduction to decision-making problems for corporate firms, and to the Modigliani-Miller theorem. Chapter 23 shows how individual risk attitudes can be aggregated to derive the society's attitude toward risk and time and its impact on asset prices.

Last, part VIII focuses on dynamic models of decisions making under uncertainty when a flow of information on future risks is expected over time. The basic tools and concepts (value of information and Blackwell's theorem) are provided in chapter 24. Chapter 25 covers the optimal timing of an investment decision when there is a quasi-optional value to waiting for future information about the investments profitability. The degree of irreversibility of the investment is taken into account. The concluding chapter 26 offers a smorgasbord of analyses related to the effect of the expectation of future information on current decisions and an equilibrium prices of financial assets.