Hahn on the Share of Wages in National Income

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Frank Hahn materialized for me in the fall of 1956, when he and Dorothy arrived from Birmingham to spend the year at MIT. I cannot remember if it was literally a case of love at first sight. By the end of that year, however, a lifelong friendship had come into being. (I am entitled to a *little* extrapolation.) By 1956 Hahn was already recognizably the economic theorist the world knows now. When we worked together or talked economics seriously, the subject was usually general-equilibrium theory, especially multimarket dynamics. It may therefore come as a surprise to many of his friends and admirers that Hahn has an excellent claim to be the originator of the "macroeconomic theory of distribution," to have been—dare I say it?—a sort of proto-Kaldor. It came as something of an embarrassment to him too. Let us see.

The scene of the crime is the London School of Economics, where Hahn was awarded a Ph.D. in 1951; his thesis was entitled "The Share of Wages in the National Income." He recorded later that Nicholas Kaldor was his supervisor for the first three months of research, to be followed by Lionel Robbins for the rest. Hahn reports that Kaldor recalled (presumably in 1971 or 1972) having suggested to his student in 1947 that "the best approach to distribution theory is macroeconomic." It is not clear to me that so vague a statement has any cash value. Nor is it clear from our hero's reporting whether he is generously sharing priority with Kaldor or fastidiously blaming the thought on someone else. I will come back to Hahn's second thoughts later.

The only immediate publication to come out of the thesis was a mere 11-page article in *Oxford Economic Papers* in 1951 with the same title as the whole thesis. The article is essentially a reproduction of chapter 3 of the thesis, called "A Simple Model" in the original.

Chapter 1, let it be said, is a critical account of the discussion that began with Hicks's *Theory of Wages* in 1932 and ended with his "Distribution and Economic Progress: A Revised Version" in 1936, and which involved most of the important names in English-speaking economics (except for Keynes) in between.¹ The young Hahn was not very much amused by this literature. He thought that the vital transition from the partialequilibrium analysis of a single industry facing a given demand to the general-equilibrium analysis of the income distribution produced by an interrelated system of (perhaps imperfectly competitive) markets had been fudged, except perhaps in certain special cases of dubious interest.

Chapter 2 was a discussion of Kalecki's model that makes the division of income between wages and profits primarily a matter of the "degree of monopoly" or excess of price over prime cost (or, more accurately, of the elasticity of demand for the representative firm).² Hahn does not care for this story at all; indeed, he takes it apart rather unsparingly for its looseness of definition and its elision of essential assumptions.

Having made his bow—if that is the word—to the literature, Hahn turns to his own entirely original contribution. I propose to devote the whole of this essay to a commentary on the model of Hahn's chapter 3 and on his discussion of it some 20 years later.

My purpose is not particularly doctrine-historical. It would not be very enlightening to trace the literature that may have influenced Hahn. My purpose is celebratory and—in a loose way—methodological. My own belief is that economics, as an applied science, is about approximations, not about theorems. So I have a deep interest in the aggregative use of microeconomic ideas. I wish everyone had. To look back at this early effort by Hahn, especially Hahn, seems therefore intrinsically interesting.

A Simple Model

The description in the thesis is considerably less explicit in laying out assumptions and deriving conclusions than the style of middle-period or late-period Hahn would lead one to expect. So I shall begin by reproducing the model, including details that its author omitted (perhaps in a gesture to the expository style of the time).

The "simple model" contains three novelties. To begin with, it is a macroeconomic model—a one-sector general-equilibrium model—whose market form is straightforward imperfect competition. Nowadays the fashion³ is to start one turtle further down, if you remember the joke, with n symetrically producible goods and a utility function that makes them imperfectly but symmetrically substitutable for each other, and then to deduce the typical firm's perceived demand curve. Hahn cannot know about that finesse. He just postulates a common demand curve whose shifts are assumed to preserve the elasticity at each price. Thus $x = AD(\pi)$ in his notation, where π is the nominal price of goods, x is output or sales, and A

is a shift parameter. (I say this a little more confidently than I ought, because the thesis does not always say whether it means "real" or "nominal.")

When I describe this as a novelty I mean only that Hahn's procedure makes a break with the immediately preceding literature on the functional distribution of income. Imperfect competition was more discussed then than it was later, but usually in a partial-equilibrium way, not as a feature of a complete model.

The second novelty is more novel. The representative firm maximizes something more complicated than profit. Hahn has in mind an entrepreneurial utility function that is increasing in profit and decreasing in the scale of output. Almost at once he assumes (with the usual apology) that the marginal utility of (profit) income is constant, so that the firm is actualmaximizing P - V(x), where P is profit and V(x) is an increasing convex function. To motivate this departure from routine, Hahn describes V itself as a "general disutility of production." He seems to have in mind that larger real output entails a larger beginning-of-period commitment of funds to the firm and thus a larger exposure to the risk of loss or bankruptcy. (There is a kinship to Kalecki's principle of increasing risk."⁴ This idea may have been in the air around the London School of Economics at the time.) One could imagine a more explicit deduction from the maximization of expected (quadratic) utility, which case the objective would be, say

$$P-aP^2-aV(x),$$

where now P is the mean value of profit and V, its variance, is assumed to increase with x. The refinement does not seem to be worth very much, so I shall keep to Hahn's assumption with the the addition of a parameter to measure the importance of this risk-aversion factor; thus, the firm maximizes $P - \lambda V(x)$.

The third novelty, the proto-Kaldorian novelty, is the explicit assumption that aggregate saving depends on the division of aggregate income between wages and profits. These are the only two sources of income considered, and the natural presumption is maintained: aggregate saving from any given aggregate income is less the greater the wage share. This effect is taken to be definite, though small; it diminishes as the wage share rises.

The last building block of the model is the demand for investment. This is discussed with some richness of detail, for a 12-page chapter, so I shall describe it accurately. Let the firm's current output, a near-normal output

for its current stock of capital, be x_0 . Investment is assumed—rather casually—to be insensitive to the interest rate. Most economists would have agreed *circa* 1950; I guess that Hahn would be less casual today, even for short-run purposes. As x increases from x_0 , investment increases more than proportionally. Thus, for $x > x_0$ we can say I = xi(x) with $i(\cdot)$ an increasing function. This superproportionality is meant as a sort of static moral equivalent to the acceleration principle. For $x < x_0$, on the other hand, only replacement and autonomous investment occur, and these may be treated as more or less independent of current output. Thus, $I = x_0i(x_0)$ for $x < x_0$.

There is neither taxation nor public expenditure in this model, so that equilibrium in the goods market requires that Saving = Investment. Hahn's description of consumption-saving behavior would naturally be formalized to say that S = s(w)x, where w is the share of the wage bill in the value of output. (He suggests that the marginal propensity to consume is approximately invariant to income as a matter of fact, and that the marginal-and-average propensity to consume is an increasing concave function of w. Thus, s(w) is decreasing and convex.) The equation of Saving and Investment yields s(w) = i(x) for $x > x_0$ and s(w)x = constant for $x > x_0$. When plotted in the (x, w) plane as an aggregate demand curve (not the conventional one, of course, but a locus of (x, w) pairs that make C + I = x, this has a peculiar shape. For $x > x_0$, dx/dw < 0; for $x < x_0$, dx/dw > 0. In the first case, higher output means a higher ratio of investment to output and requires a higher ratio of saving to output, and thus a lower wage share. In the second case, higher output leaves aggregate investment unchanged and requires unchanged saving; thus, a higher wage share is needed to offset the higher saving from higher output itself. This second phase seems a little farfetched; nor does one observe a shrinking wage share as income falls below normal in the business cycle. Hahn does not actually make much of the rising branch of the aggregate demand curve, anyway.

I have laid out the "Kaldorian" part of the model first. Now I return to the supply side of the model, which is distinctly non-Kaldorian. As already suggested, the typical firm is engaged in imperfect competition. Hahn has been careful to assume that the typical firm's demand curve undergoes only isoelastic shifts, i.e., $\pi = d(x/A)$, where d is D^{-1} . He postulates constant nominal wages and thus constant costs (= cx, say) as well. The firm's profit is P(x) = xd(x/A) - cx, but it maximizes $P(x) - \lambda V(x)$. Thus,

 $d(x/A) + d'(X/A)X/A = c + \lambda V'(x).$

From this condition it follows easily that the comparative-static derivative dx/dA > 0. (Imagine the condition satisfied, and insert a larger value of A. The LHS is just marginal revenue, a decreasing function of its argument, so it is now larger than the RHS. If x were to decrease, the RHS would be smaller and the LHS would be larger still. So x must increase to preserve equality, though x/A must fall.)

Moreover, P(x)/x, the profit margin per unit of output, is equal to d(x/A) - c. We have just seen that x/A decreases when x increases. So P(x)/x increases when A increases. Thus, P/x and x increase and decrease together when aggregate demand shifts, as parametrized by A.

The share of profits in output is

$$\frac{P}{x\pi} \doteq 1 - \frac{c}{\pi},$$

so the wage share is

$$w=\frac{c}{\pi}=\frac{c}{d(x/A)}.$$

Thus, w falls and rises as A rises and falls. An equivalent statement is that w and x move inversely under demand shocks. This is Hahn's "aggregate supply curve." It is not the conventional one at all, but it is entitled to the name. It derives entirely—well, almost entirely—from the supply side of the economy. Under these assumptions, which could obviously be relaxed a little, firms will produce more output the higher the profit margin, and only when the profit margin is higher. Notice that this is not an aggregation of individual supply curves; these firms are imperfect competitors and do not have supply curves. The correspondence between P/x (or w) and x is parametric in character. The point is that firms will produce x only when they can sell x; and sales of exactly x require that the share of wages be exactly w, higher x requiring smaller w.

Figure 1, copied from Hahn, brings the aggregate supply and demand curves together. Their intersection generates a pair of values x and w that satisfy both demand and supply conditions: when the share of wages is w, consumption and investment add up to x; when the profit share is 1 - w, firms are just prepared to produce x.

Hahn gives an informal argument to show that an intersection like that in Figure 1 is stable. In effect it goes like this: at a higher value of w, firms would wish to produce the corresponding output along AS; since this com-

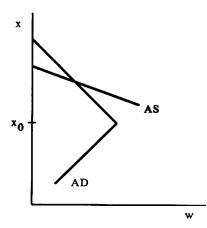


Figure 1 Aggregate demand and aggregate supply in the Hahn model.

bination lies to the right of AD, there will be excess demand for goods and the price level will rise. Since the nominal wage is fixed, the real wage and with it w will fall. Similarly for downward displacements of w.

This is not only informal but pretty special too. As so often happens, the dynamic story sharpens certain queries about the underlying model. To give only two examples in this context:

(a) The stability argument brings to the surface the previously more or less submerged constancy of the nominal wage, which looks less innocuous as soon as the price of goods starts to move.

(b) If, as is certainly plausible, investment were also a function of the profit margin (i.e., of w), then the behavior of aggregate demand would be more delicate.

A Closer Look

The thesis does a certain amount of comparative statics with this model by informally shifting the AS and AD demand curves. I want to be more nearly exhaustive, so I shall begin by formulating the model more explicitly. In this I am surely doing what the mature Hahn would do.

Since I am not going to pursue business-cycle topics, I shall forget the upward-sloping portion of the AD curve. Thus, the first equation of the model is just

$$s(w) = i(x) + e. \tag{1}$$

This differs from what I wrote before only in the addition of a parameter, e, to represent exogenous shocks to investment (or saving, for that matter). It should be noticed that in this form e is a shock to the investment *quota*, the ratio of investment to output; this is of no importance given the use I shall make of the model.

Next I want to make a minor change in Hahn's original specification of the entrepreneurial utility function. In the expression $P - \lambda V(x)$, V is supposed to represent uncertainty or the "general disutility of production" and has to do with the unwillingness to commit funds to production. In that case the argument of V is more properly total cost, cx, rather than output, x. This is an empty distinction for the thesis, since it does not ask about the consequences of variations in c. I want to do so, so I shall write V(cx). Then the typical firm's utility maximization leads to

$$d(x/A) + d'(X/A)x/A = c(1 + \lambda V'(cx)).$$

The LHS is of course the marginal-revenue function, which I shall naturally take to be downward-sloping. So it has an inverse function, to be designated $r(\cdot)$. In the notation already established, $x/A = D(\pi)$ and $\pi = c/w$. The second equation of the model can therefore be written as

$$D(c/w) = r[c(1 + \lambda V'(cx))].$$
⁽²⁾

Equations (1) and (2), taken as determining w and x as functions of e, c, and λ , form precisely the simple model illustrated in figure 1 above. One can proceed to do comparative statics with each of the parameters. Total differentiation leads to the equations of variation

$$\begin{bmatrix} S' & -i' \\ -D'c/w^2 & -\lambda c^2 r' V'' \end{bmatrix} \begin{pmatrix} dw \\ dx \end{pmatrix}$$
$$= \begin{pmatrix} de \\ cr'V' d\lambda + \left[r'(1 + \lambda V' + cx\lambda V'') - \frac{1}{w}D' \right] dc \end{pmatrix}$$

The sign pattern of the determinant Δ is

$$\begin{pmatrix} - & - \\ + & + \end{pmatrix},$$

but it can easily be checked that the "stable" configuration of the diagram, with AS flatter than AD, is equivalent to the determinant being negative. Under that assumption, straightforward calculation gives the following comparative-static derivatives:

$$\frac{\partial w}{\partial e} = \Delta^{-1} \begin{vmatrix} 1 & -i' \\ 0 & -\lambda c^2 r' V'' \end{vmatrix} < 0,$$
(3a)

$$\frac{\partial x}{\partial e} = \Delta^{-1} \begin{vmatrix} s' & 1\\ -D'c/w^2 & 0 \end{vmatrix} > 0,$$
(3b)

$$\frac{\partial w}{\partial \lambda} = \Delta^{-1} \begin{vmatrix} 0 & -i' \\ cr'V' & -\lambda c^2 r'V'' \end{vmatrix} > 0,$$
(3c)

$$\frac{\partial \mathbf{x}}{\partial \lambda} = \Delta^{-1} \begin{vmatrix} \mathbf{s}' & \mathbf{0} \\ -D'c/w^2 & c\mathbf{r}'V' \end{vmatrix} < \mathbf{0}.$$
(3d)

These derivatives are worth a comment. Inside (3b) is hidden a multiplier formula; it says that an exogenous increment to investment causes output to increase by a multiple equal to (essentially) the reciprocal of the marginal propensity to save minus the marginal propensity to invest. The qualification arises only because s is given as a function of w and has to be translated into the proper dimension. The same increment to investment-demand must lead to a lower wage share in order to elicit the additional saving. It is intuitive that a greater "general disutility of production" should reduce equilibrium production. In turn, this reduces investment, and the wage share must rise to choke off the unneeded saving.

The cost parameter c requires a word of explanation. If labor were the only input to production, c would be equal to Wn, where W is the nominal wage and n is the constant labor requirement per unit of output. Units could be chosen so that n = 1, and then c would be just the nominal wage. In that literal interpretation, however, "investment" makes no practical sense. If capital and labor were both required for production, but with labor the only variable input in the short run, constant returns to scale would imply increasing short-run unit cost, so c would be an increasing function of x. The interpretation that was pretty clearly in Hahn's mind is that there are fixed proportions in the short run. Then as long as output falls short of "capacity" (which must exceed x_0) and user costs are negligible, the labor-only version is effectively valid. Varying c is like varying W. That seems to be the correct spirit.

There remains a gray area, however. The coefficient of dc in the RHS of the equations of variation is the difference between two negative quantities; thus, it is ambiguous in sign. This, in turn, reflects the fact that both sides of (2) are decreasing in c. Without resolving the ambiguity, one does not know which way AS shifts when c varies. The direction in which this cookie crumbles is determined in part by the curvature of the demand curve. I observe that, for any constant-elasticity demand curve, AS shifts down when c increases. If we take that for the "normal" case, then

$$dx/dc < 0, \tag{3e}$$

$$dw/dc > 0; (3f)$$

however, it should be remembered that these could go the other way under reasonable assumptions. In the thesis, Hahn notes the ambiguity. His very brief discussion invokes the elasticity of substitution of an unspecified production function. This would surely matter in a more detailed model, as the later Hahn would insist.

The economics of this difficulty calls attention to a weakness in this sort of model. Think of c as standing for the nominal wage. We are accustomed to the notion that a change in c might push the share of wages in either direction. (Hence Hahn's reference to the elasticity of substitution.) If the wage share is to rise, however, the saving rate will fall and the necessary cutback in the investment quota requires a fall in output. The case that the wage share falls is analogous. Most of us—including, I suspect, the later Hahn—think of distributive shares as the resultant of a number of more fundamental events, and thus as a poor candidate to be the central concept of a theory. The point is reinforced by (3c): one is not too happy with the thought that changes in the weight of entrepreneurial risk aversion have direct and unambiguous consequences for relative shares. But that is proto-Kaldorism for you.

In this context it is natural to ask what effect a change in the degree of monopoly (i.e., in the elasticity of demand) would have. Hahn gives the answer: a lower elasticity of demand goes with lower output and a higher wage share. He does not comment on the apparent paradox here: an event that would normally be understood to strengthen the firm's hand actually reduces the share of profits. The reason, once again, is that the lower output reduces the investment quota and the higher wage share is required to induce the necessary fall in the saving rate. There might even be a mechanism for this: investment down, induced "recession," accompanying fall in profit share. But that lies outside this model, as Hahn would be the first to insist. Perhaps if we really believed that aggregate demand is highly sensitive to distributive shares we would find this story less odd-sounding. All this points up the truth and importance of a point Hahn makes very forcefully in his 1972 preface to the published thesis: factor shares may be sociologically salient facts, but that does not make them natural players on the stage of an economic model.

Commentary I

I now want to stand back and make a few general comments on this particular model. (The later Hahn will have his chance in a later section of this paper.)

The first thing to say is that it is genuinely a determinate generalequilibrium model. (But see a qualification below.) It determines w and x, aggregate output and its distribution between wages and profits, as well as the price of goods. It will be remembered that Kaldor's own paper on this subject⁵ is less complete, less "Keynesian" one might say. It reduces, in effect, to equation (1) alone, and adds the bald assertion that there is a unique level of output such that at higher levels prices rise faster than wages and the wage share falls, whereas at lower levels wages fall more slowly (or rise faster) than prices and the wage share rises. Since the demand for goods rises and falls with the wage share, this level of output is an attractor. (Is it the "natural" level of output?) The Hahn model has a theory of output determination, and an imperfectly competitive one at that.

This theory of output rests on a rigid nominal wage. Indeed, if s(w) is assumed to be independent of w and if the perceived elasticity of demand goes to infinity, the model boils down to something very like the 45°-line Keynesian cross supplemented by the usual elementary supply side. The only difference is that rising short-run marginal cost stems from entrepreneurial risk aversion and not from diminishing returns to labor.

It is also a real theory. There is no monetary mechanism at all. None is urgently needed, because the nominal wage is given and because the interest rate is assumed to have a negligible effect on investment. This characteristic extends to the whole thesis, of which I am discussing only a tiny fraction. The spirit of the time holds sway. Notice one peculiarity. The absence of the demand-shift parameter A from (1) and (2) implies that the equilibrium level of output is independent of the position of the typical firm's perceived demand curve. That is not quite the right way to put it, however. Since x = AD(c/w), A is actually determined by the model. It cannot be arbitrary, because it is pinned down by an adding-up condition. If a parameter like A is to be allowed to move the demand curve isoelastically, then it must be allowed to affect something else in the model. One-good general-equilibrium models have to be formulated with some care.

This point is worth elucidating. The model of the household is not derived from explicit utility maximization. If it were, then of course all income would have to be accounted for; what households do not spend, they save. Any parameter, like A, that changes x for given w must also generate an offsetting change in s, given w. We would all get this straight automatically now; indeed, Hahn and I use a similar but more elaborate construction—but not focused on factor shares—in our current joint work. The model of 1951 ignores this point, but the algebra compensates by "determining" A.

As things stand, the investment quota i(x) depends only on the level of output. Thus, any event that leads to a lower equilibrium price π and thus to higher output must generate increased investment regardless of profitability. That suggests making investment an increasing function of A itself; favorable shifts of demand ought to work themselves out in higher investment, higher output, and a higher price. So write I = i(x, A)x. Then a higher value of A shifts the AD curve to the left: at any given x, higher investment will require additional savings and thus a lower wage share. (I am pretending that A can be varied independently, which is not so, to give the flavor of a monetary model in which it could be.) It can be seen from the figure that this amendment to the model produces the desired result. Higher A generates higher x and lower w, therefore higher $\pi = c/w$.

I have emphasized, in praise of this analysis, that it is a complete shortrun macro-model. It determines the level of output and therefore, if implicitly, the level of employment. For instance, we could, as was noted above, treat the wage bill as the only prime cost and assume the labor requirement per unit of output to be the constant n, so that employment is nx and c is simply Wn. All the model has to say about the labor market is that W is constant. There was a time when that assumption would have been replaced by some sort of Phillips curve in real or nominal wages. One would hardly contemplate that amendment without introducing a rudimentary monetary sector. Anyway, nowadays the preferred treatment of the labor market would be different.

One simple way of patching the model, more or less consistent with efficiency-wage theory or any of several insider-outsider models, would be to make W (or $W/\pi = w/n$) an increasing function of employment or just of x itself. Suppose Wn = g(x), for example. Then (2) becomes

$$D(g(x)/w) = r[g(x)(1 + V(xg(x)))].$$
(2')

It is straightforward to check that (2') defines a downward- sloping curve, as in (2), only steeper. Along the AS curve, an increment to x will correspond to a higher MC because the nominal wage, as well as the normal risk premium, will be higher. Thus, the increment to x will generate a larger increment to π than it would have if the wage were rigid, and thus a larger fall in w. This extra lift to aggregate demand makes the model less stable. Otherwise the structure of the model is much as it was, only a bit more contemporary.

With the few changes I have suggested, we have a working model with reasonable properties. Its distinguishing feature is that it accommodates fluctuations of investment, as Hahn says, by "explicitly postulat[ing] that the burden of adjustment will fall on the share of wages and not the rate of interest." The later Hahn might not wish to do just that, but he can look back and say that if it were to be done, this was an intelligent way to do it.

Second Thoughts

The whole thesis was published,⁶ essentially unchanged, in 1972—more than 20 years after it was written—with a new preface by the middle-period Hahn. What does he think of his handiwork?

It is striking that he pays no attention to the details of his thesis. There is much more of it than I have mentioned here, much of it still fresh and interesting. But Hahn gives the impression that he cannot be bothered. Not that he dislikes it. To the contrary, although there are some things he would wish to do differently, on the whole he approves of the earlier work. This is not surprising, partly because it is a good thesis and partly because star economic theorists usually like what they see when they read what they wrote. Self-doubt is not much to be found among the successful.

The preface is really about something altogether different. In the intervening years, some of Hahn's most eminent colleagues at Cambridge had come to regard social-class differences in savings propensities as a central stylized fact and to believe that they could somehow use it as a stick with which to beat "neo-classical" economic theory. The form in which the beating was to be administered was the development of a theory of interest or profit that was entirely independent of technology. (The motive for this was presumably the belief that the linking of capitalists' income with the marginal productivity of something or other could be taken as the major step in an apologia for the distribution of income in a capitalist economy. Hahn is quite properly scornful of every link in this argument.) So here is the author of what may be the first decent theory of distribution built explicitly on saving-rate differences, who happens also to be (a) an eminent but never naive neo-classical economist, within the meaning of the act, (b) a person who neither admires the distribution of income in developed capitalist economies nor believes that anything in neo-classical economics provides the slightest ethical justification for it, and (c) a professor at Cambridge. He knows, indeed he has shown by example, that there is no conflict between paying attention to differences in savings propensities and paying attention to productivity considerations.

The preface dismisses this claptrap (Hahn's word) forthrightly. Hahn points out that rigorous theory does not usually classify agents according to social class or make much of the distinction if it does. This may be a serious deficiency in practice, if social class matters for supply or demand, but it has nothing to do with the logic of economic theory. He then describes succinctly what it means to find an equilibrium vector of input prices and quantities, emphasizing the ahistorical character of the construction. The only problem for a reader today is to imagine that this once needed to be said to famous economists in one of the world's centers of learning.

About his youthful effort, Hahn insists that his main concern in 1950 had been with disequilibrium anyway. There is truth in that. The bulk of the thesis, after the first few chapters, is given over to sequence analysis. But I leave it aside because I have wanted to emphasize the basic macromodel, and that is a piece of temporary equilibrium theory. The middleperiod Hahn wishes that he had been less aggregative: "... there is too much in the book which relies on the 'representative firm' and occasionally on aggregate production functions. I now believe this to be a vital flaw. It may well be that it is an illusion to hold the view that there are any macroeconomic shortcuts." Carried to its logical end, this would probably entail abandoning the very idea of a theory of distributive shares. Building that quantity up from its constituents is not a serious practical pursuit for the theorist. Assumptions that would make it so would probably also make rigorous macroeconomics possible. Absent those assumptions, Hahn seems to hold out only the prospect of an austere theory that does not speak directly to the national income and product accounts. I find no fault with his analysis; but I hold to a different aesthetic.

Commentary II

I have left that last sentence standing so that I can now point out that it is a dodge. The doctrine that there is no point arguing about tastes provides a temptation to classify differences of opinion as aesthetic because one is relieved of the necessity to defend one's own views seriously. (I once said to Ronald Dworkin that I found gross income differences "unaesthetic." He asked if I didn't mean "immoral." I gulped.) The truth is that I resist the 1972 Hahn's rejection of loosely aggregative economics, and I would defend the approach of the younger Hahn even against his maturer self.

The later Hahn says: "On purely theoretical grounds there is nothing to be said [in favor of an aggregative version of neo-classical models]. The view that nonetheless it 'may work in practice' sounds a little bogus and in any case the onus of proof is on those who maintain this. I am glad to see that I did not believe this argument in 1951 and sorry to see that I nonetheless every so often slipped into the aggregative version of the neo-classical model."

That is certainly a legitimate stance. I do not think it is the only legitimate stance. I try out the following alternative on my beloved and admired friend. A model is a logically consistent representation of a possible economy (or part of an economy). Apart from purely technical amusement, which is not to be sneezed at, a model becomes interesting if it can defensibly be said to be an adequate representation of some real economy. "Adequate" does not mean perfect or nearly perfect; standards of adequacy are discussable, and a model may be adequate for some purposes and not for others. This means, I suspect, that it is not at all "bogus" to say of a model that it works pretty well in practice even if in some or many ways it is transparently "unrealistic" or theoretically lacking. It is a risky defense because its use suggests that one does not have any good idea of the circumstances in which it works, or whether any particular test of its working is stacked in the model's favor. But that is not bogus.

There are two ways in which it can be checked if a model "works reasonably well." One is to see if the elementary assumptions of the model correspond approximately to the facts. They may still be combined in fruitless ways. (That is, some nonelementary assumptions—e.g., "all markets clear" or "there is excess supply of labor"—may be inappropriate.) But it is surely better that the building blocks be realistic than not. The other and more interesting way is to take seriously the point long emphasized by Paul Samuelson. The main empirical implication of a model is its comparativestatic properties—in this case, the inequalities (3a)–(3f). That is the sort of "prediction" a laboratory science finds it easier to check than we ever can. The model "works reasonably well" to the extent that those inequalities seem to hold in data, to the extent that the relative size of the comparativestatic derivatives matches up with what we can guess about the basic parameters, and to the extent that the basic parameters enter in an empirically plausible way.

We now know, as the young Hahn did not, how to construct a representative-firm model of imperfect competition as a "true" aggregation of an economy with n imperfectly but symmetrically substitutable goods. Hahn and I have used that device in our joint work. I suppose that lends a little additional interest to the model of chapter 3 of the 1951 thesis. But only a little, I would say. In my view it would be much more compelling to be able to produce evidence that all or most of the comparative-static properties of (3) hold for five- or ten-year averages of the modern British economy.

Of course there may be other inoffensive but different models that iead to the qualitative results of (3). Then reasonable people can continue the argument. They can find places where the alternative models have different implications and consider the weight of the evidence. They can debate the factual plausibility of assumptions. They can, in short, do all the things that Frank Hahn does superbly well in addition to the one thing he likes to pretend to emphasize.

Notes

^{1.} Hicks' famous book was published by Macmillan. The article appeared in Review of Economic Studies 4 (1936): 1-12, and was republished with the second edition of Theory of

Wages in 1963. Articles by Friedman, Hicks, Kahn, Lerner, Machlup, Meade, Robinson, Sweezy, and Tarshis appeared in *Review of Economic Studies* in the years 1933–1936.

2. Kalecki's article "The Determinants of Distribution of the National Income" appeared in *Econometrica* 6 (1938): 97–112, and was reprinted in *Essays in the Theory of Economic Fluctu*ations (Allen & Unwin, 1939).

3. The origin is A. K. Dixit and J. E. Stiglitz, "Monopolistic Competition and Optimum Product Diversity," American Economic Review 67 (1977): 297-308.

4. See "The Principle of Increasing Risk," *Economica* N.S. 4 (1937): 440–447, reprinted in the volume cited in note 2.

5. "Alternative Theories of Distribution," Review of Economic Studies 23 (1955-56).

6. The Share of Wages in the National Income (Weidenfeld and Nicolson, 1972).