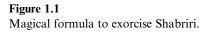
## שברירי ברירי רירי ירי ירי



hostile and sometimes favourable to him and in this fact he sees a manifestation of the anger of the spirits—or of their good disposition. One must, then, treat the spirits as one would treat men whom one *cannot dispense with*" (Bouisson 1960, 95; emphasis mine). Because speech and music were known to influence the actions of other people, sometimes almost miraculously, many people felt there was a strong basis for believing that they would influence the "spirits," the forces that generated the natural phenomena that ruled their lives.

An example of an incantation is the formula (fig. 1.1) from the Babylonian Talmud used to exorcise the demon Shabriri, who was said to cause blindness. The magical formula, repeated before drinking at night, consists of the demon's name (written here in Hebrew letters on the top line of the figure), pronounced six times, with ever fewer of its syllables (Shabriri, Briri, Riri, Iri, Ri, I). The demon is supposed to waste away as the syllables of his name are removed and thus not be able to do his evil work of blinding unsuspecting drinkers from the water jar.

As illustrated by the Shabriri formula, incantational magic involves formulas of rhythms, intonations, and words performed *in the correct order*. Of course order is important even in ordinary speech. Indeed, it is a truism that word order can determine meaning, for example, "sharkeating man" versus "man-eating shark." And intonation can also affect meaning. For example, rising pitch toward the end of an English sentence indicates a question; falling pitch indicates a statement of fact. Listen to yourself pronounce the following sentences: first, "The eggs are done?" and then, "The eggs are done." In poetry, the effect of word order, rhythm, and intonation is even more striking. Read the following poem, by Robert Bringhurst (1995, 100) out loud:

II Parable of the Harps

In the drum of the heart Are the hoofbeats of horses—the horse Of the muscles, the horse of the bones.

In the flutes of the bones are the voices Of fishes—the fish of the belly, The fish of the fingers and limbs.

In the streams of the limbs We are swimming with fishes And fording with lathering horses.

Love, in this bed full of horses And fishes, I bring to the resonant gourds Of your breasts the harps of my hands.

You can almost feel the horses thundering in your heart, or feel yourself in the river, swimming fluidly with the fishes. But try reading aloud the following permutation of the poem:

the swimming In bones. Are the Love, of the horses—the horse the muscles, the horse Of the breasts

heart In the bones I bring of the fish Of fishes—the lathering of belly, fording The resonant gourds. flutes of the

harps In the drum of fishes limbs We are And fishes, are the streams with limbs And fish with horses.

in this of the bed of fingers to the horses and of the voices Of your hoofbeats the full of my hands.

Not only has the meaning of the poem been destroyed, but also the meaning of its rhythm and intonation. Indeed, it is no longer a poem, only a word salad. A good poem can bring tears to the eyes. Word salad just confuses, and can even signal mental illness. For example, people with untreated schizophrenia often utter strings of words that resemble the "tossed" version of Bringhurst's poem.

## 1.2 Dynamics

The unfolding of behavior in time, especially that of human speech, has been a dominant theme in our understanding and subjugation of nature, as reflected by the dominant role played by dynamics in science, especially in physics, from Aristotle to Galileo to Kepler to Newton to Einstein and into the modern era. Quantum electrodynamics (QED), the foundation stone of modern physics, is essentially dynamical; indeed, according to Feynman (1985), it consists of only "three little *actions*" (emphasis mine): electrons and photons move in space-time and electrons scatter photons.

Given the historical importance of magic, music, and rhythm, the preeminent role of dynamics in physics, and the emphasis on change over time in the other sciences (e.g., reactions in chemistry and evolution and development in biology), it is somewhat surprising that the dominant approaches to cognitive science, and to psychology as a whole, are statical rather than dynamical. That is, empirical laws and theoretical statements, when expressed formally, are written as

$$B = f(x_1, x_2, x_3, \dots, x_n), \tag{1.1}$$

where time is not a relevant variable, rather than as

$$B = f(x_1, x_2, x_3, \dots, x_n, t), \tag{1.2}$$

where time enters as an important variable.

A good example of a statical law in psychology is the well-known psychophysical power law, sometimes called "Stevens's law," expressed in the following equation:

$$R = cS^m,\tag{1.3}$$

which describes reasonably well how responses (R) in a psychophysical scaling experiment vary with the intensity of the stimulus (S). For example, we might ask a subject to give a number (a "magnitude estimation") that indicates how loud a sound seems each time it occurs. When we play several sounds, each of a different intensity, several times each, and plot our subjects' average responses on a graph (see fig. 1.2), the results closely fit the curve of equation 1.3, which we can then use to calculate a value of R close to the one our subjects would give for any sound, even those to which they did not give a number. Because different people give similar numbers to the same sounds (Stevens 1975), this procedure is used routinely to predict people's responses to sounds when designing music halls, airports, and hearing aids.

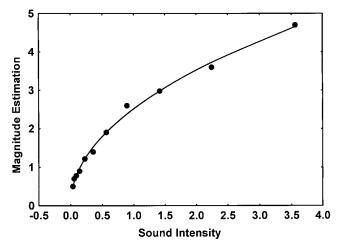


Figure 1.2 Stevens's law (curve) summarizes psychophysical scaling data (dots).

As useful as Stevens's law is, it does not describe everything of interest about subjects' responses in a psychophysical scaling experiment. Subjects do not always give the same numbers when presented with the "same" physical stimulus (the error bars were omitted in fig. 1.2). Of course, as the quotation marks around "same" indicate, all stimuli differ, no matter how hard we try to get them to be the same. Air currents, voltage fluctuations in a sound generator, increasing fatigue of loudspeaker cones, rising or falling temperature and air pressure, and so forth together produce trial-to-trial fluctuations in sound pressure at the eardrum even for the same nominal sound stimulus. And subjects change, too. Blood pressure rises and falls, attention wavers, digestion proceeds, viruses and bacteria multiply and die, and so forth. It would be a miracle if subjects and stimuli were exactly the same on any two occasions. In fact, subjects often give dramatically different responses to the "same" stimulus at different times, and often give the same response to dramatically different stimuli presented on different occasions. A psychophysical scaling experiment is necessarily extended in time, and the behavior involved in making psychophysical judgments fluctuates over time, even when the experimenter strives to make the situation identical from moment to moment.

How should we deal with these fluctuations? In psychophysics, one tradition, attributed to S. S. Stevens (1975), has been to ignore them. The justification is that we are not usually interested in the fluctuations in subjects' blood pressure, digestion, sleepiness, and so forth over the

course of a psychophysical scaling experiment. Nor are we interested in the unavoidable differences in the stimulus magnitude from trial to trial. These effects reflect our incomplete control over the experimental situation and are nuisances. We therefore agree simply to call these fluctuations "error variance," to be "averaged away" (as in fig. 1.2). A different tradition, beginning with Fechner (1860) and continuing through Thurstone (1927) and Green and Swets (1966), is to use the amount (but not the timing) of the fluctuations to measure sensation and sensory discrimination. Thus Fechner built his famous law of sensation on what we now call "Weber's law," that the difference threshold (the smallest intensity difference that can be reliably detected) is proportional to the stimulus intensity at which it is measured,  $\Delta I = kI$ , where  $\Delta I$  is the difference threshold and I the stimulus intensity. This proportionality arises because response variability generally increases with stimulus intensity. In signal detection theory (and related approaches), an important measure of stimulus discriminability, d', is defined as the difference between the means of two assumed probability distributions of sensory effects of stimuli divided by their common standard deviation, the latter representing the amount of fluctuation in those sensory effects. Just how those fluctuations are distributed over time is irrelevant (but see Link 1994). Thus, whether a nuisance or a fundamental concept, psychophysical variability is usually treated as "error" rather than as temporally distributed information about cognitive processes. Both traditions ignore time, even though fluctuations over time can provide fundamental information about the processes generating the behavior in question. This point applies not only to psychophysics, but also to most of the work being done in cognitive science today.

It is undeniable that in some cases the temporal distribution of response variability adds nothing to our understanding of the phenomenon we are studying. On the other hand, however uninteresting the temporal distribution of these fluctuations might seem to be, it takes on new meaning when juxtaposed with several significant observations from physics and biology and from physical methods applied to human behaviors. First, such temporally distributed fluctuations are ubiquitous; indeed, in quantum mechanics and statistical physics, they are fundamental (see chap. 13). Second, these random processes can actually "drive" physical and biological phenomena by falling into one or the other of several "basins of attraction," depending on random fluctuations early in the process. They can determine, for example, which of two equally rich food sources will be exploited by an ant colony (e.g., Beckers et al. 1990). Random

choices made by the first ants to reach a choice point bias the probabilities of choices made by ants reaching the choice point at a later time (via the pheromones they deposit), leading to a strong "preference" for one food source over the other. Importantly, random choice fluctuations of the same size occurring at a later time in the process, once a preference has developed, have no effect on which source is preferred. In this and in many other cases, the *timing* of the fluctuations is all important. Third, the random fluctuations can convey valuable information when observed in human behavior. For example, fluctuations that occur with a certain type of periodicity, called "1/f" or "pink" noise (see chap. 15), are diagnostic of stochastic processes that interact at several scales of time; 1/fnoise characterizes the residual response variability in standard experimental tasks of cognitive science (see chap. 16). Finally, even deterministic processes can create fluctuations that appear random (Chan and Tong 2001). For example, the logistic difference equation

$$Y_i = a Y_{i-1} (1 - Y_{i-1}), (1.4)$$

although simple and completely deterministic, demonstrates extremely complicated, effectively random behavior for values of a > 3.58. Only dynamical analysis of time series of behaviors can reveal the differences between such deterministic processes and processes characterized by white noise.

In making these same points, other authors (e.g., Gregson 1983, 1988; Port and van Gelder 1995; Kelso 1995) have also argued that dynamics should be central to cognitive science and to psychology as a whole. Nevertheless, even though more and more cognitive scientists are using dynamical metaphors, and even though some are using dynamical system theory to analyze behavioral experiments, the practice of dynamics is still not widespread. This is so partly because dynamics is complicated and difficult, requiring considerable sophistication in mathematics and other disciplines with a large technical content, and partly because some practitioners of the dynamical approach (e.g., Gregson 1983, 1988) have set it against the more traditional approaches, arguing for a Kuhnian paradigm shift in psychology. Unfortunately for the latter, not all statical theories of psychology can be subsumed as special cases under dynamical theories (see chaps. 8 and 9). Finally, psychology's reluctance to embrace dynamics lies partly in our having few well-worked-out examples of the benefits that accrue when the dynamical approach is taken (see, for example, Kelso 1995). In a discipline such as economics, where prediction of the next values of a time series (e.g., stock market prices) has obvious benefits,

dynamics is understandably central. In psychology, however, the benefits of dynamical analysis are not so clear.

Sometimes a dynamical analysis simply generalizes a statical theory to other situations and data, particularly where values fluctuate in time, as in the psychophysical case (see chap. 11). Sometimes, however, the dynamical analysis provides a fundamentally different theoretical approach to an old problem, with novel predictions following from the dynamical theory. Beer (2000) describes one such dramatic case in the "A-not-B" error in infant reaching: infants 7 to 12 months old continue to reach for an object in the one of two opaque containers they have been trained to reach for, even after they have seen the object being hidden in the other one. Whereas Piaget argued that the error is the result of applying an immature concept of object permanence to the task, a dynamical analysis suggests that it is caused by an immature goal-directed reaching system (Thelen et al. in press). The dynamical model also accounts for the dramatic context effects that have been observed in this task. Moreover, it makes the novel prediction that, under the right conditions, the error should be observed in older children as well because it arises from general properties of the perceptual-cognitive-motor system controlling goal-directed reaching. Thus, if confirmed, the dynamical theory would dramatically change our understanding of infants' performance in goaldirected reaching, and perhaps even our assessment of concepts such as object permanence in child development.

In light of the possible benefits, this book aims to overcome the considerable obstacles to the centrality of dynamical analysis in cognitive science. It aims to describe some tools of dynamical analysis simply and clearly, so that even mathematically unsophisticated researchers can see how they are used. It aims to show how dynamical and statical approaches are complementary and mutually informative. And finally, it aims to provide examples of the increases in understanding that accrue when cognitive science is informed by both dynamical and statical analyses. Chapter 2 Sequence

The various sections of the Torah were not given in their correct order. For if they had ... anyone who read them would be able to wake the dead and perform miracles. For this reason the correct order of the Torah [is] known only to the Holy One....

-Rabbi Eleazar ben Pedath, quoted in Gershom Gerhard Scholem, On the Kabbalah and Its Symbolism

The mind which any human science can describe can never be an adequate representation of the mind which can make that science. And the process of correcting that inadequacy must follow the serial steps of an infinite regress.

-J. W. Dunne, The Serial Universe

Just about all of the interesting hypotheses we have about how social systems function imply at their base an imagined scenario of interaction, a scenario invariably sequential in character.

-John M. Gottman and A. K. Roy, Sequential Analysis

## 2.1 The Serial Universe

It is often best to begin at the beginning. In the case of sequence, the beginning has to do with the nature of the universe itself. In his charming little book, from which the second epigraph above is taken, Dunne (1934, 28) argued that the universe "*as it appears to human science* must needs be an infinite regress" (emphasis his). In this context, a "regress" is a question that can be answered only by asking another, similar question, which of course leads to asking still another, similar question, and so forth, to infinity. For example, a child learning arithmetic might attempt to answer the question "What is the largest integer?" The child first considers whether there is a larger number than "1" and discovers that "2" is larger than "1," which leads to another question, whether there is a number larger than "2," and so forth. Because, "1" can always be added to any