CHAPTER 1

The Basic Electrical Quantities and Their Measurement

1.1 ELECTRICAL FORCES; COULOMB'S LAW

Electrical and magnetic phenomena have been observed in one form or another for thousands of years. Until the end of the eighteenth century, however, these observations were largely qualitative and unrelated. Such phenomena as the orientation of the compass needle or the electrical displays during thunderstorms were merely curious and strange aspects of nature. With the advent of quantitative measurements, an ever-deepening insight into the nature of the structure of matter was obtained. The universal attribute, mass, had to be expanded to include the notion of charges out of which atoms are made. Finally, during the twentieth century, the detailed study of charged particles led to the formulation of quantum mechanics and a satisfactory description of atomic structure. The first step in this evolution was an adequate description of the two forces at a distance which the charged components of matter can exert on each other. The simpler of these is the electrostatic force measured and analyzed by Charles A. Coulomb in 1785.

One force at a distance, namely, gravity, is described by Newton's famous law

$$F \propto \frac{mm'}{r^2}$$

illustrated in Fig. 1.1*a*. Electrical forces follow a similar law, called *Coulomb's law*

$$F \propto \frac{qq'}{r^2} \tag{1.1}$$

illustrated in Fig. 1.1*b*. Here q and q' represent quantities of electrical charge, just as m and m' in Newton's law represent quantities of mass.

Charges may be produced in a variety of ways, the oldest of which is simply by friction. Rubbing a glass rod with a piece of silk, or a hard-rubber rod with a piece of fur, for example, will leave the rubbed surfaces charged. These charges may then be transferred by contact to other light bodies to demonstrate electrostatic forces.

Although Newton's and Coulomb's equations are so similar, the physical manifestations of gravitational and electrical forces are very different. There are two reasons for this difference. The first is the

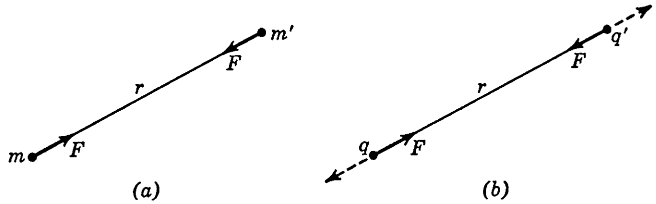

FIG. 1.1. (*a*) The gravitational force F acting between two masses, m and m'. (*b*) The electrical force F acting between two charges, q and q'. The electrical force may be attractive or repulsive, but it always acts along the line joining the two charges.

magnitude of the forces that can be produced in the laboratory. An illustration of the differences encountered may be taken from everyday experience. The weight of a human hair is a small but measurable force. It is a gravitational force exerted on the hair by an adjacent object, namely, the earth. The gravitational force exerted on a hair by a nearby comb can readily be computed. It is negligible compared to the weight of the hair, yet a comb can, under certain circumstances, exert a different force at a distance which is many times the weight of the hair. It can lift the hair up against the gravitational force and make it stand on end. This effect is due to an electrical force. Friction can separate electric charges, leaving some on the hair and some on the comb, and it is these separated charges that attract each other and produce the observed effects.

The second important difference between gravitational and electric forces is that, whereas the former are invariably attractive, the latter may be either attractive or repulsive, as is illustrated in Fig. 1.2. There is only one kind of mass, and all masses attract each other. There are, however, two kinds of electric charge; positive and negative. Positive charges repel each other, as do negative charges, but positive charges attract negative charges. It is a consequence of these facts that a given quantity of positive charge can be canceled by an equal quantity of negative charge. Such a cancelation is possible electrically, but not gravitationally. In the hair and comb experiment, for instance, the hairs stand up separately. They repel each other because they have attached to them charges of like sign. They are all attracted to the comb because it has attached to it charges of the opposite sign. The material bodies involved, namely, hair and comb, seem to exert forces on each other because of the electric charges that are attached to them. The electric charges are not permanently and irrevocably attached to

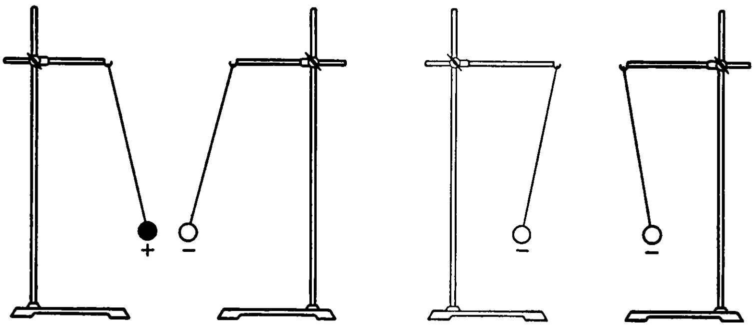

FIG. 1.2. (*a*) The attractive force between oppositely charged spheres. (*b*) The repulsive force between similarly charged spheres.

the hair and comb. The electric forces of attraction will now and again pry some of the charge loose, and this may flow through the air or along the arm toward the charge having the opposite sign, and so produce a partial cancelation.

The quantitative application of Eq. 1.1 requires the specification of units. In general, Coulomb's law may be written

$$F = k\frac{qq'}{r^2} \tag{1.2}$$

where the magnitude of the constant k depends on the units used. Perhaps the simplest procedure is that adopted in the *centimeter-gram-second (cgs) electrostatic system* of units. The constant $k = 1$, so that Coulomb's law becomes

$$F = \frac{qq'}{r^2} \tag{1.3}$$

Here q is in electrostatic units (esu), r is in centimeters (cm), and F is in dynes. The esu of charge is called the *statcoulomb.* It is defined as that quantity of charge which, placed 1 cm away from a like charge, will experience a force of 1 dyne. This system is very convenient when we are concerned with purely electrostatic interactions, but it has certain disadvantages when we come to consider moving charges and magnetic effects.

The system of units used in this book is the *rationalized meter-kilogram-second (mks) system.* It was adopted in 1935 by the International Electrotechnical Commission and is gradually replacing other systems in scientific and technical work. The unit of charge is the *coulomb.* It is defined in terms of electrical currents in a way which we shall take up further on. The magnitude of the coulomb is such that the proportionality constant k in Coulomb's law has the value

$$k = 8.98776 \times 10^9 \frac{\text{newton-meter}^2}{\text{coulomb}^2}$$

if the charges are in a vacuum. We shall in general round this off to

$$k = 9 \times 10^9 \frac{\text{n-m}^2}{\text{coulomb}^2} \qquad (1.4)$$

Coulomb's law is usually not written in terms of this proportionality constant k but of a related constant, called the *permittivity,* and defined as follows:

$$\epsilon_0 = \frac{1}{4\pi k} \approx 8.85 \times 10^{-12} \frac{\text{coulomb}^2}{\text{n-m}^2} \qquad \text{(approx.)} \qquad (1.5)$$

The form of Coulomb's law that we finally adopt is then

$$F = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2} \text{ newtons} \qquad ▶(1.6)◀$$

In this equation ϵ_0 has the value* of Eq. 1.5, the charges q and q' are expressed in coulombs, and the distance r between them in meters.

Coulomb's law specifies the magnitude of either one of two equal and opposite forces, namely, that which the charge q exerts on the charge q', or the equal but oppositely directed force which the charge q' exerts on the charge q. If the two charges under consideration have the same sign, the forces are repulsive. If they have opposite signs, the forces which they exert are attractive.

* When the factor 4π is written out explicitly in the definition of ϵ_0, as has been done here, one obtains the so-called rationalized mks system. Some writers define ϵ_0 by the equation $k = 1/\epsilon_0$. This definition leads to the so-called unrationalized mks system. One must be careful when reading works on electricity to determine whether rationalized or unrationalized units are being used. We shall use only the former in this book.

In the form shown in Eq. 1.6, Coulomb's law describes the force between charges in a vacuum. It is found experimentally that, if charges are embedded in non-conducting substances such as oil or glass, for example, Coulomb's law must be modified. The permittivity will have a value differing from that specified in Eq. 1.5. The first two chapters will be confined, however, to the case of charges in a vacuum. The permittivity of air is so nearly equal to that of a vacuum that Eq. 1.6 will also be applied to charges in air.

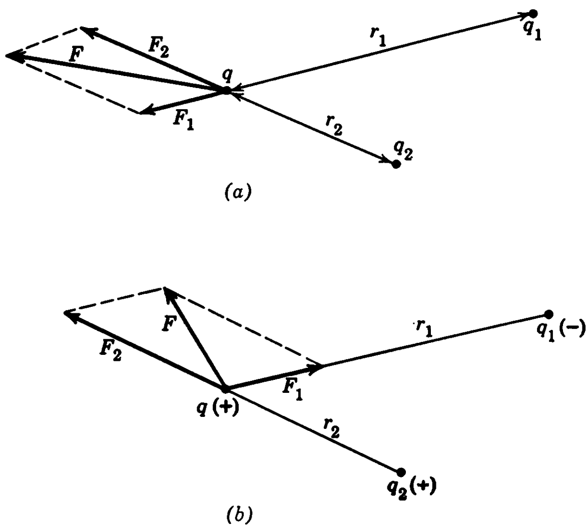

FIG. 1.3. (*a*) The resultant force F exerted on a charge q by charges q_1 and q_2 having the same sign as q. (*b*) The resultant force F exerted on a positive charge q by a negative charge q_1 and a positive charge q_2.

If several charges are present, the resultant force exerted on some one charge may be found through adding the forces exerted by the separate charges vectorially. This is illustrated in Fig. 1.3. Thus, if we are concerned with the force on a charge q in the presence of a charge q_1 of like sign at a distance r_1 from q, and another charge q_2 of like sign at a distance r_2 from q, we have the situation shown in Fig. 1.3*a*.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{\overrightarrow{qq_1}}{{r_1}^2} + \frac{\overrightarrow{qq_2}}{{r_2}^2}\right) \qquad (1.7)$$

This procedure may readily be extended to any number of charges. It must be remembered, however, that unlike charges attract each other. Hence if we modify the situation illustrated in Fig. 1.3*a* by reversing the sign of q_1 so that we might suppose q and q_2 to be positive, but q_1 negative, we should have the situation shown in Fig. 1.3*b*.

The force exerted by n charges $q_1, q_2, q_3, q_4, \cdots, q_s, \cdots, q_n$ on a charge q may be written

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0} \sum_{s=1}^{n} \frac{\overrightarrow{qq_s}}{r_s^2} \tag{1.8}$$

where the summation is to be a vector sum for all the possible values of s from 1 to n.

As will become apparent from quantitative applications of Coulomb's law, the coulomb is an extremely large quantity of charge seldom physically realized in the laboratory. One more often deals with microcoulombs (μc) or micromicrocoulombs ($\mu\mu$c). The use of such prefixes as scale factors is described below.

Note about Scale Factors

The following prefixes have these meanings:

Prefix	Abbreviation	Meaning
Mega or meg	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
micromicro	$\mu\mu$	10^{-12}

Examples: Megavolt, megohm, megacycle; kilovolt, kilometer, kilowatt; millivolt, milligram, milliampere; microampere, microcoulomb, micromicrocoulomb.

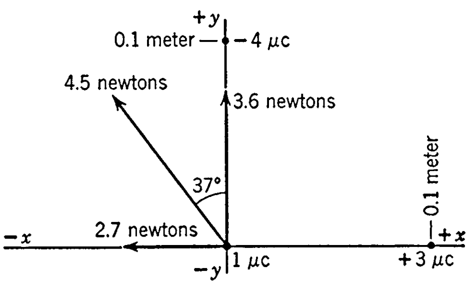

Fig. 1.4.

An Example of the Application of Coulomb's Law

The application of Eq. 1.7 is illustrated in the following example. A charge of 1 μc is located at the origin of a rectangular coordinate system as shown in Fig. 1.4. A charge of 3 μc is located at a point 0.1 meter from the origin on the positive x axis, and a charge of -4 μc 0.1 meter from the origin on the positive y axis. What is the resultant electrical force on the charge at the origin?

Let us consider first the force produced by the 3-μc charge on the x axis. The force on the charge at the origin due to this charge is repulsive, since both have the same sign, and in the direction of the x axis, since both charges are on this axis. We have for the magnitude of the force due to the first charge

$$F_1 = 9 \times 10^9 \frac{(1 \times 10^{-6})(3 \times 10^{-6})}{(0.1)^2} = 2.7 \text{ newtons}$$

Similarly, the force due to the second charge is attractive, and in the direction of the positive y axis. Its magnitude is

$$F_2 = 9 \times 10^9 \frac{(1 \times 10^{-6})(4 \times 10^{-6})}{(0.1)^2} = 3.6 \text{ newtons}$$

The resultant of these two mutually perpendicular forces is

$$F = \sqrt{F_1^2 + F_2^2} = 0.9\sqrt{3^2 + 4^2} = 4.5 \text{ newtons}$$

The angle which the resultant makes with the positive y axis is θ, where $\tan \theta = \frac{3}{4}$, or approximately 37°.

1.2 INSULATORS AND CONDUCTORS

Matter is composed of *atoms*, having diameters of the order of 10^{-10} meter. At the center of an atom is a very small positively charged *nucleus*. Nuclear diameters range from 10^{-15} to 10^{-14} meter. Around atomic nuclei there is a cloud of *electrons*. In electrically neutral atoms the negative charge of the electrons just balances the positive charge of the nucleus.

Small clusters of atoms are called *molecules*. Thus oxygen gas is made up of pairs of oxygen atoms held together by chemical forces, vibrating and rotating about their centers of mass. Hydrogen, nitrogen, and oxygen are examples of gases, under normal conditions, made up of diatomic molecules, designated H_2, N_2, and O_2 respectively. Water molecules consist of three atoms, two hydrogen and one oxygen, designated H_2O. Ammonia contains four atoms, NH_3, and so on.

Atoms and molecules need not be electrically neutral. If for some reason there is an excess of charge of one sign, the particle is called an *ion*. Ions in a gas or liquid will move in response to electrical forces. The details of the motion will depend on the applied force and on the collisions between the ions and their neighbors.

Matter may be classified electrically into two categories, according to whether electrical forces will produce moving charges or not. Ionized gases or liquids are examples of electrical conductors, because electrical forces acting on the charged particles will produce motion, as illustrated in Fig. 1.5. Gases and most liquids containing only neutral atoms are insulators.

In solids, atoms are so closely packed together that it is difficult for two atoms to change places. Although such exchanges do occur, they are so rare that for the purposes of the present discussion we shall suppose that ions in a solid do not move in response to electrical forces. The mechanism of electrical conduction in solids, hence, is different from that in gases or liquids. Metals are electrical conductors, and they conduct because they contain free electrons. In an insulator, each atom may be thought of as holding its electrons in a cloud around its nucleus. In a metal, in addition to these bound electrons, there are some free electrons that can move about. If a metallic wire is connected into an electrical circuit, the free electrons can flow out at one terminal and are replaced by other free electrons entering the wire at

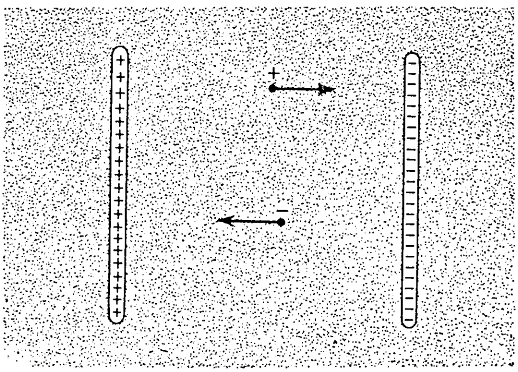

FIG. 1.5. The forces acting on positive and negative ions in the presence of charged bodies. An ionized gas is an example of a conducting medium.

the other terminal. If an isolated piece of metal is given an excess charge, for example by having a charged object rubbed against it, the excess charge is free to move around on or in the metal. Excess charge on an insulator, such as a glass plate, for example, will tend to remain where it is placed. It is bound and not free to move.

1.3 ELECTRICAL ENERGY; THE VOLT

In the first section we introduced the concepts of a quantity of charge, measured in terms of statcoulombs or coulombs, and of the forces which charges exert on each other. In this section we shall take up the volt. The definition of the volt follows quite simply when we consider electrical phenomena from the point of view of the energies involved rather than from the point of view of the forces which charges exert on each other. The one follows from the other. In teaching mechanics, the usual procedure is to start with a discussion of forces and first express a relationship between force and motion in the form $F = ma$. Subsequently it is shown that for many problems a much more useful formulation involves not forces but energy and "quantity of motion," or momentum. Without concerning ourselves with the detail involved in the description of varying forces, we can compute many important characteristics of the motion of objects by applying the principles of the conservation of energy and momentum. For example, it is possible to compute the maximum altitude reached by a ball thrown upward with a given initial velocity, or the changes in velocity produced in an elastic collision between two particles, without having to solve for the motion in detail. We shall find, similarly, that considerations involving electrical energy will enable us to compute much about electrical phenomena that would be difficult if we were required to explore the electrical forces in full detail.

In electrical problems the electrostatic potential energy is an important quantity. Let us explore the space surrounding the charged terminals of a current generator or battery with a very small charge. This small charge, or "test body" as we shall call it, is assumed to have a charge sufficiently small so that its presence will not alter the position of other charges as it is moved about. The test body will experience a force at any given point in space. The magnitude and direction of this force can be computed from Coulomb's law. If we move the test body from point to point, its potential energy will change. If we move the test body in a direction opposite to that in which the electrical forces act, we must do work, and this work goes to increase the potential energy of the test body. The mechanical analog would consist of lifting a mass against the downward pull of gravity. In lifting the mass we move it in a direction opposite to the downward pull, and the work we do in lifting it is precisely equal to the increase in its potential energy. The electrical forces will in general vary in magnitude and direction from point to point, so that we cannot write a simple expression like mgh for the potential energy, as we could for gravitational forces in the laboratory, which were uniform in magnitude and direction. But in spite of this complication, we may assign a given potential energy to our test body at every point in space (except for an arbitrary constant specifying the point at which we shall designate the potential energy to be zero). The change in potential energy in going from some point A to some point B will be the work required to move the test body from A to B.

This potential energy concept is particularly useful when we come to consider electrical circuits. We shall be considering charges flowing through various electrically interconnected circuit elements from one terminal of a battery to another. We shall find that electrical forces must be present in order to make the charges move, but that it is almost never necessary to resort to a detailed description of the forces. Instead, we consider the potential energy of a small quantity of charge at various points in the circuit and from this deduce the resultant flow of charge. It is most fortunate for our study that this potential energy of a quantity of charge is, first of all, so revealing and, second, so easy to measure.

Let us again return to a consideration of mechanical problems. Suppose we plan to dig a deep ditch from one end of Kansas to the other, connecting two water reservoirs, and we wish to decide in advance how much water is going to flow along the ditch and in which direction. It would, of course, be necessary to know something of the size and construction of the proposed ditch, but the most important point of all, the point which determines the direction of the flow and for a canal of any given size also the rate of flow, would be the difference in the water level of the two reservoirs. Stated more generally, the direction of flow and the rate of flow will be determined by the difference in the gravitational potential energy mgh of a test body m, or the difference in the gravitational potential gh, at the two reservoirs.

Similarly, if we have two points in a circuit, A and B, and we wish to predict how much current will flow through a wire connected to A and B, we must, first of all, know the difference in potential energy of a coulomb of charge at A and at B. If the potential energy at A is higher than at B, then positive charge will spontaneously move "downhill," as it were, from A to B. The potential energy per coulomb is the important quantity. It is called the *electrostatic potential.* Its dimensions are joules/coulomb in the mks system of units, and 1 *joule/coulomb is called a volt.*

Potential, like potential energy, has an arbitrary absolute magnitude, but differences in potential energy are unambiguous and it is differences in potential that are significant. Thus, if V_A and V_B are the potential energies of 1 coulomb at points A and B, then the difference in potential between A and B, usually written V_{AB}, is just $V_A - V_B$. The work $W_{B\to A}$ required to carry a charge q from B to A is the potential energy of the charge q at A, or qV_A minus the potential energy of the charge q at B, or qV_B. We have, therefore

$$W_{B\to A} = qV_A - qV_B = qV_{AB}$$

or

$$V_{AB} = \frac{W_{B\to A}}{q} \text{ volts} \qquad (1.9)$$

In general, there is a potential difference between charged objects and the difference in potential is equal to the work that must be done by external mechanical or other non-electrostatic forces in carrying a unit positive charge from the lower to the higher potential. The terminals of a battery, or the outlets on a d-c switchboard, are examples of such charged objects. The negatively charged terminal is at a lower potential, and the positively charged terminal is at a higher potential.

It follows from our previous definitions that all points on an isolated metal object must be at the same potential. If they were not, charge would move spontaneously along or through the conducting object from points of high to points of low potential until the postulated equality of potential was established. We can use this fact to devise an instrument

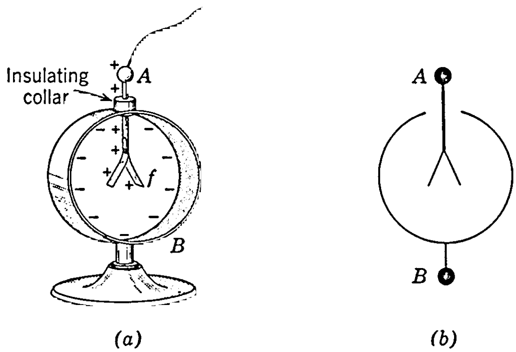

FIG. 1.6. (*a*) A gold-leaf electroscope. (*b*) Symbol for an electrostatic voltmeter.

for measuring the voltage or potential difference between two points. The electrostatic voltmeter based on the gold-leaf electroscope illustrates one simple way in which this can be done. See Figs. 1.6*a* and *b*.

The main features of a gold-leaf electroscope are shown in Fig. 1.6*a*. A cylindrical metallic cylinder with transparent ends has mounted in it a conducting rod A held by means of an insulating collar in B. At the end of the rod is a very thin gold foil f folded in the middle. When the instrument is uncharged, the two halves of the foil hang straight down. If now a certain amount of positive charge is removed from B and placed on A, a difference of potential will be established between A and B. The more charge transferred, the greater will be the difference of potential V_{AB}. But the positive charge on A will spread out, and part of it will be found on the foil f. The two halves of the foil will repel each other and will be attracted by the container B. The degree of separation of the two halves will be a measure of the amount of charge on A and B and therefore also of the potential difference V_{AB}. If the instrument is calibrated and provided with a scale, it becomes an electrostatic voltmeter. Its use as such is illustrated in Fig. 1.7.

Two parallel metal plates are connected to terminals a and b respectively. The plate connected to a has a charge $+Q$, and the plate connected to b a charge $-Q$. As a result, there is a difference of potential V_{ab} between the plates. These plates are connected through the dashed lines shown to the electroscope terminals A and B. The charge is now redistributed to include the electroscope, and, when equilibrium is established, the plate connected to a and the side of the electroscope A is at one potential throughout and the plate connected to b and the outer container of the electroscope B is at another. If the electroscope

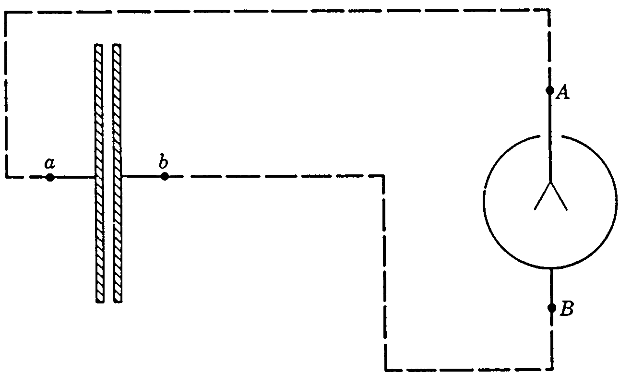

FIG. 1.7. A gold-leaf electroscope used as an electrostatic voltmeter to measure the potential difference between two charged plates.

is calibrated, it shows the potential difference $V_{AB} = V_{ab}$. This is, of course, the potential difference V_{ab} after the electroscope has been connected and is approximately the same as the potential difference originally present if only a small fraction of the charge originally on the parallel plates is taken up by the electroscope. We shall study this aspect of the problem in more detail in the next chapter.

Note about Units of Potential Difference

The potential difference between two points has the same units as the potential itself. According to the convention mentioned at the end of Section 1.1, large and small units of potential differences are:

The megavolt (Mv) $= 10^6$ volts.
The kilovolt (kv) $= 10^3$ volts.
The volt.
The millivolt (mv) $= 10^{-3}$ volt.
The microvolt (μv) $= 10^{-6}$ volt.

In the cgs electrostatic system of units the unit of potential is the *statvolt* = 1 erg/statcoulomb, which is approximately equal to 300 volts.

1.4 ELECTROMOTIVE FORCE

If two charged objects, such as the metal plates *a* and *b* of Fig. 1.7, are electrically connected to each other by means of a wire or other conductor, charge will flow through the conducting wire until the objects *a* and *b* are uncharged. The difference in potential will then be zero. The moving charge in the conductor is called an electric current. Two oppositely charged objects can maintain a current in a wire joining them only until the charge initially present has been dissipated.

A device capable of maintaining a current in a circuit is said to be a *source of electromotive force* (emf). It is a reservoir of energy, or a transformer of energy from one form into another. A source of emf is a device having electrical terminals between which a difference of potential is maintained even while a current is being delivered.

Among such devices are certain types of rotating machines called electric generators. Torque is applied to a shaft, and the work done by the forces exerting the torque is the source of electric energy supplied to a circuit connected to the generator.

Storage batteries are also sources of emf. The principles involved in their operation may be briefly described as follows: If two oppositely charged copper plates are dipped into a conducting solution, the free charges in the solution will move and a current will be established. The current will be such as to discharge the plates. If the plates are initially uncharged, there will be no force between them and therefore no current when they are put into the solution.

This is true only if the two plates are made of the same metal. If the plates are made of different metals, copper and zinc, for instance, the initially uncharged plates will be found to acquire a charge when placed in the solution. A difference of potential is produced by chemical forces. In the above example the copper is positive with respect to the zinc. The chemical action will be to remove positive charge from the zinc and transfer it to the copper and negative charge from the copper and transfer it to the zinc until equilibrium is established. Before equilibrium is established, chemical forces will act in such a way as to produce the charging process already described. As charge accumulates on the plates, electric forces will be set up in such a way as to oppose the continuation of the chemical process. Equilibrium is reached when the electrostatic forces are strong enough to stop the chemical processes from continuing, that is, when the work done by the chemical forces in carrying unit positive charge from the negative to the positive terminal is just equal and opposite in sign to the work done by the electrical forces. The total work done on a positive charge in being carried through a charged battery from the negative to the positive terminal is zero.

A further example of a source of emf is the thermocouple. A current will be generated in a loop of wire consisting of two pieces made of different metals, if the junctions are kept at different temperatures, T_1 and T_2. If one of the wires is cut, as in Fig. 1.8, a difference of poten-

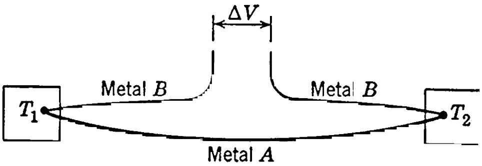

FIG. 1.8. A thermocouple.

tial will be found between the two ends. The magnitude of this potential difference for any given combination of metals is a measure of the temperature difference between the junctions. At present this device is not used commercially to generate current, but it is important as an instrument for measuring temperature.

Most thermocouples produce only millivolts for differences in temperature of their junctions of the order of 100° centigrade, but even such small potential differences can be measured accurately. It is most convenient to be able to put a junction of two small wires in such widely

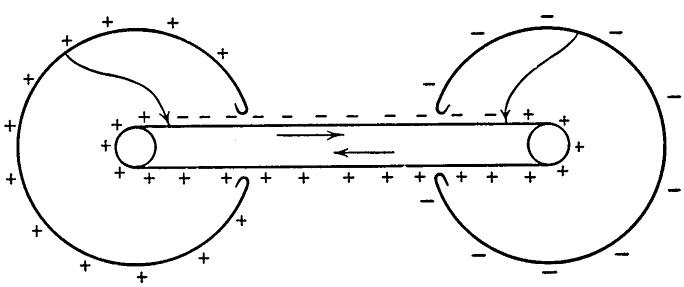

FIG. 1.9. The Van de Graaff generator. (See also Fig. 4.17 for a more modern version of a Van de Graaff generator.)

diverse and inaccessible places as, say, the low temperature chamber of a cryostat, or some muscle or organ of the human body, or a crucible of molten iron, and to be able to measure the temperature of this junction by maintaining the other at some fixed temperature, usually 0°C, and observing the voltage difference between two conveniently protruding bits of wire.

As a final source of emf we shall take up the Van de Graaff generator. The functioning of the instrument may be simply described in terms of the schematic diagram in Fig. 1.9. This is very much like the first large generator built about 20 years ago, consisting of two insulated spheres with an endless belt running from the inside of one to the inside of the other. Inside of each sphere there is a device for spraying charge of one sign onto the belt and transferring charge of opposite sign to the sphere itself, where it appears on the outer surface. The device for spraying charge onto the moving belt is not shown in the illustration,

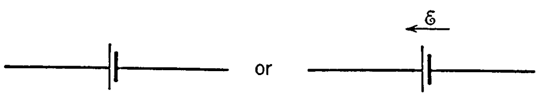

Fig. 1.10.

but is indicated by the arrowheads inside the spheres. In the left-hand sphere negative charge is being sprayed onto the belt, and in the right-hand sphere positive charge is being sprayed onto the belt. The potential difference between the spheres can be increased until electrical breakdown occurs. For instance, at sufficiently high voltage differences, a spark may jump through the air from one sphere to the other.

If the charges on the terminals of a source of emf are continuously replaced by some means, for example, by the moving belt of a Van de Graaff generator, the source becomes a continuously operable device for maintaining an electric current, and the device is said to have an emf designated by ℰ. The vector is considered to be in the direction that the device drives positive charges, or from the positive terminal through an external circuit to the negatively charged terminal. In general, the *emf of a generator is the terminal voltage when no current is being drawn.*

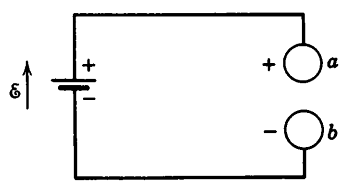

Fig. 1.11. Relationship between the emf of a battery and the current which it can produce.

A source of emf is designated in circuit diagrams by the symbols shown in Fig. 1.10. Of the two terminals the one on the long thin side of the symbol is positive, and the arrow indicates the direction in which positive charges are driven by the emf. Thus, in the case illustrated in Fig. 1.11, V_{ab} is a positive quantity and a positive charge released at a will go down in potential through the intervening space toward b. We may then think of the charge as being pushed back up to the higher potential by the battery or other source of emf represented by the symbol.

1.5 ELECTRIC CURRENTS AND MAGNETIC FORCES

The difficulties which Coulomb had to overcome to obtain reliable data on electrostatic forces were very considerable. In the first place, the forces produced by the quantity of charge that could be placed on a small object by available methods were small. In the second place, the charge on an object tends to leak off, particularly if there is considerable humidity in the air. It might seem that magnetic forces would be simpler to deal with. Anyone who has held two small permanent magnets is aware that the forces between them may be very appreciable. They are independent of such factors as atmospheric humidity. Further, since such forces had been known to exist for a long time, one might have expected the magnetic forces to have been analyzed much earlier in history. The magnetic compass, for example, was used as an aid to navigation before the twelfth century. Nevertheless, quantitative observations of both magnetic forces and electric currents were not made until the same period of history, namely, the end of the eighteenth century.

The magnetic force is quite different in character from the electrostatic or gravitational force. In the first place, it is not in general directed along a line joining two magnets and it is not inversely proportional to the square of the distance between them. If we were to examine the force between two magnetized objects, we should find that their orientation with respect to each other was an important variable. In other words, each magnet is to be described, not in terms of a scalar "quantity of magnetism," as was the mass in the gravitational case or the charge in the electrical case, but in terms of a directed vector quantity, describing its state of magnetization. Further, we should find that the force may be repulsive or attractive, depending on the mutual orientation of the two magnets, and need not be directed along the line joining them. For a fixed mutual orientation of the magnets, we should find that it varies inversely as the cube of the distance between the magnets when their separation is large compared to their dimensions. Finally, we should find that, in addition to the resultant force already described, each magnet may exert a torque on the other. The results, in other words, would be exceedingly complicated and hard to describe.

These difficulties are resolved by the introduction of a new concept which breaks the problem into two parts. This is the concept of a magnetic field. We consider a magnet as producing, or as being surrounded by, a magnetic field, and we then describe the forces in terms of the field of one magnet and the magnetization of the other. This field is perhaps best introduced through considering the interaction of the magnetized earth and the compass needle. We do not know what sort of thing the magnet inside the earth may be, but even without this knowledge we can describe the force. Actually we shall consider only the magnetic torque, or twist, which acts on the compass needle at any point on the earth's surface. The field may be thought of as exerting

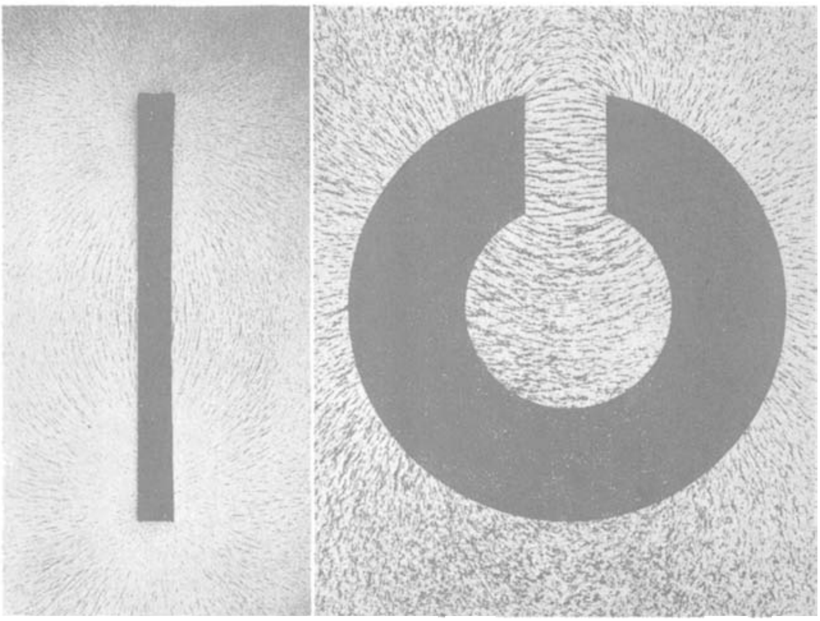

FIG. 1.12. The magnetic field surrounding (*a*) a bar magnet and (*b*) a horseshoe magnet, as revealed by iron filings.

forces which tend to make the compass needle point in the direction of the field. A further well-known example is shown in Fig. 1.12, which shows the orientation of small magnetized iron fragments in the presence of a magnetic field. The chains of magnetized particles serve as a graphic representation of the field surrounding the magnets. In Fig. 1.12, it can be seen that the lines of the magnetic field extend from one part of a magnetized object to another. These regions in which lines of force originate or terminate are called "poles." When a piece of iron, for example, is polarized, or magnetized, it develops a north pole at one end and a south pole at the other. In the illustration we cannot distinguish north poles from south poles, but by bringing up a compass needle whose poles have been marked we could tell whether the bar magnet in Fig. 1.12*a* had its north or south pole at the upper end, because, as in electrical phenomena, like magnetic poles repel each other whereas unlike poles attract each other. We shall not here attempt to define magnetic poles more precisely, but it should be noted that isolated magnetic poles of one sign are not found in nature. A magnetized object always contains both north and south poles.

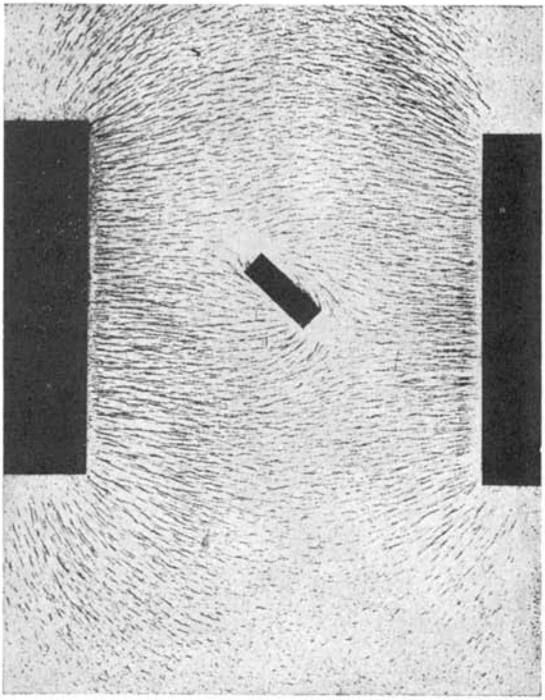

FIG. 1.13. A complex magnetic field revealed by iron filings.

Figure 1.13 shows a complex magnetic field revealed by iron filings. We now take up the problem of relating the electric and magnetic forces to each other, and shall show how these basic forces are related to phenomena that occur in electrical circuits. Everyone is familiar with the words "ampere" and "volt" and with the meters which are used to measure these quantities, but there is often a certain amount of confusion regarding their precise significance. We have already considered the concept of electrostatic potential energy and the related volt. We shall now review the concepts associated with electrical currents and their measurement.

Let us return to our discussion of the historic sequence of events and consider how the concepts of current and ampere, on the one hand, and potential difference and volt, on the other hand, came to be developed.

At the end of the eighteenth century experimenters had only permanent magnets and electrostatic machines to experiment with. It was felt that somehow the phenomena which they produce must be related, but just how was not clear. In 1770 the Bavarian Academy sponsored a series of prize essays on "the identity of the two great mysterious forces, electricity and magnetism," but no connection was established. Regarding the functioning of an electrostatic generator, certain significant ideas were beginning to evolve. Henry Cavendish, the man who used an apparatus much like Coulomb's to measure the gravitational attraction between objects in his laboratory, described two ways of making electrical tests. One was to discharge charged objects through his own body and estimate the severity of the shock. The other was to observe the distance through which a spark would jump from one to the other of two charged objects. It was clear that these two tests measured different things, because the distance a spark could be made to jump from one object to another was by no means proportional to the severity of the shock felt when the two objects were grasped, one in each hand. The nature of the difference was, however, obscure. In order to clarify the issue, it was necessary to define electrical currents and electrical potential differences and find means of measuring them.

The series of events that led to this development took place during the first 30 years of the nineteenth century. Electrical batteries were produced. These made it possible not only to send pulses of charge through a wire but to drive charges around a metallic circuit in a continuous flow. The current so produced was observed to generate heat, to produce chemical changes if passed through a solution, and to give rise to light in an arc between electrodes.

Then in 1819 Hans Christian Oersted made the crucial discovery of a relation between electricity and magnetism. He showed that a compass needle was deflected by a current, that is, by electric charges in motion along a wire. In fact, a current produces a magnetic field, which can be demonstrated by means of iron filings, just as we demonstrated the field of a magnet. The filings, as shown in Fig. 1.14, form circles around the current-carrying wire. This qualitative fact is so familiar that its significance has often been underestimated. Its most obvious significance was that it made possible the construction of electrical meters. With the advent of these precise measuring instruments, the science of electricity and magnetism could begin to grow. But perhaps of even greater significance was the fact that the first step had been taken toward establishing a new set of laws of nature. The refinement of Oersted's ideas led, about 50 years later, to Maxwell's precise specification of the laws of electricity and magnetism, and enabled Maxwell to relate electrical phenomena to the transmission of energy through empty space.

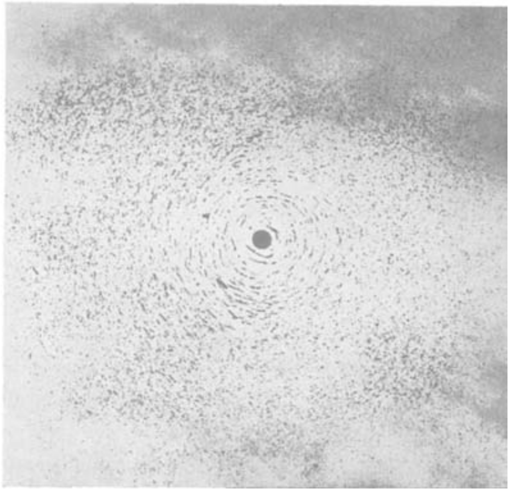

FIG. 1.14. The magnetic field of a current.

Crude current-measuring devices were made soon after Oersted's discovery. From the concept of quantity of charge q, responsible for electrostatic forces, was developed the concept of the rate of flow of charge. *An ampere is a rate of flow of charge along a conductor at the rate of 1 coulomb per second.*

$$i = \frac{dq}{dt} \frac{\text{coulombs}}{\text{seconds}} \quad \text{or amperes} \tag{1.10}$$

There is a certain ambiguity about the sign of the charge used in this expression for the current. If we have two charged objects having equal and opposite charges, we can discharge them by moving all the positive charge to the negatively charged object, or all the negative charge to the positively charged object, or by moving some positive and some negative charge. The electrical effects depend only on the total charge moved, not on which charge was moved. The same is true of the magnetic effects due to moving charges. Moving positive charge from left to right along a wire produces the same magnetic effects as the movement of negative charge at the same rate from right to left. In order to compute electrical changes, therefore, it is not necessary for us to know whether conductors actually carry moving positive or negative charge. The choice has been arbitrarily made that we consider the direction of a current as the direction of motion of positive charge producing the observed effects. Thus, if a wire is connected to the charged objects just discussed, the direction of the discharging current will be from the positively charged to the negatively charged object.

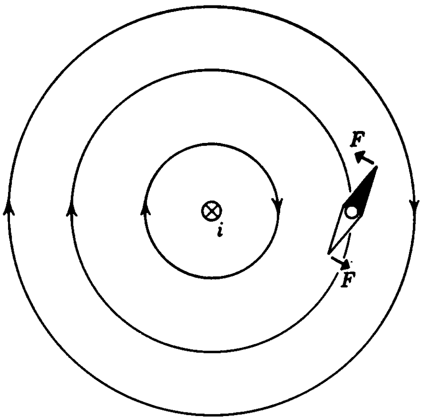

FIG. 1.15. Torque on a compass needle near a conductor carrying a current.

It was eventually established that the torque on a compass needle held in some fixed place and orientation near a wire was proportional to the current flowing through the wire. For example, Fig. 1.15 represents a current flowing down into the paper along a conductor. Iron filings would form circular chains around the current, as shown in Fig. 1.15. A compass needle near such a conductor would have forces acting on it in the direction shown in the figure. The resulting torque would tend to make the needle align itself with the field. The magnitude of the torque in any position is proportional to the current. A very simple, though insensitive and clumsy, current-measuring device could be produced in this way.

Instead of measuring the forces acting on a magnetized compass needle near a conductor carrying a current, it is possible to determine a current in a conductor by measuring the forces on the conductor in the presence of a magnet. This is merely the reaction to the forces exerted on the compass needle. If the conductor exerts forces on a magnet, such as a compass needle, then the magnet must exert equal and opposite forces on the conductor. These forces are most easily described in terms of the field which the magnet produces at the conductor. A particularly simple case is shown in Fig. 1.16 in which current flows along a conductor at right angles to the uniform field between the poles of a magnet. The force acting on the conductor is proportional to the current and is at right angles to both current and field.

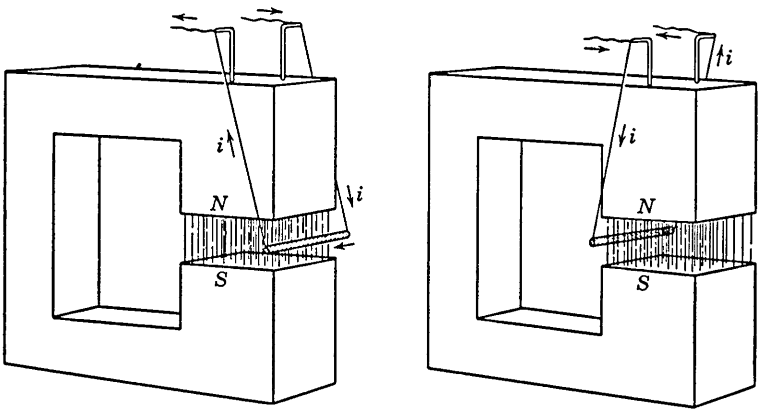

FIG. 1.16. The electromagnetic force exerted by a magnetic field on a conductor carrying a current.

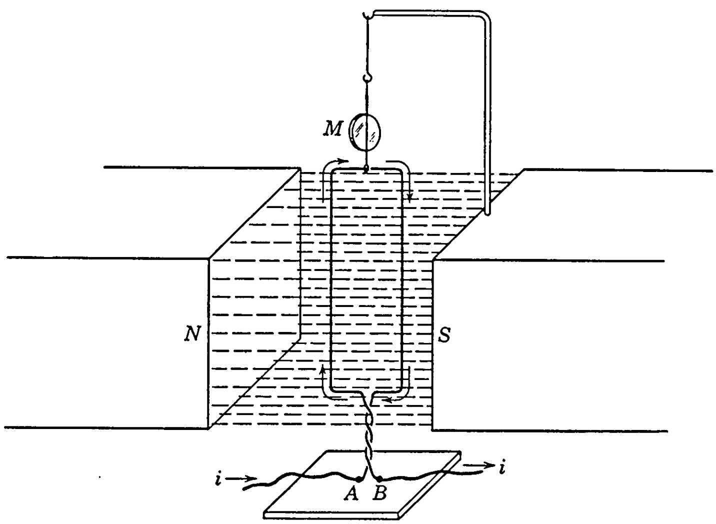

FIG. 1.17. A simple form of galvanometer.

As shown in Fig. 1.16, reversing the current reverses the direction of the force.

This electromagnetic force is incorporated in a variety of ways in instruments called *galvanometers*. They contain either a fixed coil and a movable magnet or, more usually, a fixed magnet and a movable coil. The movable part is elastically suspended in such a way that it is rotated by an amount proportional to the torque, and therefore proportional to the current.

A simple form of galvanometer is shown in Fig. 1.17. A loop of wire is connected to terminals AB of a source of current between the poles of a magnet. The system is held taut from above and has a mirror M fastened to it. A light beam from a fixed source reflected onto a scale from the mirror shows the position of the system. When a current flows through the wire, a torque is exerted on the loop and the mirror is consequently rotated. The amount of rotation is a measure of the current. When an instrument of this sort is provided with a calibrated scale reading amperes, milliamperes, or microamperes, it is called an *ammeter*, a *milliammeter*, or a *microammeter*. The construction of actual instruments may be more complicated than the device shown in Fig. 1.17, but the fundamental principle involved is always the same.

Ballistic Galvanometers and the Measurement of Charge

Galvanometers may be used to measure not only current but also quantity of charge. To see how this can be done we must remember that the forces acting on the wires of the suspended coil of a galvanometer are proportional to the current. Symbolically, we may put

$$F = ki \tag{1.11}$$

Suppose now that we pass a current i for some short time Δt through the galvanometer. The total charge Δq which has passed through the galvanometer will be

$$\Delta q = i\,\Delta t \tag{1.12}$$

The forces will have given an impulse Ft to the coil. This impulse produces momentum. Actually the forces are so arranged that they tend to rotate the coil about its suspension. The impulse is an angular impulse and produces angular momentum about the suspension. This angular momentum will cause the galvanometer to swing, and if the impulsive force acts for a time short compared to a quarter period of the instrument then the subsequent total deflection, or maximum swing of the instrument, is proportional to the initially acquired momentum.

Going over the arguments again, we have:

1. A quantity of charge Δq flowing through a galvanometer is equivalent to a current pulse.

2. A current pulse through a galvanometer produces an impulsive torque whose magnitude is proportional to the charge Δq which passes through the galvanometer coil.

3. An impulsive torque will produce angular momentum. The angular momentum acquired by the coil, and therefore also its initial angular velocity ω_0, will be proportional to the charge Δq.

$$\Delta q \propto \omega_0 \tag{1.13}$$

4. If the impulsive force is removed before the instrument is appreciably deflected, the total energy acquired is $\frac{1}{2}I\omega_0^2$.

5. The instrument, if left undisturbed, now swings to a maximum deflection, where it comes to rest. The kinetic energy is zero and has been converted into potential energy. The potential energy of a suspension twisted through an angle θ is equal to $\frac{1}{2}k\theta^2$, where k is the torsion constant of the suspension. We have therefore

$$\frac{1}{2}I\omega_0^2 = \frac{1}{2}k\theta^2$$

$$\omega_0 \propto \theta$$

and therefore, using Eq. 1.13, the maximum deflection θ is proportional to the charge sent through the instrument.

$$\Delta q \propto \theta \tag{1.14}$$

This instrument is the electrical analog of the ballistic pendulum used to measure the momentum of bullets.

1.6 RESISTORS, OHM'S LAW, JOULE'S LAW

A metallic wire is characterized by the fact that, if its temperature is kept constant, the current which it will pass is directly proportional to the voltage difference applied to its terminals. The proportionality

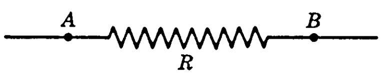

FIG. 1.18.

constant is called the resistance R of this wire. If the terminals A and B of a resistor R, as shown in Fig. 1.18, have across them a voltage V_{AB}, then the current i flowing through the resistor will be given the relation

$$iR = V_{AB} \tag{1.15}$$

$$i = V_{AB}/R$$

This is called *Ohm's law.* The resistance R has the dimensions of volts/ampere. The unit of resistance is called an *ohm.* A symbol commonly used to designate ohms is a Greek omega, ω (sometimes Ω). One ohm is equal to 1 volt per ampere. Thus a 5-ohm resistor carrying a current of 10 amp must necessarily have a potential difference of $10 \times 5 = 50$ volts appearing across its terminals. Conversely, if 50 volts are applied to its terminals, the resistor will pass a current $50/5 = 10$ amp. The direction of the current will always be from the terminal at the higher potential through the resistor to the terminal at the lower potential. These current-voltage relationships are graphically represented in Fig. 1.19. An electrical resistance may be computed from voltage and current measurements made with instruments connected as shown in Fig. 1.20.

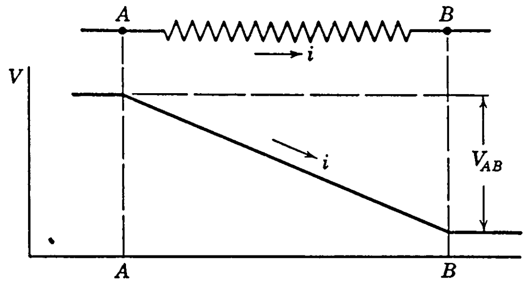

Fig. 1.19. Voltage-current relationships in a resistor.

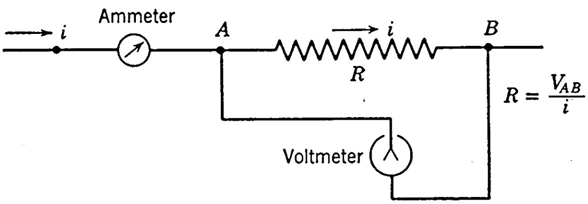

Fig. 1.20.

Variable resistors are represented by one of the symbols shown in Fig. 1.21. In the second designation the arrow represents a movable

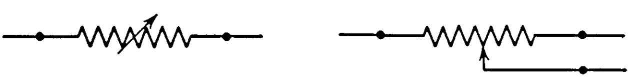

Fig. 1.21.

sliding contact. The resistance between this sliding contact and either one of the other terminals of the resistor depends on the position of the slider.

The Internal Resistance of a Source of EMF

As an example of the application of Ohm's law we shall consider the circuit shown in Fig. 1.22 involving a battery connected to a resistor. A battery or other source of emf will in general have an internal resistance r. This is often small compared to the external resistance and, if so, plays no part in limiting the current. But, if we short-circuit the terminals of a battery, that is, connect them to the terminals of a conductor whose resistance is small compared to the internal resistance of the battery, the observed current will be entirely limited by the internal resistance. This is the maximum current that the source of emf can deliver and is called the short-circuit current. Its magnitude is

$$i \text{ (short circuit)} = \mathcal{E}/r$$

The internal resistance of a battery must, in general, be included in the description of a circuit. If it is not explicitly given, it may be assumed to be negligible.

The total resistance of the circuit in Fig. 1.22 is $(R + r)$, and the current in this circuit will therefore be given by the equation

$$\mathcal{E} = i(R + r). \tag{1.16}$$

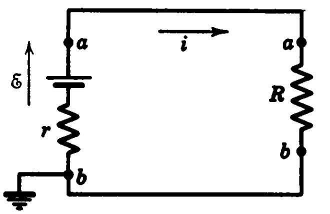

FIG. 1.22. A diagram of an electric circuit consisting of a battery whose emf is $\mathcal{E}$ and whole internal resistance is r, connected to an external resistance R and grounded at point b.

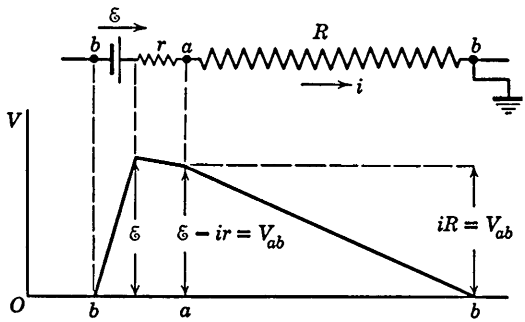

FIG. 1.23. Schematic diagram of the potential at various points of the circuit shown in FIG. 1.22.

Figure 1.23 represents the current-voltage relationships in this circuit. The circuit is here represented as a straight line for convenience, but the two end points are really one and the same point. Below the circuit diagram is a schematic plot of the potential at various points of the circuit. The terminal b is here assigned zero potential, since this point of the circuit is shown grounded, or connected to the earth, which is commonly used as a reference body at zero potential. In Fig. 1.23, there is a drop in potential, in going through any resistor in the direction of the current, equal to the current times the resistance.

It should be remembered as a universally applicable rule, already emphasized, that if a current i is passing through a resistance R there will necessarily be a potential difference of iR volts across its terminals, and that, conversely, if there is a potential difference of this magnitude across the terminals, there must be a current i flowing through it. Moreover, the current always flows through the resistor from high to low potential. Note that the potential difference between the battery terminals V_{ab} is no longer equal to the emf, but that when the battery is discharging

$$V_{ab} = \mathcal{E} - ir.$$

The symbol $\mathcal{E}$ is reserved for the open-circuit potential difference across the terminals when no current is flowing through the battery.

Finally, in order for the voltage at the two ends of the diagram to be the same, the current must have such a value that the sum of the potential drops through the resistors is just equal to the emf of the battery. If, for example, a 6-volt battery having an internal resistance of 1 ohm is connected, as in Fig. 1.22, to a resistor having a resistance of 11 ohms, we have from Eq. 1.16,

$$6 \text{ volts} = i \text{ amp} \times (11 \text{ ohms} + 1 \text{ ohm})$$

or

$$i = 0.5 \text{ amp}$$

A variant of this situation is obtained by considering the same elements with which we have been dealing, but connected as shown in Fig. 1.24 at points b and c to some other circuit which will maintain a current i flowing from c to b. We again draw the voltage diagram, using the procedure described above, and arrive at the results shown in the figure. One of these results is

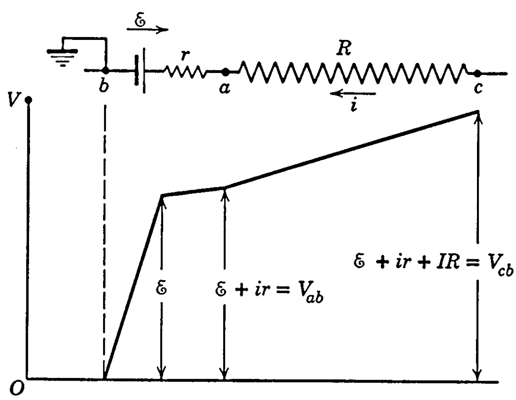

FIG. 1.24. Voltage diagram for a part of a circuit.

that the potential difference across the battery terminals is now greater than the open-circuit emf of the battery. This is generally true when a current is driven through a battery from its positive to its negative terminal, for example, when the battery is being charged.

Knowing the magnitude of the current in the circuit illustrated in Fig. 1.24, we can compute V_{cb}. We have, from Fig. 1.24,

$$V_{cb} = \mathcal{E} + ir + iR \tag{1.17}$$

Thus if we connect a 6-volt battery having an internal resistance of 1 ohm to a 114-volt line through an external resistor having a resistance of 11 ohms, and make the connection so that the positive terminal of the battery is connected to the positive terminal of the line, a charging current will be delivered to the battery. Its magnitude follows from Eq. 1.17.

$$114 = 6 + i(1) + i(11).$$

Solving for the current. we get $i = 9$ amp.

We shall now take up the energy and power relations in a simple circuit, for example, that shown in Fig. 1.25. In the circuit in question the battery is connected in series with a load *CE* and a switch *ED*. By a load we mean a device, such as a heater or a motor, which removes energy from the circuit when the battery drives a current through it. By a series connection we mean an arrangement in which the current flows successively through the units connected in series: that is, the current flows first through one; then without branching, the same

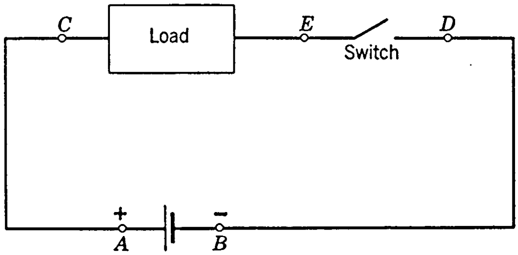

FIG. 1.25. A simple circuit diagram.

current flows through the second; and then successively through any other units in the series connection.

Let us as an exercise first consider the potential differences between various points of the circuit when the switch is open. Suppose that the battery is a 6-volt storage battery. Then $V_{AB} = 6$ volts. A is at a potential 6 volts higher than B. B is at a potential 6 volts lower than A. We might also write $V_{BA} = -6$ volts, since V_{BA} is equivalent to $V_B - V_A$. The straight line connecting terminal A of the battery to terminal C of the load is assumed to involve no change of potential. Thus $V_{AC} = V_{CA} = 0$, and $V_{AB} = V_{CB}$. Similarly $V_{BD} = 0$, $V_{AB} = V_{AD} = V_{CD}$. If now the switch is closed, current will flow through the load. The switch again is assumed to carry the potential at D to E, so that $V_{ED} = 0$. Therefore the voltage across the load, or in other words the potential difference V_{CE} between its terminals, is equal to the potential difference between the battery terminals V_{AB}.

We have seen that the battery terminal voltage when we draw a current may be appreciably less than the open-circuit voltage, particularly if we draw large currents. We assume for the present, however, that the battery voltage remains 6 volts, and that when the switch is closed 6 volts appear across the load. This means that we should still have to do 6 joules of work on every coulomb of charge forced by some means from terminal E to terminal C, and, conversely, that every coulomb of charge flowing through the load from C to E loses 6 joules of energy. These 6 joules are converted into heat if the load is a heater, or into mechanical energy if the load is an efficient motor. The energy converted by q coulombs is qV_{CE} joules, and, if we have a current i amperes $= dq/dt$ coulombs per second, then we have iV_{CE} joules of energy per second or watts of power being converted. The power delivered by the battery is

$$W = iV_{AB} \text{ watts} \tag{1.18}$$

The power converted by the load is

$$iV_{CE} \text{ watts}$$

Since the same current is flowing in both of these units, and the voltages across the two are equal, we conclude that the power delivered by the battery is consumed in the load. If the load draws 0.5 amp, the power delivered by the battery $iV_{AB} = 0.5 \times 6 = 3$ watts.

If the load is a resistor, the above considerations may be amplified. For example, in the resistor shown in Fig. 1.19, the power being converted to heat must be $W = iV_{AB}$. But from Ohm's law, as expressed in Eq. 1.15, we have $V_{AB} = iR$, or combining these results, we have

$$W = iV_{AB} = i^2R \text{ watts} \tag{1.19}$$

This is called *Joule's law.* For example, a 5-ohm resistor drawing 10 amp will generate $5 \times 10^2 = 500$ watts, or 500 joules/sec of heat.

Power Relations in a Charging or Discharging Battery

To illustrate the application of Joule's law, we consider first the battery connected to a resistor as in Fig. 1.22. The power delivered by the battery is $i\mathcal{E}$, and this is converted into heat in the resistor R and in the battery itself. We have

$$W = i\mathcal{E} = i^2R + i^2r = i(iR + ir)$$

This checks with Eq. 1.16, which states that the open-circuit emf of the battery must equal the sum of the voltage drops in all the resistances present.

If the battery is being charged by an externally applied voltage connected in series with a resistor R, as in Fig. 1.24, the power delivered by the external circuit iV_{cb} must equal the rate of generation of heat $i^2(r + R)$ plus the rate of storage of energy in the battery, $i\mathcal{E}$. Here the current i is opposed to the emf $\mathcal{E}$, and $i\mathcal{E}$ represents a rate of storage of energy in the battery.

If, as in the discussion of Fig. 1.24, a 6-volt battery having an internal resistance of 1 ohm is connected through an 11-ohm resistor to a 114-volt line, we find that the charging current will be 9 amp. The rate at which chemical energy is accumulated in the battery is 9×6 or 54 watts. The rate of evolution of heat in the battery is $i^2r = 81 \times 1 = 81$ watts. The total power supplied to the battery is $54 + 81 = 135$ watts. The heat generated in the resistor R is at the rate $i^2R = 81 \times 11 = 891$ watts. The total power supplied to the circuit is 135 + 891 = 1026 watts or 1.026 kilowatts. This must check with the power delivered by the line $iV_{cb} = 114 \times 9 = 1026$ watts.

Wire Sizes and Current-Carrying Capacity

To give some idea of the magnitudes of the actual resistance of wires and the magnitude of the currents they can carry, we quote a few numbers which can be found in handbooks concerning copper wires. Wire sizes are conventionally given in B&S, or Brown and Sharpe, gage numbers. The diameters are given in mils, or thousandths of an inch. Areas are given in circular mils (cir mils). If the diameter of a wire having a circular cross section is D mils, its area in circular mils is D^2 cir mils. To convert to square inches we note that

$$\frac{\pi D^2}{4} \times 10^{-6} \text{ sq in.} = D^2 \text{ cir mils}$$

$$1 \text{ cir mil} = \frac{\pi}{4 \times 10^6} \text{ sq in.}$$

The "safe" current-carrying capacity of a wire is a fairly arbitrary quantity. What is permissible for a bare wire in an experiment in the laboratory would certainly be unsafe for an insulated wire in a conduit in a house. Approximate conservative values for confined insulated wires are given in Table 1.1.

TABLE 1.1 ALLOWABLE CARRYING CAPACITIES OF COPPER WIRE
(Regulation of the National Board of Fire Underwriters)

			Amperes	
B&S Gage	Diameter, mils	Cross Section, cir mils	Rubber Insulation	Other Insulation
0000	460.0	2,111,600	225	325
000	409.6	167,800	175	275
00	364.8	133,100	150	225
0	324.9	105,500	125	200
1	289.3	83,690	100	150
2	257.6	66,370	90	125
4	204.3	41,740	70	90
6	162.0	26,250	50	70
8	128.5	16,510	35	50
10	101.9	10,380	25	30
12	80.8	6,530	20	25
14	64.1	4,102	15	20
16	50.8	2,583	6	10
18	40.3	1,624	3	5

Note that, according to Table 1.1, to carry 1 or 2 amp a wire roughly $\frac{1}{32}$ in. in diameter is needed, whereas a wire $\frac{1}{8}$ in. in diameter will carry around 50 amp. For 500 amp something more like a copper bar around $\frac{1}{2}$ in. in diameter is necessary.

SUMMARY

Matter is composed of electric charges that exert forces on each other described by Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{qq'}{r^2}$$

The force F is in newtons if the charges q and q' are in coulombs, the distance r in meters, and

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ coulomb}^2/\text{n-m}^2$$

Charges are of two kinds, positive and negative. Like charges repel each other; unlike charges attract each other.

Magnetized matter and electric currents are surrounded by magnetic fields which can be studied by means of iron filings.

An electric current is related to the movement of charge in a conductor or other conducting medium by the equation

$$i = \frac{dq}{dt} \frac{\text{coulombs}}{\text{second}} \qquad \text{or amperes}$$

The current through a conductor in amperes is equal to the number of coulombs passing any cross section in 1 sec. The direction of the current is taken as the direction of motion of positive charge which would produce the same electrical effects as the observed current. If a current is due to the motion of negative charge, then the direction of motion of this negative charge is opposite to the direction of the current.

Currents are produced by potential differences. The terminals of a battery, for example, have a potential difference which is numerically equal to the work required to carry 1 positive coulomb from the negative to the positive terminal of the battery (along a path outside of the battery). Potential differences are expressed in volts, which have the dimensions of joules per coulomb. From this definition it follows that the power delivered by a battery to a load is

$$W = iV_{AB} \text{ watts}$$

where i is the current sent through the load by the battery and V_{AB} is the potential difference between the battery terminals while it is delivering the current i.

The current through a metallic conductor at any fixed temperature is proportional to the impressed voltage. The proportionality constant is called the resistance R in ohms, and the relation

$$iR = V_{AB}$$

where i is the current through the conductor in amperes and V_{AB} is the potential difference between its terminals, is called Ohm's law. The rate of generation of heat in a conductor having a resistance R, usually called a resistor, is

$$W = iV_{AB} = i^2R \text{ watts}$$

This is called Joule's law.

The voltage or potential difference between two points is measured by connecting the terminals of a voltmeter to the two points in question. Galvanometers are instruments which contain moving parts that produce observable deflections when currents are passed through them. Currents are measured by being passed through a calibrated galvanometer, or ammeter. Galvanometers may also be used ballistically to measure quantity of charge.

PROBLEMS

1. Two charges of 5 μc each are 0.01 meter apart. What is the force on each?

2. If two isolated charges of 1 coulomb each could be placed 1 meter apart, what would be the force between them expressed in tons?

3. A charge of 1 μc and one of 25 μc are 0.1 meter apart. Find the places where there would be no force on a third charge.

4. A charge of 1 μc and one of -25 μc are 0.1 meter apart. Find the place where there would be no force on a third charge.

5. From the value of $\epsilon_0 = 8.85 \times 10^{-12}$ coulomb^2/n-m^2, and the definition of the statcoulomb, prove that 1 coulomb $= 3 \times 10^9$ statcoulombs (approx.).

6. Positive charges of 1 μc are placed at two of the corners of an equilateral triangle 0.3 meter on a side, and a negative charge of 1 μc is placed on the third corner. (*a*) What are the magnitude and direction of the force on the negative charge? (*b*) What are the magnitude and direction of the forces on the positive charges?

7. Assume that it is possible to place a charge of 0.005 μc on each of three pith balls suspended from a common point by threads 0.3 meter long. What must the mass of the pith balls be if they hang at the corners of an equilateral triangle 0.03 meter on a side?

8. Two charges of 25 μc each are placed at the points (-0.05 meter, 0) and (0.05 meter, 0) in the x-y plane. (*a*) Find the force on a third charge of 1 μc placed at any point on the x axis; (*b*) at any point on the y axis.

9. Two charges of Q coulombs each are placed at two opposite corners of a square. What additional charges q placed at each of the other two corners will reduce the resultant electric force on each of the charges Q to zero? Is it possible to choose these charges so that the resultant forces on *all* of the charges is zero?

10. Make a sketch showing the distribution of poles in Fig. 1.13.

11. A storage battery delivers a current of 1.5 amp to a load for $\frac{1}{2}$ hour. (*a*) How many coulombs of charge have passed through the load? Make a sketch showing the polarity of the battery and the direction of the current. Show how an ammeter and a voltmeter should be connected to measure the current delivered by the battery and its terminal voltage. (*b*) If the current is actually carried by negatively charged electrons, the charge on each electron being -1.6×10^{-19} coulomb, how many

electrons would have passed through the load? In what direction would they have moved? What would then have been the direction of the current?

12. A 2-ohm resistor and a 10-ohm resistor are placed in series across a 6-volt battery having a negligible internal resistance. (*a*) Find the current through each resistor. (*b*) Find the potential across each resistor.

13. In Problem 12, how much power is delivered by the battery? How much heat is generated in each resistor?

14. (*a*) What is the resistance of a 100-watt lamp under ordinary operating conditions? (*b*) How much power would you expect to be drawn by each of the two 100-watt lamps connected across a 110-volt line in series?

15. A 10,000-ohm resistor and a 1000-ohm resistor are connected in series to a 200-volt power source. (*a*) What is the current drawn? (*b*) What is the voltage drop across each resistor?

16. The equation of motion of the moving coil of a galvanometer may be expressed in the form

$$\tau = \frac{d}{dt}(I\omega) = I\alpha = I\frac{d^2\theta}{dt^2}$$

stating that the torque τ applied to the moving coil is equal to the rate of change of angular momentum. If the system is given a deflection by some means and is then allowed to oscillate freely, the only torque acting will be that due to the suspension. This is proportional to the deflection θ of the system and is always in such a direction as to restore the system to its equilibrium position. The equation of motion of a freely swinging suspended system of this kind therefore is

$$I\frac{d^2\theta}{dt^2} = -k\theta$$

The solution of this equation will be a periodic function having an arbitrary amplitude but a given frequency.

(*a*) Assume that the solution has the form $\theta = A \sin 2\pi ft$, and find the equation for the frequency f with which the system will oscillate by substituting the expression suggested for θ into the equation of motion.

(*b*) Passing a steady current through this galvanometer produces a torque proportional to the current

$$\tau = ci$$

and this in turn produces a deflection which we have already defined

$$\tau = -k\theta$$

By observation, it is found that a particular ballistic galvanometer has a period of 10 sec, and that a steady current of 100 microamperes produces a steady deflection of 13 cm on a scale 1 meter away. The moment of inertia of the moving parts is 10^{-8}kg-m^2. How much torque is exerted on the moving coil when a current of 5 μa flows through it?

(*c*) When the instrument is used ballistically, how many coulombs must be discharged through it to produce a deflection of 10 cm on a scale 1 meter away?

★17. Show that the maximum possible rate at which a battery can deliver energy to an external resistor is realized when the external resistance is equal to the internal resistance of the battery