Preface

In this thesis we propose a new approach to study the depth of boolean circuits: The *Communication Complexity approach.* The approach is based on an equivalence between the circuit depth of a given function, and the communication complexity of a related problem. The bottom-line of the new approach is that it looks at a computation device as a *separating* device; that is, a device that separates the words of a language from the non-words. This allows us to view computation in a *Top-Down* fashion and makes explicit the idea that *flow of information* is a crucial term for understanding computation.

We demonstrate that the communication complexity approach is both useful and intuitive. We do so by

- Giving new simpler proofs to old results which help us understand the results in the correct setting.
- Proving a super-logarithmic monotone depth lower bound for the function st-connectivity.

We present, in our new setting, results of Berkowitz and Dunne relating monotone and non-monotone computation. Also, we present upper bounds to some functions by giving *protocols* for the communication problems associated with them, and we introduce the notion of universal relations which, in a sense, correspond to universal circuits.

What best exemplifies the first item above is, perhaps, a new proof of a theorem of Khrapchenko. The original proof of the theorem gave no clue whatsoever to the fact that its truth stems from a simple information theoretic fact: *One needs* $\log d$ *bits to distinguish among* d *possibilities.*

We present a tight $\Theta(\log^2 n)$ depth lower bound for monotone circuits computing the function *st-connectivity*, a function which has $O(n^3 \log n)$ size monotone circuits. This is our main technical contribution: A monotone depth lower bound which is *super-logarithmic* in the size of the best circuit for the function considered. That is, our techniques apply to depth rather than to size. Thus, our results complement those by Andreev and Razborov who obtained exponential size lower bounds for monotone circuits computing some functions in NP. As a consequence, we get both super-polynomial ($n^{\Omega(\log n)}$) size lower bounds for monotone formulas computing st-connectivity, and a separation of the monotone analogues of NC^1 and AC^1.