Preface

General equilibrium theory in the modern sense was first developed in the second half of the nineteenth century by Francis Edgeworth, Alfred Marshall, and Léon Walras, most systematically by Walras. In the first half of the century some earlier moves in the direction of formal analysis of competitive markets using mathematics had been made by Augustin Cournot and Jules Dupuit. Then in the early twentieth century Vilfredo Pareto and Gustav Cassel added some additional formulations to this theory. However, the modern elaboration and rigorous development of general equilibrium theory from these foundations was begun in the 1930s and 1940s by John Hicks and Paul Samuelson, in the tradition of academic economics but with liberal appeal to mathematics, and by Abraham Wald and John von Neumann, from a rigorous mathematical viewpoint. Frank Ramsey in the late 1920s and von Neumann in the 1930s had laid the ground for optimal growth theory, which I relate to general equilibrium over time. However, the general equilibrium theory that this book is concerned to present was developed in the second half of the twentieth century primarily by Kenneth Arrow, Gerard Debreu, and me but with many contributions from others. In particular, Tjalling Koopmans should be mentioned for his activity analysis and optimal growth theory. Morgenstern, Samuelson, Hicks, and Koopmans were my teachers. Of the authors whose work is cited here Hiroshi Atsumi, Robert Becker, Sho-Ichiro Kusumoto, Leonard Mirman, Tapan Mitra, Anjan Mukherji, Kazuo Nishimura, José Scheinkman, and Makoto Yano were my students. I apologize to my many students whose valuable contributions to economics happened not to be relevant to this book. However, I must mention Jerry Green (1977) and Charles Wilson (1976) who were pioneers in the study of markets with asymmetric information. General equilibrium is far from the whole of economics.

I characterize the general equilibrium theory that I will discuss as classical to indicate that it is the theory developed in the 1950s and 1960s along with continuations in the period after that. It was then that theorists began to derive theorems in a more satisfactory way from the same basic assumptions and to provide natural extensions of the original results. The assumptions that I refer to, in the case of the existence, optimality, and turnpike theorems, are perfect foresight for each future state of the world, or equivalently one initial market in which all transactions are made for the whole future and for all states of the world. The traders in both models are assumed to continue to live throughout the period,

finite or infinite, to which the market refers. On the other hand, for the stability theory which is the Walrasian tâtonnement, it is assumed that equilibrium is reached before transactions are made final. This theory received much attention in the 1930s, 1940s, and 1950s While not realistic, it gives an indication of the conditions for stability in the very short run. It may also be relevant to later theories of temporary equilibrium where the question of how expectations are formed is important.

In the literature after 1970 these assumptions were generalized in some fundamental ways. In the existence theory the case was treated of repeated markets in which assets including stocks, bonds, and money are traded. However, perfect foresight of future prices in each state of the world is still assumed. Thus what is achieved is the description of the relations between asset prices and other prices. These relations depend on asset payoffs for different states of the world whose objective probabilities are unknown and will be estimated differently by different traders (see Magill and Quinzii 1996). This elaboration of the model may be compared with the elaboration in optimal growth theory that retains the assumption of perfect foresight but examines the progress of capital accumulation in these circumstances leading to turnpike theorems. (See Becker and Boyd 1997 for many extensions of this theory beyond the scope of this book.) Perfect foresight means that the future state of the world is known. Thus the assumption is actually stronger than that used in the classical existence theory where trading takes place for goods that include a specification of the state in which they are to be delivered but perfect foresight of the future state is not assumed. In another direction the assumption that traders live through the whole future that is covered by the market is replaced by the assumption of an infinite sequence of overlapping generations. (See Balasko, Cass, and Shell 1980 for an existence proof.) In macro models of optimal capital accumulation uncertainty was introduced by Brock and Mirman (1972; see also Stokey and Lucas 1989). Finally in recent years much attention has been given in one sector models to chaotic paths of capital accumulation (see, for example, Majumdar and Mitra 1994; Nishimura and Sorger 1999).

This book does not attempt to cover these many amendments of the classical theory. It is aimed rather at presenting a detailed and rigorous treatment of the classical model itself in which proofs of the basic theorems are given step by step. This does not mean that the argument is easy. Every step of the proofs is given, but in many cases the individual steps

require some elaboration by the reader to achieve a full understanding. I believe this is the only way to obtain a mastery of the method that will allow the student to go beyond what has been done already and derive new results. The class notes that are the original form of the material of the book owe a great deal to the suggestions of my students over the years. Also I am grateful to many of my former students for their assistance in removing errors and misprints from earlier versions of my manuscript. These are too numerous to list, but I owe a special debt to Kazuo Nishimura and Makoto Yano who used some of the chapters in their own general equilibrium seminars and to Hajime Kubota who came to Rochester during several summers to give my chapters their most careful reading. Of course, I know from experience that not all errors have been removed or ever will be removed, but I think that unfortunate circumstance should be laid at my door.