1 An Exploration of the Eductive Justifications of the Rational-Expectations Hypothesis

Roger Guesnerie

The rational-expectations hypothesis plays a central role in modern economic theory. To the skepticism of critics, who often view it as a deus ex machina, proponents have opposed two types of justifications. Following Kenneth Binmore's (1987) suggestive terminology, these justifications can be grouped in the categories of "eductive" and "evolutive" justifications. Eductive explanations rely on the understanding of the logic of the situation by economic agents; they are explicitly or implicitly associated with mental activity of participants aiming at "forecasting the forecasts of others." Evolutive explanations put the emphasis on the learning possibilities offered by the repetition of the situation; they are associated with the study of convergence of more or less ad hoc learning processes.

The present study is primarily concerned with the eductive point of view. It starts from the examination of one of the most popular eductive justifications found in the literature (implicitly present in John Muth [1961] and repeatedly evoked by successors): the rational-expectations hypothesis is nothing else than *the extension of the rationality hypothesis* to expectations. In other words, the rational-expectations forecast is the rational forecast; people make the right forecast because this forecast is in their own interest. It is known that this latter assertion is both right and wrong: it is right in that it is in the interest of agents to make correct forecasts; it is wrong in assuming that perfect coordination of forecasts is the necessary outcome of an independent optimizing effort of isolated agents. A right forecast must take into account the possibly wrong forecasts of others. In game-theoretical terms, the rational-expectations hypothesis is associated with a Nash equilibrium of beliefs and not with a dominant strategy as Muth's assertion seems to suggest.

Indeed, the formation of beliefs is analyzed in the present chapter in a game-theoretical framework. However, instead of taking for granted the Nash conjectures which sustain a rational-expectations equilibrium, the chapter attempts to derive them from more basic principles. Following a modern stream of the game-theoretic literature (originating from earlier work associated in particular with the names of Richard Luce and Howard Raiffa [1957], Robin Farquharson [1969], and Hervé Moulin [1979a]), the paper focuses attention on beliefs that are rationalizable in the terminology of Douglas Bernheim (1984) and David Pearce (1984). As made clear by T. Tan and S. Werlang (1988), rationalizable solutions (which will generate rationalizable beliefs) essentially derive from two more fundamental principles: the first one is individual Bayesian rationality; the second one is the fact that individual rationality is common knowledge.[1] An assessment of the validity of the rational-expectations hypothesis relying on such principles—although in the context of the models of the paper, the analysis makes intuitive sense so that it could be reasonably well understood and defended without direct reference to abstract principles—leads to the recognition of two types of cases. In "good" cases, the rational-expectations forecast will appear as the necessary outcome of agents' mental activities which have clear and appealing economic grounds; the rational-expectations outcome will then be explained and not merely assumed. In such cases, the above Muthian justification will be rehabilitated once conveniently reformulated: *the rational-expectations hypothesis is a consequence of rationality and of common knowledge of rationality*. In "bad" cases however, no such unique outcome will emerge, and the Muthian case for the rational-expectations hypothesis will have to be reformulated in the much weaker terms suggested above.

As indicated in the title, the approach of the present chapter has an exploratory dimension; it is based on a very stylized model, which is in fact a variant of Muth's original model. Stylization concerns the institutional framework in which decisions take place (where coordinating *institutions* are a priori ruled out), the nature of mental activities that are analyzed (which again suppose, in game-theoretical terms, that rationality is common knowledge), and also the specific connection between decisions and expectations that is assumed (today's decisions will tomorrow affect the price on which they are based). Keeping in mind the exploratory dimension of the study, I will stress its two main messages.

First, it explains the nature and power of the "eductive" game of guessing, second-guessing, and so forth, through which agents attempt to predict the outcome of the system (here the equilibrium price). It is shown that in the one-good model there is a very close connection between this mental process (which takes place in virtual time) and the traditional cobweb "tâtonnement" (which is normally assumed to describe a real-time evolution). This connection, which is demonstrated in a simple version of the model (linear and nonnoisy), is shown to extend to nonlinear and stochastic versions of the model. With several goods, the eductive argument becomes more complex but is still conclusive in a large subset of situations. The eductive approach also has the advantage of highlighting the role of credible policy interventions in promoting the stability of expectations.

Second, it argues that the conditions that determine the success of the eductive coordination of beliefs have strong economic relevance. The eductive criterion under scrutiny here can be understood as a "predictability" criterion: the equilibrium is "predictable" whenever it can be "educed." As will be checked later case by case, conclusions on predictability drawn from my approach have an appealing economic flavor. In the one-good case the elasticity conditions favoring eductive coordination (high elasticity of demand or low elasticity of supply) are in close line with the economic intuition that can be straightforwardly gained from limit cases (vertical or horizontal demand or supply). Also, it is particularly satisfactory that "predictability" in the above sense increases (a) when suppliers make their decisions sequentially (taking advantage at later dates of observations of previous decisions) rather than simultaneously and (b) when the range of product diversification in an industry increases.

The chapter is organized as follows. Section 1.1 presents the model (1.1.1), the concept of strongly rational expectations equilibrium (1.1.2), and the basic insights of the analysis (1.1.3). Section 1.2 checks the robustness of the intuition developed in subsection 1.1.2: Subsections 1.2.1 and 1.2.2 respectively extend the basic findings to the nonlinear and noisy framework. Subsection 1.2.3 contains a very brief discussion of the connections between evolutive and eductive learning. Section 1.3 explores two important directions for extension of the analysis (sequential timing and multidimensional decisions) and stresses the coherence and economic appeal of the basic message in settings of broader range.

1.1 Model and Concepts

1.1.1 The Model

I start from a variant of the model originally considered by Muth (1961). It is a partial-equilibrium formalization of a market in which producers have to make production decisions one period before their product is sold (e.g., farmers having to decide on the size of the crop, firms having to decide on the production level of a homogeneous good). Each of these producers is small with respect to the size of the market; I adopt the standard formulation that there is a continuum of such producers indexed by f (farmers or firms) where f belongs to the segment $[0, 1]$. In order to normalize, I put the Lebesgue measure on [0,1].

Agent f has a cost function $C(q,f)$, which I still denote $C_f(q)$, where q is the production decision (the size of the crop or the production level); when the product is sold at price p, price-taking behavior leads to maximization of $pq - C(q,f)$. When C is strictly convex and differentiable, the corresponding supply function, $S(p, f)$, equals $(\partial_q C_f)^{-1}(p)$ where $(\partial_q C_f)^{-1}$ is the inverse of the marginal cost function of f. Aggregate supply is

$$S(p) = \int S(p,f)\,df. \tag{1.1}$$

Note that, if p is not known, the supply of agent f depends a priori on the probability distribution over p, which agent f forms. This distribution is denoted $d\mu(p)$, so that the supply function should be written $\tilde{S}(d\mu(p),f)$. Here, agents are risk-neutral so that

$$\tilde{S}(d\mu(p,f)) = S(E(p), f), \tag{1.1'}$$

where $E(p)$ is the expected value of p associated with $d\mu(p)$, and the aggregate supply is defined accordingly.

The demand side is described through an aggregate downward-sloping demand function $D(p)$. One may assume that it comes from a continuum of identical consumers so that the model describes a standard competitive situation in which all individual agents are "small" with respect to the size of the market.[2]

I will now present the linear specification of this basic deterministic model. The production side has a cost function $C(q, f) = q^2/2C(f)$ where $C(f)$ is a parameter which may depend on farmer f. The maximand is $pq - q^2/2C(f)$ so that $S(p, f) = C(f)p$, and aggregate supply is given by[3]

$$S(p) = \left[\int C(f)\,df\right] p = Cp. \tag{1.1''}$$

Similarly, the aggregate demand function is linear:

$$D(p) = \begin{cases} A - Bp & \text{if } A - Bp > 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1.2}$$

The timing of decisions is the following: at date t, producers decide on the production level; at date $t+1$ they sell their product on the competitive market. All the objective characteristics of the situation (cost function, demand curve, and individual payoffs) are presumed to be public information. More precisely, I will assume later that these elements are common knowledge.

1.1.2 Concepts

I will now define the game-theoretic concept of *rationalizable solution*. For that, one views the farmers' problem just described as a normal-form game. The strategies of farmers are the sizes of their crops; hence the strategy set of farmer f is the set of positive numbers, which is denoted S_f. Given the profile of production decisions $s_{f'}$ ($s_{f'} \in S_{f'}$), the total crop is $\int s_{f'}\, df'$, and the equilibrium price is $p = D^{-1}(\int s_{f'}\, df')$. The payoff of farmer f as a function of the decisions of others (and of his own decision) is then

$$\left\{ D^{-1}\left(\int s_{f'}\, df' \right) \right\} s_f - C(s_f, f).$$

Given a strategy profile of others $s_{f'}$ ($f' \neq f$), the best response of farmer f is the maximand of the former expression. Note that the integral does not depend on his own action.

The farmers' problem being embedded in the normal-form framework just described, I will provide first a loose explanation and then a formal definition of the rationalizability concept.

I start with the loose explanation. As explained in the introduction, rationalizability is derived from the hypothesis of rationality and common knowledge of rationality. The implications of these hypotheses can be exhausted through the following sequence of considerations:

i. Each farmer is rational: agent f only uses strategies that are best responses to some possible profile of strategies that can actually be played by the others. Hence, rationality implies that strategies in S_f that are not best responses, in the sense just sketched, will never be played.

ii. Each farmer knows that all the other farmers are rational. Then each farmer knows the conclusion of statement (i), that the other farmers never use a (possibly) nonempty subset of their initial strategy sets. Taking that into account, farmer f may discover that some of his (remaining) strategies are no longer best responses. He will eliminate them.

iii. Each farmer knows that all farmers know that all farmers are rational.

⋮

p. Each farmer knows that all farmers know that all farmers know ... that all farmers are rational.

The following formal definition proposes a description of the iterated elimination of non-best-response strategies which has just been suggested. This definition is a variant of the one proposed by Pearce (1984).[4]

I proceed, starting from $S(0, f) \equiv \mathrm{S}_f$, to an iterated elimination of strategies that are not best responses of agents. The precise rule is described through formula (1.3)

$$S(\tau, f) = S(\tau - 1, f) \Big/ \Big\{ \bar{s} \in S(\tau - 1, f) \,|\, \bar{s} \text{ is not a "best response" to any } \prod_{f' \neq f} s_{f'} \text{ where } s_{f'} \in S(\tau - 1, f') \Big\}, \tag{1.3}$$

where τ is an index of "virtual" time or the steps of the iterative process. (Again, a strategy $\bar{s}$ is a best response for f to the strategies

$\prod_{f' \neq f} s_{f'}$ if $\bar{s}$ maximizes the utility of f among his admissible strategies when all $f' \neq f$ play $s_{f'}$.)

The set of *rationalizable strategies* is, by definition,

$$\mathfrak{R} = \prod_{f} \left(\bigcap_{\tau=0}^{+\infty} S(\tau, f) \right). \tag{1.4}$$

Again, (1.3) and (1.4) comprise only a formal restatement of the previous argument: at "time" 0 in $S(0, f)$, agent f eliminates "useless" strategies (i.e., those which are never best responses, whatever the strategies played in $\prod_{f'} S(0, f')$ by his opponents). This generates $S(1, f)$. At "time" 1, each agent knows that his opponents only play in $S(1, f')$; then he may find other "useless" strategies so that his set of "useful" strategies may shrink, and the process continues. Note that the iteration describes a mental (rather than a real) process; it takes place in people's minds. The time which is referred to is "notional" time.

A *rationalizable-expectations equilibrium* is defined as a (measurable) function $Q(f)$ of producers' supplies, where each individual strategy $Q(f)$ is rationalizable in the sense just defined.

To each rationalizable-expectations equilibrium one may associate a rationalizable-expectations equilibrium price, $p = D^{-1}(\int Q(f)\, df)$, the market-clearing price associated with the profile $Q(f)$ of rationalizable strategies.

The rationalizable-expectations equilibrium that has just been defined has to be compared with more standard concepts. A *competitive equilibrium* consists of a price $\bar{p}$ such that

$$S(\bar{p}) = D(\bar{p}). \tag{1.5}$$

A *rational-expectations equilibrium* consists of a probability distribution on $\{p\}$ denoted $d\mu\{p\}$ which is indeed generated by the market-clearing equation at time $t+1$ when it is believed by all agents at time t. Now in this model, for every probability distribution on expected prices, aggregate supply is deterministic. As there is no noise in the market-clearing equations, the market-clearing price cannot be random. The rational-expectations equilibrium is then a *perfect-foresight equilibrium*; it immediately follows that it coincides with the competitive equilibrium (which is unique with my assumptions).

It is well known and easy to check that the rational-expectations equilibrium (here the perfect-foresight equilibrium or the competitive equilibrium) is the unique Nash equilibrium of the farmers' game just described. (Note, incidentally that although I have identified strategies with production decisions, one could have identified them with the equivalent deterministic market-clearing price that each farmer expects, p_f^e, since there is a one-to-one correspondence between the two formulations.)

Also, it follows from the above definition that every Nash equilibrium is rationalizable.[5] Then, the *rational-expectations equilibrium is necessarily a rationalizable-expectations equilibrium*. I will say that an equilibrium is associated with *strongly* rational expectations if the converse holds true.

DEFINITION A strongly rational-expectations equilibrium (SREE) is a rational-expectations equilibrium that is the *unique* rationalizable-expectations equilibrium of the producers' game. Equivalently, an SREE is a rationalizable-expectations equilibrium that is unique.

The competitive equilibrium describes the usual Walrasian outcome. It could obtain either from a Walrasian tâtonnement undertaken at time t with all economic actors being present or from the computation of a perfectly informed central planning board. It insures a full coordination of plans of economic agents.

At the other extreme, the concept of a rationalizable-expectations equilibrium attempts to describe some kind of minimal coordination which can take place in the absence of an explicit coordinating institution. Farmers have to be envisioned as being isolated (e.g., in a closed room) and deciding simultaneously about the size of their crops. This is obviously an extreme situation. In counterpart I assume that a powerful mental process associated with the common knowledge of rationality can be set into action. It should be understood that this assumption goes much beyond standard individual rationality. It reflects something that can be viewed as a strong form of collective rationality.

The aim of the present chapter is to attempt to understand when the mental process of coordination which underlies the rationalizable-expectations equilibrium can reach the full-coordination outcome (or semifull coordination when the rational-expectations equilibrium does not itself achieve full coordination; cf. section 1.2). When full coordination cannot be achieved, the chapter asks what are the minimal coordinating interventions that are required. Particular emphasis will be put on the elaboration of an economic intuition concerning the factors that are favorable (or unfavorable) to "eductive" coordination.

1.1.3 Basic Insights from the Linear Model

This section provides the basic insights on what makes a rational-expectations equilibrium strongly rational. The argument describes a collective thought process, whose economic meaning is intuitive enough to be understood without full reference to the formal definition of rationalizability stated above. The argument makes clear how and to which extent elastic demand on the one hand and inelastic supply on the other hand favor "eductive" coordination.

Consider the linear version of the above model; that is, assume

$$S(p) = Cp. \tag{1.1}$$

$$D(p) = \begin{cases} A - Bp & \text{if } p \leq p_0 \equiv A/B \\ 0 & \text{otherwise.} \end{cases} \tag{1.2}$$

Then the perfect-foresight equilibrium price is $\bar{p} = A/(B + C)$. Is it an SREE? This question has here a very simple answer.

PROPOSITION 1.1 (i) $B > C \Leftrightarrow \bar{p}$ is a strongly rational-expectations equilibrium. (ii) $B \leq C \Leftrightarrow \bar{p}$ is not strongly rational, and the set of rationalizable-expectations price equilibria comprises the segment $[0, p_0]$.

Proof (i) Consider the iterative process of elimination of strategies associated with the rationalizability idea. At (notional) time 0, all agents realize that the equilibrium price cannot be higher than p_0 (since there is no demand for prices higher than p_0).[6] Then each of them deletes from his strategy set any offer $s_f \geq S(p_0, f)$. This defines $S(1, f) = [0, S(p_0, f)]\ \forall f$. From the consideration of $S(1, f')$, $f' \neq f$, every farmer f realizes that total supply cannot be greater than $\int S(p_0, f')\, df' = S(p_0)$ (note that I use here the continuum assumption which implies that each agent is infinitesimal). Then from the market-clearing equation, it follows that the equilibrium price cannot be smaller than $p_1 = D^{-1}[S(p_0)]$. Then, agent f deletes from his strategy set any offer $s_f \leq S(p_1, f)$. This leads to $S(2, f) = [S(p_1, f), S(p_2, f)]$.

It then follows that total supply cannot be smaller than $S(p_1)$ so that everybody realizes that prices cannot be higher than $D^{-1}[S(p_1)]$. The process goes on from $D^{-1}[S(p_1)]$ as it went from p_0 and leads through a new deletion of strategies to $S(3, f)$, and so on. The convergence to equilibrium is diagramed in figure 1.1.

Now, taking into account the linear structure, after changing the axis in such a way that the origin is at the equilibrium shown in figure 1.1 (prices in the new system are denoted p'), one obtains

$$p'_1 = D^{-1}(S(p'_0)) = -\frac{C}{B} p'_0,$$

$$p'_2 = D^{-1}(S(p'_1)) = \frac{C^2}{B^2} p'_0,$$

$$p'_n = (-1)^n \frac{C^n}{B^n} p'_0,$$

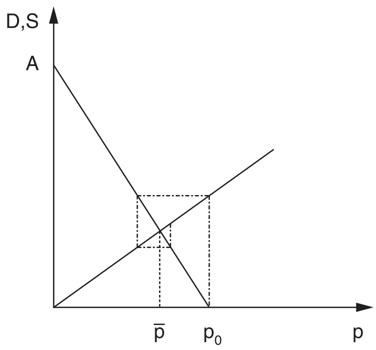

Figure 1.1
Convergence to a strongly rational-expectations equilibrium.

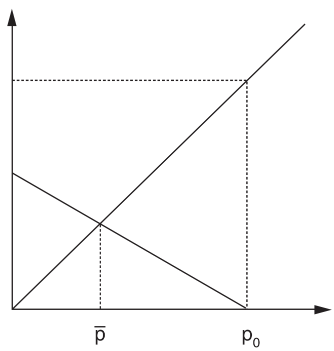

Figure 1.2
The set of rationalizable equilibria consists of $[0, p_0]$.

which is a sequence that converges to $\bar{p}' = 0$, under part (i) of the proposition.

Property (ii) is clear from figure 1.2. The first step of the above process leads to a deletion of offers $S(1, f) \subset S(0, f)$, but the second step does not. Then,

$$S(1, f) = S(2, f) = S(t, f) \cdots = \{s_f \mid s_f \leq S(p_0, f)\}. \tag{1.6}$$

It follows that any price in $[0, p_0]$ is a rationalizable price. ■

A striking feature of the above proof is the role played by the aggregate competitive supply (and demand) functions. Here, contrary to the standard textbook situation, agents do not attach any special significance to competitive data, the relevance of which is a priori dubious.

Furthermore, note that the process defined is nothing else than the familiar cobweb tâtonnement. However, here it does not take place in real time on the market place, but in "notional time" in the agent's mind.

Now one should try to get more economic intuition on why the mental process implicit to the rationalizability concept converges to the competitive equilibrium. The step that initiates the whole story is that some "bad" news from the agent's point of view (i.e., the fact that p is necessarily smaller than p_0) is known. The fact that everybody knows this and everybody knows that the others know has the happy consequence that everybody knows that supply will be lower than $S(p_0)$ and hence that prices will be higher than $D^{-1}[S(p_0)]$. Hence, initial pessimism has generated some optimism, which in turn will generate some kind of pessimism (prices cannot be greater than p_2). When does the process of alternate "optimism" and "pessimism" converge?

Figure 1.3A shows three positions of the demand curve for a fixed supply curve: in position 1 the dotted demand curve is flat; demand does not react much to prices. With such inelastic demand the competitive equilibrium is not strongly rational. Position 2 is the borderline case. When the demand curve is steeper than position 2, as it is in position 3, then the equilibrium is strongly rational. The limit case of a vertical demand curve is enlightening; in such a case, the equilibrium price $\bar{p}$ is fixed by the demand conditions, and the mental process leading to $\bar{p}$ is trivial.

Figure 1.3B considers a fixed demand curve together with a variable supply curve. With the flat supply curve (position 1), the

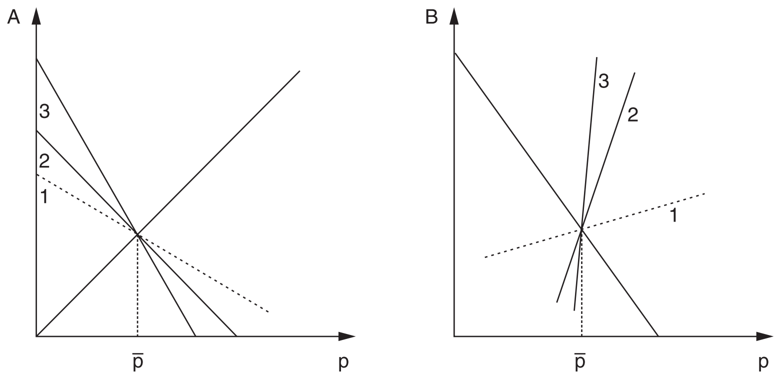

Figure 1.3
The effects of changes in (A) demand curves and (B) supply curves on the rationalizability process.

equilibrium is strongly rational, while it is not with the steep supply curve (position 3).[7]

The intuitive content of the comparative-statics exercise may be summarized as follows. With a vertical demand curve (i.e., an infinitely elastic demand around some price $\bar{p}$), the equilibrium of the system is easy to predict: it can only be $\bar{p}$. By continuity, the prediction remains fairly accurate when the demand curve is almost vertical. The argument shows that, in fact, sophisticated guessing allows the prediction to be completely accurate, even if the demand curve is far from being vertical, as soon as $B > C$.

Also, the equilibrium price of a system with a horizontal supply curve is easily predictable. In such a system the aggregate production does not depend upon price expectations: the market-clearing price only depends on demand. With an almost horizontal supply curve, there would be little uncertainty on the equilibrium price. The argument shows that, in fact, sophisticated guessing allows the prediction to be completely accurate, even if the supply curve is no longer horizontal, as soon as $C < B$.

1.2 Testing Robustness of the Basic Intuition

In this section, I will show how the basic insights of the linear case of subsection 1.1.3 extend to a nonlinear case (subsection 1.2.1). Then, I will also extend the conclusions to a noisy version of the basic model (subsection 1.2.2). Finally, I will compare briefly the "eductive-learning" viewpoint of the chapter with the more standard viewpoint of "adaptive learning" (which takes place in real time; subsection 1.2.3).

1.2.1 The Nonlinear Model

Here the demand and supply functions are no longer assumed to be linear. Aggregate demand is supposed to be decreasing on some interval $(0, p_0]$ (after p_0 it can stay at zero). Both supply and demand are continuous and, whenever necessary, differentiable. There is a unique competitive equilibrium price $\bar{p}$.

The linear case corresponds to the case in which the derivatives of supply and demand (i.e., S' and D') are constant (at least on $[0, p_0]$). Note that in this case, success of eductive coordination requires $S' < |D'|$ or $|S'/D'| < 1$. In the general case, such a condition (the derivatives being evaluated at $\bar{p}$, the competitive equilibrium) will be shown to play an important role.

As in the linear case, one can define

$$\varphi(p) = D^{-1}[S(p)]$$

and call $\varphi : p \to D^{-1}[S(p)]$ the cobweb function.

The basic argument of subsection 1.2.3 can be transposed here in order to give the following statement:

Fact 1 If it is common knowledge that the equilibrium price is smaller (greater) than p, then it is common knowledge that it is greater (smaller) than $\varphi(p)$.

The proof replicates the central argument of proposition 1.1. If it is common knowledge that the equilibrium price is smaller than p, then no farmer will supply more than $S(p, f)$ so that aggregate supply will be smaller than $\int S(p, f)\, df = S(p)$. Then the equilibrium price will be greater than $D^{-1}[S(p)] \equiv \varphi(p)$, a fact that is common knowledge.

Clearly, as in the linear case, the mental iterative process will lead to the iteration of the above statement. This suggests the following definition:

$$\varphi^2(p) = \varphi(\varphi(p))$$

$$\varphi^3(p) = \varphi(\varphi^2(p)) = \varphi^2(\varphi(p))$$

$$\vdots$$

$$\varphi^n(p) = \varphi(\varphi^{n-1}(p))$$

$$\vdots$$

where the function φ^i is the ith iteration of the cobweb function.

Iterating the above statement (fact 1), one obtains the following.

Fact 2 If it is common knowledge that the equilibrium price is smaller (larger) than p, then it is common knowledge that it is smaller (larger) than $\varphi^{2n}(p)$, $\forall n > 1$.

This statement obtains for $n = 1$ (φ^2) from the iteration of fact 1. Again, the $n = 1$ statement can be iterated n times.

The two above statements make clear that the outcome of the eductive process in the nonlinear case relates (as in the linear case) to the properties of the cobweb function φ and its iterates. A number of properties of φ or φ^2 are listed and proved in the appendix. For example φ is a decreasing function of p and satisfies $\varphi(\bar{p}) = \bar{p}$. Also, $\varphi'(\bar{p}) = (S'/D')_{(\bar{p})}$. Furthermore, $\varphi^2(p)$ is an increasing function of p; it satisfies $\varphi^2(\bar{p}) = \bar{p}$ and $(\varphi^2)'_{(\bar{p})} = [\varphi'(\bar{p})]^2$.

From fact 2, it can be seen that the success of eductive learning can be assessed entirely from the knowledge of φ^2: it is subject to two conditions:[8]

i. there exists some common-knowledge initial information on prices (for example that $p \leq p_0$);
ii. $\lim_{n\to\infty}[(\varphi^2)^n(p_0)] = \lim_{n\to\infty} \varphi^{2n}(p_0) = \bar{p}$.

Note that (i) introduces a difference with respect to the case of subsection 1.1.3. In that case the initial restriction was embedded in the definition of the problem (due to the fact that it was known that the equilibrium price could not be greater than p_0, a price from which demand was zero). This is no longer necessarily the case. Hence, the initial price restriction has to be introduced exogenously. This leads to the introduction of the concept of *credible price restriction*.

I will say that there is a *credible floor (ceiling) price restriction* $\underline{p}_0$ ($\bar{p}_0$) *if, at the moment when the agents decide, it is common knowledge that everybody believes that the price will be greater than* $\underline{p}_0$ (*smaller than* $\bar{p}_0$).

A credible price restriction may derive from the characteristics of the system as in subsection 1.1.3. More generally, strong drops in demand above some “high” prices (due to accelerated substitution) could be substitutes for credible price restrictions. However, credible restrictions can also come from a “government” credible commitment to support prices if they go below $\underline{p}_0$ or to depress them if they go above $\bar{p}_0$.

Naturally, the fact that there exists a credible price restriction makes the existence of an SREE more plausible. Formally the definition of an SREE given in section 1.1 did not refer to the possibility of a credible price restriction (which would not be embedded in the data of the economy). One has then to define an SREE subject to a credible price restriction. This is left to the reader.[9]

PROPOSITION 1.2 (i) If $|\varphi'(x)| < 1$ [i.e., if $S'(x)/|D'(S(x))| < 1$] $\forall x$ and if there is a credible price restriction (either a specified floor or a specified ceiling), then $\bar{p}$ is an SREE subject to the given price restriction. (ii) If $|\varphi'(\bar{p})| < 1$ [i.e., $S'(\bar{p}) < |D'(\bar{p})|$], there is a credible price restriction (either a floor $\underline{p}_0$ or a ceiling $\bar{p}_0$ [both $\neq \bar{p}$]) such that $\bar{p}$ is an SREE (subject to this restriction). (iii) If $|\varphi'(\bar{p})| > 1$ [i.e., $S'(\bar{p}) > |D'(\bar{p})|$] and if the graph of φ^2 intersects transversely the 45-degree line more than one time, then there exist credible restrictions (floor or ceiling) such that the set of rationalizable prices (conditional to the given price restriction) consists of the segment $[p_1, p_2]$ such that $p_2 = \varphi(p_1)$ ($\varphi^2(p_2) = p_2$, $\varphi^2(p_1) = p_1$).

The formal argument is in the appendix. It can be intuitively understood from figure 1.4 where different forms of the second iterate of the cobweb function are depicted. Case (i) of proposition 1.2 is depicted in figure 1.4A. In case (i), the slope of φ^2 is always smaller than 1. This will hold if, as assumed here, the slope of φ in x [i.e., $S'(x)/D'(S(x))$] is always smaller than 1 in absolute value. In such a case, the sequence $\varphi^2(p_0), \varphi^4(p_0), \ldots, (\varphi^2)^n(p_0)$ will converge to $\bar{p}$, whatever the starting point p_0. Any floor or ceiling restriction will provide such a starting point for the eductive process.

Statement (ii) corresponds to figure 1.4B. The shaded area around $\bar{p}$ on the horizontal axis is the "basin of attraction" of $\bar{p}$ (for φ or φ^2)

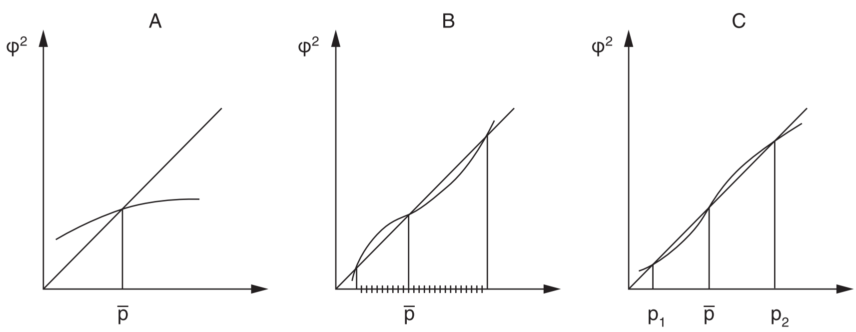

Figure 1.4
(A) The equilibrium is (globally) strongly rational; (B) the equilibrium is (locally) strongly rational; (C) the set of rationalizable-expectations equilibria is a connected segment.

[i.e., the set of p_0 such that $\varphi^{2n}(p_0) \to \bar{p}$]. Starting from some price restriction p_0 when p_0 belongs to this area (a ceiling restriction if $p_0 > \bar{p}$; a floor restriction if $p_0 < \bar{p}$), the iterative mental process described above will converge to $\bar{p}$. Note that the condition $S'(\bar{p}) < |D'(\bar{p})|$, a local condition around $\bar{p}$, does coincide in the linear case of subsection 1.2.3 with the global condition $C < B$.

Finally statement (iii) corresponds to figure 1.4C. In this case, the iteration of any point in the interval $[p_1, p_2]$ where p_1 and p_2 are values where $\varphi^2 = 0$ (i.e., cycles of order two of φ) converges either to p_1 or p_2 and not to $\bar{p}$. Statement (iii) says that the whole interval can be viewed as a set of rationalizable price equilibria, at least when adequate initial price restrictions (outside $[p_1, p_2]$) are given.

The results have the same flavor as those of proposition 1.1. In the linear case, the global condition and the local condition coincide and reduce to $S' < |D'|$ or, with the notation of subsection 1.1.3, to $C < B$. However, proposition 1.2 stresses some important aspects of the general case.

First, even in good cases, in which the cobweb tâtonnement converges whatever its starting point [case (i)], a credible price restriction may be needed in order to obtain an SREE. However, the price restriction required to initiate the elimination process may be in some sense as "innocuous" as desired. For example, the ceiling restriction may be very high.[10]

Second, part (ii) of proposition 1.2 puts additional emphasis on the role that exogenous credible price restrictions may have in insuring the stabilization of beliefs toward the price equilibrium. However, since $S' < |D'|$ holds at equilibrium, but not necessarily in a large interval around it, the required price restriction may have to be tight and even close to the equilibrium price. (Note that a floor and a ceiling restriction are not simultaneously needed. One of them is sufficient in the present model.)

Finally, in the case of part (iii) of proposition 1.2, no price restriction but the trivial restrictions that the floor and the ceiling equal the equilibrium price can assure that the equilibrium is an SREE. However, part (iii) of proposition 1.2 identifies sets of rationalizable prices (included within the cycles of order two of the cobweb function) which are closest to the equilibrium price.

1.2.2 A "Noisy" Version of the Basic Model

I am going to consider in this section that supply and demand are affected by noise. The noise on supply is associated with the discrepancy between the initial production decision of the agents and the actual volume of the crop that is affected, for example, by random climatic events. The noise on demand reflects the fact that the demand curve is not exactly known at the decision time.

The most obvious consequence of the introduction of noise is that the perfect-foresight assumption has to be given up. The equilibria of interest are nondegenerate rational-expectations equilibria. Hence, as noted in section 1.1, the supply function of farmers should depend a priori on the whole distribution of prices and be denoted $\tilde{S}$. Having that in mind, one can formulate the simplifying assumptions that are made in this section.

ASSUMPTION 1.1 (on the noisy model)

i. $\tilde{S}(\,\cdot\,) = S(E(p)) - \varepsilon_0$

ii. $D(p) = \begin{cases} A - Bp - \varepsilon_D & \text{if } p \leq (A - \varepsilon_D)/B \\ 0 & \text{otherwise} \end{cases}$

iii. ε_0 and ε_D are independent random variables of zero mean. (iv) If $\bar{p}$ is the perfect-foresight price equilibrium of the system, then with probability 1, $S(\bar{p}) - \varepsilon_0 + \varepsilon_D$ is strictly positive, and $S(0) - \varepsilon_0 + \varepsilon_D$ is strictly smaller than A.

Besides the technical requirements of part (iv) (which requires a noise with compact support) assumption 1.1 has three main implications. First the noise under consideration is additive. Second, producers are risk-neutral; their decision only depends on the expected value of prices. Third, demand is linear as in section 1.1. The assumptions are intended to facilitate the analysis. It will be rather easy to assess the direction in which their relaxation modifies the analysis.

With the above assumption, the rational-expectations equilibrium can be computed. Ignoring boundary problems (caused by the kink in demand when demand becomes zero) the random price equilibrium should satisfy

$$A - B\tilde{p} = S(E(\tilde{p})) - \varepsilon_D + \varepsilon_0. \tag{1.7}$$

Taking expectations on both sides leads to

$$E(\tilde{p}) = \bar{p}, \tag{1.8}$$

$$\tilde{p} = \bar{p} - \frac{\varepsilon_0 - \varepsilon_D}{B}. \tag{1.9}$$

From part (iv) of assumption 1.1, the boundary problems do not arise for the candidate price equilibrium defined by (1.9). This shows that (1.9) indeed defines the rational-expectations equilibrium.

One of the reasons why the introduction of noise in the model is interesting is that it brings into the picture ingredients that allow a more realistic analysis of the conditions in which a "government" would intervene through credible price restrictions. Previously, the announcement of a price restriction, if it were found credible by the "farmers" and sufficiently well chosen to induce convergence of beliefs toward the SREE, had no cost for the government. Although the price restriction had to be more or less severe [a ceiling restriction, however high, was sufficient for case (i); the restriction was more or less severe according to the size of the basin of attraction in case (ii); in case (iii), a *simultaneous* extremely severe restriction (i.e., setting simultaneously $\underline{p}_0 = \bar{p}$, $\bar{p}_0 = \bar{p}$) was required for coordination on $\bar{p}$], the *ex post* cost of these price restrictions was zero. For example, the price support required for a floor intervention, which requires that the government buy the product when its price falls below the floor, never had to be exercised. Here, when noise is introduced, a price restriction has a cost in the sense that even if it leads to an adequate coordination of beliefs it has to be enforced in some events (e.g., when demand is low for a floor restriction).

Then, in the present model, stabilization of beliefs through price restrictions induces a cost of intervention (whatever the precise way it is defined). This cost of intervention can also be viewed as a cost of stabilization of beliefs which differs according to the characteristics of supply and demand in the situation under consideration. The present theory should allow situations to be ranked according to the costs of stabilization that they require.

Notice a first difficulty: a credible price restriction, since it affects the distribution of prices associated with a rational-expectations equilibrium, in general affects the mean of the rational-expectations equilibrium and then the associated supply decisions. There are two ways of dealing with such a difficulty. The first consists of accepting the change in the producers' actions induced by price restrictions and introducing the social cost induced by this change in the cost–benefit analysis of "stabilization" policies. This is the more satisfactory procedure, but its implementation in the present context would require a more careful specification of the demand side of the model (in order to analyze the social costs of price changes) and would increase complexity more than in proportion with the additional insights it would allow. The second procedure, the one adopted here, involves restricting attention to credible price policies that lead to full stabilization in the sense that they make the rational-expectations equilibrium an SREE without modifying the mean of the price distribution. This imposes simultaneously a floor restriction $\underline{p}_0$ and a ceiling restriction $\bar{p}_0$ in such a way that the truncation of the rational-expectations equilibrium distribution induced by these restrictions leaves the mean price unaffected.

Formally, let $\bar{p}_0 > \bar{p}$ and define $\nu(\bar{p}_0)$ through the following formula

$$\nu(\bar{p}_0)\int_{-\infty}^{\nu(\bar{p}_0)-\bar{p}} dF(x) + \int_{\nu(\bar{p}_0)-\bar{p}}^{\bar{p}_0-\bar{p}} (\bar{p}+x)\, dF(x) + \bar{p}_0 \int_{\bar{p}_0-\bar{p}}^{+\infty} dF(x) = \bar{p}, \quad (1.10)$$

where dF is the density function of the random variable $\varepsilon \equiv (\varepsilon_D - \varepsilon_0)/B$. Assuming that this random variable has a positive density on the interior of its support, this formula defines a unique $\nu(p_0)$.

Then, a neutral pair of price restrictions consists of a ceiling restriction $\bar{p}_0$ and a floor restriction $\underline{p}_0$ such that

$$\underline{p}_0 = \nu(\bar{p}_0). \quad (1.11)$$

As ν is strictly decreasing in p_0, one can also write (10) as

$$\bar{p}_0 = \nu^{-1}(\underline{p}_0). \quad (1.11')$$

Now consider the "product mapping" $\varphi = D^{-1} \circ S$ (i.e., the cobweb function, as defined in section 1.2) for the nonnoisy system.

PROPOSITION 1.3 Assume the four parts of assumption 1.1. Then (i) the rational-expectations price equilibrium is the random variable

$$\tilde{p} = \bar{p} + \frac{\varepsilon_0 - \varepsilon_D}{B}, \quad E(\tilde{p}) = \bar{p}. \quad (1.9)$$

(ii) Consider φ, the (deterministic) cobweb function and suppose

$$\varphi'(\bar{p}) < 1$$

(i.e., $S' < |D'|$). Then there is a connected set $V(\bar{p})$ such that if $(\underline{p}_0, \bar{p}_0)$ is a pair of neutral restrictions and if either $\underline{p}_0$ or $\bar{p}_0$ or both belong to $V(\bar{p})$, the rational-expectations equilibrium associated with the price distribution

$$
\begin{aligned}
\tilde{p} &= \bar{p} + \frac{\varepsilon_0 - \varepsilon_D}{B} && \text{if } \underline{p}_0 \leq \bar{p} + \frac{\varepsilon_0 - \varepsilon_D}{B} \leq \bar{p}_0 \\
&= \underline{p}_0 && \text{if } \bar{p} + \frac{\varepsilon_0 - \varepsilon_D}{B} < \underline{p}_0 \\
&= \bar{p}_0 && \text{if } \bar{p} + \frac{\varepsilon_0 - \varepsilon_D}{B} > \bar{p}_0
\end{aligned}
$$

is a strongly rational-expectations equilibrium conditional on the given credible neutral price restriction.

The proof of proposition 1.3 is in the appendix. It should be mentioned here, however, that the central argument of the proof involves proving the following assertion:

If at some stage of the rationalizability process, it is common knowledge that the *expected value* of equilibrium price $E(p)$ must be smaller than p_0 (with $\bar{p} \leq p_0$), then it is common knowledge (at the next step of the process) that $E(p)$ must be greater than $\varphi(p_0)$ where φ is the *deterministic* cobweb function.

Proposition 1.3 provides a neat extension of proposition 1.2. It says that things are not basically different without noise than with noise. In both cases, the success of eductive coordination depends on the convergence of sequences $\varphi^n(p_0)$, where φ is the same deterministic cobweb function. There are however two differences.

On the one hand, in a noisy system, the credibility of price restrictions requires intervention, and intervention will modify the distribution of the equilibrium price. Hence, in order to preserve the mean of the price distribution, one ceiling and one floor restrictions have to be set simultaneously.

On the other hand, it is noteworthy that the presence of noise together with the existence of a neutral couple of credible restrictions, rather than a single one, makes the convergence of the iterative mental process of rationalizability faster when noise is considered than when it is ruled out. The reason is that, in case of noise, the floor restriction (ceiling) generates an upward (downward) move of expectations, when compared with what happens in the nonnoisy case, in the iteration step starting from the statement that $E(p) \leq p_0$ ($\geq p_0$). This move is positively correlated with the variance of the noise.[11]

The essence of the argument presented until now is that, in the absence of explicit institutions of coordination, sophisticated agents face difficulties in forecasting future prices. It is a conclusion of the analysis that, even in very favorable cases when the cobweb function converges whatever the starting point, a minimum outside intervention may be needed to initiate a mental process converging towards the rational-expectations beliefs [see part (i) of proposition 1.2]. I have called "government" the outside agent providing the minimal coordinating signal. The present section has associated the emission of the coordinating signal with an active intervention aimed at maintaining the credibility of the signal. The fact that intervention is normally costly suggests defining a cost of stabilization which would measure the cost of "price stabilization" required to guarantee the eductive coordination of beliefs in a given system.

1.2.3 Some Remarks on Evolutive versus Eductive Learning

In this section, as well as in the following, I return to the nonnoisy model studied in section 1.2. As stressed in the introduction, the justification of the rational-expectations hypothesis I focus upon is "eductive" in the sense that it relies on mental activities of agents. In this section I am considering justifications that are "evolutive" in Binmore's terminology (i.e., they are based on real-time "learning" activities).

First note that an evolutive explanation requires repetition. I will suppose that the "game" played by the farmers and described in section 1.1 is repeated T times (the economy has $2T$ subperiods, since the initial period was subdivided into two subperiods). I assume that the t-period version of the game is entirely analogous to the one period version. This means more precisely that I assume *stationarity of the data* and *absence of inventories*: neither the farmers, nor the consumers are allowed to hold inventories. Without this assumption the present conditions of the economy could depend on its entire history, and the problem would exhibit memory.

In this sequential setting, the most common extension of the rationalizability concept to extensive-form games (i.e., in the spirit of Pearce [1984]) would lead to the same conclusion as in the static case: the sequence of rationalizable-expectations equilibria would only be the repetition of the static rationalizable-expectations equilibria. Therefore, I will consider some "evolutive" learning process taking place in real time; for example the standard *adaptive learning* rule:

$$p^{\mathrm{e}}_{t/(t+1)} = \alpha p^{\mathrm{e}}_{(t-1)/t} + (1-\alpha) p_t.$$

The price forecast for tomorrow (common to all agents) $p^{\mathrm{e}}_{t/(t+1)}$ is a convex combination of the forecast of yesterday $p^{\mathrm{e}}_{(t-1)/t}$ and the realization today. This rule describes a revision procedure that puts more or less weight on the present realization according to the value of α. It has no "full rationality" justification but can rather be viewed as reflecting a bounded rationality approach to the coordination problem.

What relationship is there, if any, between the convergence (in real time) of the adaptive learning rule and the convergence (in notional time) of the eductive process mentioned above? The answer is formally stated in proposition 1.4 and is based on the following remarks:

i. For $\alpha = 0$, starting from the same initial p_0, both processes lead to the same sequence of prices; one merely has two different interpretations of the cobweb tâtonnement.

ii. When the process converges for $\alpha = 0$, it also converges for any $\alpha \in [0, 1)$.

PROPOSITION 1.4 A competitive equilibrium $\bar{p}$ is an SREE conditional on a ceiling price restriction $p \leq \bar{p}_0$ if and only if every α adaptive learning process (for $\alpha \in [0, 1)$) starting from $\bar{p}_0$ converges to it.[12]

Proposition 1.4 puts the emphasis on the fact that, when success is required for every possible α, the conditions of (instantaneous) success of eductive learning or (asymptotic) success of adaptive learning are the same. The rationale for the comparison made in proposition 1.4 is that, as the choice of α is ad hoc, the convergence of the evolutive learning process will be certain only if it is known to occur for every α.

Naturally, proposition 1.4 can also be read in a different way: the "success" of eductive learning implies the "success" of adaptive learning ($\forall\alpha$), but the converse (i.e., the success of adaptive learning for some α) does not imply the success of eductive learning.

1.3 Extending the Initial Framework

In this section, I extend the basic analysis in two directions. First, I suppose that the timing of decisions is sequential (instead of simultaneous). This situation seems to be favorable to the eductive coordination of beliefs, since agents who have to decide later can take advantage of actual observations of previous decisions. The conclusion of subsection 1.3.1 strongly supports this conjecture. Second, I introduce market interdependencies. The previous analysis suggests that too strong market interdependencies are likely to disturb eductive learning. Again the analysis supports this intuition and gives it a clear-cut formulation in the two models considered in subsection 1.3.2.

1.3.1 Sequential Timing as an Argument for the Eductive Validity of the Rational-Expectations Hypothesis

In this subsection, I still consider the model of section 1.1 but modify it in the following way. During the first subperiod of the game, decisions are made at two different moments. First, half of the producers decide, given the same implicit conditions as in section 1.1. Then, *observing the decision* of the first half, the second half makes a decision, again given the same conditions, but with the additional knowledge of the actual decision made by the first group. The new procedure may describe a situation in which, as in Muth, agents are farmers but there are two types of wheat which are perfect substitutes but are planted at two different periods of the year. This situation is illustrated by the option that is available in some regions of having winter wheat and spring wheat.[13]

As argued in the introduction, the two-step procedure under consideration is "favorable" to the "eductive" coordination I am concerned with. Proposition 1.5 indeed supports this intuition.

PROPOSITION 1.5 Assume that at the competitive equilibrium $\bar{p}$, $|S'/D'| < 2$. Then, in the two-step procedure just described, one can find a pair of ceiling and floor restrictions $(\bar{p}_0, \underline{p}_0)$ $(\bar{p}_0 > \bar{p}$, $\underline{p}_0 < \bar{p})$ such that the equilibrium is an SREE conditional on these restrictions.

Before the proof, some comments are in order. In the original model, with a one-step decision procedure, conclusions similar to the conclusion of proposition 1.5 obtain for $|S'/D'| < 1$. The present criterion $|S'/D'| < 2$, which can also be viewed as the condition for strong rationalizability of the competitive equilibrium in the economy $(S/2, D)$ (i.e., in an economy with a flatter supply curve more favorable to eductive coordination; see discussion in section 1.2), is much weaker. The set of economies for which some kind of eductive coordination (obtaining an SREE conditional to a nontrivial price restriction) can be expected is significantly enlarged in passing from a one-step procedure to a two-step procedure.

For the proof, I shall present an informal sketch only (the rigorous argument being slightly tedious and consuming more space).

Proof of Proposition 1.5 (sketch) i. Consider the function $\tilde{\varphi}(p) = D^{-1} \circ \{[S(\bar{p}) + S(p)]/2\}$. Then $\tilde{\varphi}'(\bar{p}) = (S'/2D')$. This shows that the hypothesis assures that the basin of attraction of $\bar{p}$ in the economy whose demand curve is $D(p)$ and whose supply curve is $S(\bar{p})/2 + S(p)/2$ is nonempty. Note that by continuity this nonemptiness property remains true in an economy $(D, K + S/2)$ when K is close to $S(\bar{p})/2$.

Then choose $\underline{p}_0$ and $\bar{p}_0$ ($\underline{p}_0 \neq \bar{p}, \bar{p}_0 \neq \bar{p}$) such that when $K \in [S(\underline{p}_0)/2, S(\bar{p}_0)/2]$, either $\underline{p}_0$ or $\bar{p}_0$ is in the basin of attraction $B(K)$ of the equilibrium of the economy $[D, K + (S/2))$. The fact that one can do that is intuitively plausible and can be deduced from the local continuity of $B(K)$ which implies that $\bigcap_K B(K)$ for K close enough to $S(\bar{p})/2$ is of nonempty interior.

ii. Consider the "mental" process of a member of the group of "farmers" who have to decide first. Starting from price restrictions consisting of $\underline{p}_0$ and $\bar{p}_0$, it is common knowledge that the supply (over the two periods) will be between $S(\underline{p}_0)$ and $S(\bar{p}_0)$. Then, it is common knowledge that farmers having to decide at the second step will observe a first-step supply between $S(\bar{p}_0)/2$ and $S(\underline{p}_0)/2$.

iii. When K is within the bounds just defined, the "economy" $(D, K + S/2)$ has an equilibrium that is an SREE conditional to a restriction (either $\underline{p}_0$ or $\bar{p}_0$). Farmers in the group having to decide first know that the equilibrium price will not be smaller than $\underline{p}_1$ where $\underline{p}_1$ is defined by

$$\frac{S(\bar{p}_0)}{2} + \frac{S(\underline{p}_1)}{2} = D(\underline{p}_1).$$

iv. The continuation of the above argument generates a sequence of lower and upper bounds for prices $(\underline{p}_n, \bar{p}_n)$ (which are common knowledge) such that

$$S(\bar{p}_{n-1}) + S(\underline{p}_n) = 2D(\underline{p}_n);$$

$$S(\underline{p}_{n-1}) + S(\bar{p}_n) = 2D(\bar{p}_n).$$

This dynamic system whose current state is $(\underline{p}_n, \bar{p}_n)$ has a fixed point $(\bar{p}, \bar{p})$; the dynamics of the system around this point is governed by $[S'/(S' - 2D')]^2$. The system is converging for a starting point close enough to $\bar{p}$ whenever $-1 \leq S'/(S' - 2D') \leq +1$, which is always the case. ■

Two remarks are in order. First, the careful reader will have noticed that the requirement of having a couple of price restrictions, rather than a single one, is not really needed. It only provides a more symmetrical treatment (and also allows a $\bar{p}_0$ farther away from $\bar{p}$ than it would be in the case of a single ceiling restriction). Second, the argument suggests that the result could be extended to an n-step procedure where $1/n$ of the farmers decide sequentially on (observable) crops with the condition $|S'/nD'| < 1$ [instead of $(S'/2D') < 1$]. I have not proved the conjecture, although an induction argument seems to work.

1.3.2 On Eductive Justification of the Rational-Expectations Hypothesis for Multidimensional Production Decisions

I now introduce two variables into the model. To illustrate the new version of the model in terms of the Muthian story of farmers, let the farmer's choice concern two different crops, wheat and corn. The previous model is unaffected, except for the definition of individual and aggregate supply and aggregate demand functions.

Competitive aggregate supply when all agents have common point expectations of prices tomorrow (p_1^e, p_2^e) is a two-dimensional vector denoted $\mathbf{S}(p_1^e, p_2^e)$ where

$$\mathbf{S}(p_1^e, p_2^e) = \begin{bmatrix} S_1(p_1^e, p_2^e) \\ S_2(p_1^e, p_2^e) \end{bmatrix}.$$

From now on, I assume that the supply, which comes from a continuum of *identical* agents, function is differentiable and that the standard symmetry conditions of cross-derivatives hold, which I denote as

$$S'_{12} = S'_{21}.$$

The demand function is also assumed to be differentiable. For the sake of simplicity I will assume in the first part of this section that there are no cross price effects in demand:

independent demand (ID):

$$\mathbf{D}(p_1, p_2) = \begin{bmatrix} D_1(p_1) \\ D_2(p_2) \end{bmatrix}.$$

In the present context, a price restriction consists of a subset of $\mathbb{R}_+^2$ of “forbidden prices.” Equivalently, I will associate such a price restriction with the complement of such a subset (i.e., with the set R of “authorized” prices).[14]

I say that R is a *nontrivial price restriction* if R has a nonempty interior in $\mathbb{R}_+^2$. This definition generalizes the previous one and is intuitively appealing. Price restrictions leading to a set of “authorized” prices of measure zero (consisting of either the competitive equilibrium or a curve in $\mathbb{R}_+^2$) are clearly too severe and are likely to make conditional rationalizability, as defined below, a trivial phenomenon.

In the present system, the definition of a competitive equilibrium is formally similar to that of section 1.1, but $\bar{p}$ is now a vector ($\bar{\mathbf{p}}$), the price equilibrium vector $[\bar{p}_1, \bar{p}_2]$. The competitive equilibrium $\bar{\mathbf{p}}$ is an SREE conditional on the credible price restriction R, if it is the unique rationalizable outcome of the game played by the agents when initially it is common knowledge that the price cannot belong to R^C, the complement of R.

Now, I can define again the cobweb function $\varphi = D^{-1} \circ S$. The next proposition shows that this function is again relevant for the study of strongly rational-expectations equilibria.

PROPOSITION 1.6 The competitive equilibrium is an SREE conditional to a local credible price restriction R, if and only if the cobweb tâtonnement $p_n = \varphi(p_{n-1})$ converges to $\bar{p}$, whatever the initial starting point p_0 in R.

Proof (sketch) Note that agents are identical and the restriction local, and reformulate the proof of proposition 1.1 (see also chapter 2). Consider the iterative mental process. Call p, the market-clearing price:

i. Initially, it is common knowledge that $p \in$ R.

ii. At the first step, it is common knowledge that $p \in \varphi$(R).

n. At the *n*th step, it is common knowledge that $p \in \varphi^n$(R).

The conclusion would follow then (for example) from lemma 1 in Moulin (1984).[15] ■

It follows that a competitive equilibrium will be an SREE for some nontrivial price restriction if and only if the cobweb tâtonnement is locally asymptotically stable.

The next proposition provides conditions guaranteeing such a property.

PROPOSITION 1.7 Assume independent demand (ID) and also assume that at the competitive equilibrium, $[\bar{p}_1, \bar{p}_2]$, the following holds true: $|S'_{11}/D'_1| = |S'_{22}/D'_2| = k$.

If

$$\left[\frac{|S'_{12}|}{\sqrt{D'_1 D'_2}}\right]_{\bar{p}} \leq 1 - k \tag{1.12}$$

then there exists a nontrivial price restriction such that the competitive equilibrium is an SREE conditional on this price restriction.

Proof According to proposition 1.6 and the general results concerning dynamical systems, the desired property is equivalent to the fact that the Jacobian matrix $(\nabla\varphi)_{\bar{p}}$ (where $\varphi = D^{-1}S$) has eigenvalues of norm smaller than 1.

This Jacobian is computed as follows:

$$(\nabla\varphi)_{\bar{p}} = \begin{bmatrix} \dfrac{1}{D'_1} & 0 \\ 0 & \dfrac{1}{D'_2} \end{bmatrix} \begin{bmatrix} S'_{11} & S'_{12} \\ S'_{21} & S'_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{S'_{11}}{D'_1} & \dfrac{S'_{12}}{D'_1} \\ \dfrac{S'_{21}}{D'_2} & \dfrac{S'_{22}}{D'_2} \end{bmatrix}.$$

The sum of the roots of the characteristic polynomial is $(S'_{11}/D'_1) + (S'_{22}/D'_2)$, and the product is $(S'_{11}/D'_1) + (S'_{22}/D'_2) - [(S'_{12})^2/D'_1 D'_2]$. Since $S'_{11}/D'_1 = S'_{22}/D'_2 = -k$, the roots are necessarily of the form $-k - \alpha$ and $-k + \alpha$, and their product is $k^2 - \alpha^2$. This implies

$$k^2 - \alpha^2 = k^2 - \frac{(S'_{12})^2}{D'_1 D'_2} \Leftrightarrow \alpha = \pm \frac{|S'_{12}|}{\sqrt{D'_1 D'_2}}.$$

Then, inequality (1.12) insures that both roots are smaller in norm than 1. ■

A more general statement could be given, wherein one would not assume either independence of demand or that $|S'_{11}/D'_1| = |S'_{22}/D'_2|$. However, I have chosen to stress the above result because it captures neatly the economic intuition.

The term $|S'_{11}/D'_1|$ or $|S'_{22}/D'_2|$ relates to the "eductive stability" of markets 1 and 2, as it can be assessed from proposition 1.1, 1.2, or

1.3 if these markets were "separated." The index $(1-k)$, which is positively correlated with the asymptotic speed of convergence of the eductive process *in each of the "separated" one-good systems* (which I have assumed to be the same in both markets), can be viewed as the index of eductive stability of the system when cross effects of supply are ignored. Now $|S'_{12}|/\sqrt{D'_1 D'_2}$ is an index that measures the intensity of cross effects of prices on aggregate competitive supply (where it is normalized by reference to the demand effects). Then, formula (1.12) can be read, roughly speaking as follows:

When the "normalized" cross price effects are smaller than the eductive index of stability common to each one-dimensional market, then there exists adequate price restriction to generate an SREE.

Roughly speaking again, eductive stability of the two-dimensional system increases with the specific eductive stability of the two markets considered separately and decreases when cross effects of prices on production decisions increase. The fact that such cross effects are destabilizing fits well with the intuition built from proposition 1.1 where it was argued that sensitivity of production decisions to prices was the crucial destabilizing factor in the one-dimensional system.

Now consider the case symmetric to the preceding one. Assume independence of supply (IS):

$$\mathbf{S}(p) = \begin{bmatrix} S_1(p_1) \\ S_2(p_2) \end{bmatrix}.$$

Demand now depends on both p_1, and p_2. This case illustrates competition between producers in a case of product differentiation: producers either produce product 1 or product 2; they care about the price of their own product only.

Proposition 1.7 has a counterpart, which is easier to write when demand is symmetric (i.e., $D'_{12} = D'_{21}$).[16]

PROPOSITION 1.8 Assume IS and suppose that at the competitive equilibrium $\bar{\mathbf{p}}$, $|S'_1/D'_{11}| = |S'_2/D'_{22}| \equiv k$. Let λ be the value of $(D'_{12})^2/D'_{11}D'_{22}$ in $\bar{\mathbf{p}}$. Then if $\lambda \leq \lambda^{\text{lim}}(k)$ where $\lambda^{\text{lim}}(k)$ is a number that decreases with k [and such that $\lambda^{\text{lim}}(0) = 1$, $\lambda^{\text{lim}}(1) = 0$] the equilibrium is an SREE for some nontrivial price restriction.

Proof (sketch) The proof parallels the one of proposition 1.7. One solves for the Jacobian matrix of φ:

$$(\nabla\varphi) = \frac{1}{\Delta}\begin{bmatrix} D'_{22}S'_1 & -D'_{12}S'_2 \\ -D'_{12}S'_1 & D'_{11}S'_2 \end{bmatrix}$$

with $\Delta = D'_{11}D'_{22} - (D'_{12})^2$. Using notation k and λ one finds that the sum of the roots of the characteristic polynomial is $-(2k/1 - \lambda')$, and the product is $[k^2/(1-\lambda)^2] - [\lambda k^2/(1-\lambda)^2]$. Both roots are smaller than 1 if $+k/(1-\lambda) + \sqrt{\lambda}/(1-\lambda)k \leq 1$ [i.e., $k(1+\sqrt{\lambda}) \leq 1 - \lambda$]. Considering the graph of the functions of λ on both sides, one sees that the inequality holds for $\lambda \leq \lambda^{\text{lim}}(k)$ where the function satisfies the conditions of the theorem. ■

Once again, the statement illustrates ideas with flavor analogous to the ones stressed previously. It is easy to check that when $D'_{12} = 0$, the conclusion of the theorem holds true for $k \leq 1$. This is not surprising because one obtains the standard result of section 1.2 for independent markets. An increase of D'_{12} or of λ is associated with less product differentiation. It affects negatively the stability of eductive learning. Put another way, *increasing product differentiation increases the success of eductive learning*. This is consistent with previous intuition again.

The results of propositions 1.5, 1.7, and 1.8 are, in some sense, the most striking in the chapter. The general questions that I try to analyze with the concepts introduced here could be formulated in a very loose way as "how difficult is coordination of beliefs in an economic system?" The nature of the results obtained in propositions 1.5, 1.7, and 1.8 suggests that the attempts at giving a precise meaning to the above question have been successful. After all, if a criterion of "coordination difficulties" can be defined, it should intuitively lead to comparative-statics properties in line with those of the above propositions: *more observability is better for coordination; also less production substitutability and more product differentiation are better for coordination*.

1.4 Conclusion

1.4.1 Previous Literature

The connections with the game theoretical literature have already been mentioned, and some of the methods used for the present analysis have general counterparts which have been assessed in the work of Bernheim (1984) and Pearce (1984). However, the present chapter is more closely related to the part of the economic literature that uses rationalizability ideas for the analysis of coordination. The work of John Bryant (1987) is a typical example focusing on macroeconomic coordination issues. The contributions closest in spirit to the present chapter are those of Gabay and Moulin (1980), Moulin (1984), and Bernheim (1982). Gabay and Moulin (1980) and then Moulin (1984) have analyzed what they called "dominance solvability" in the Cournot-oligopoly problem; in spite of the difference of framework and concept, several results of sections 1.1 and 1.3 are closely related to Moulin's (1984) results. The second chapter in Bernheim's (1982) thesis, whose objectives are similar to the objectives of the present chapter, puts emphasis on negative results. To the best of my understanding his findings have no significant overlap with the results presented here. A paper by Robert Townsend (1978), although it focuses on a different and somewhat more complex problem (Bayesian learning of an unknown parameter), provides early insights related to the focus of the present chapter (the game of guessing, second-guessing, etc.).

Finally, it should be noted that "local" rationalizability as defined and analyzed in the microeconomic setting under consideration has a close connection with the concept of expectational stability proposed by Robert Lucas (1978) and S. DeCanio (1979), which has been analyzed in different macroeconomic contexts by George Evans (e.g., Evans, 1985). Expectational stability has been criticized for lacking conceptual foundations (see e.g., Guillermo Calvo, 1983).[17] The present chapter is mainly concerned with such conceptual foundations. Its analysis could be routinely extended to contexts in which expectational stability has been considered. I conjecture that such transposition of the game-theoretical perspective of the present study to these other settings would prove fruitful by providing different insights and suggesting different questions.

1.4.2 Limitations of the Present Framework

The stylized framework adopted for the analysis may raise objections, which I must now discuss. The first criticism involves the stylized (and extreme) description of eductive learning provided by the rationalizability construct. Such a criticism suggests two remarks. On the one hand, if the intention of the objection is to promote evolutive approaches, it is particularly ill-founded in view of proposition 1.4. In the present context, the eductive approach can be viewed as the ultimate phase of demanding evolutive studies that face the arbitrariness of learning processes. On the other hand, if one should accept the fact that rationalizability provides an idealized picture of mental processes at work when eductive learning takes place, it should also be understood (a) that eductive learning is likely to take place in actual situations and (b) that forces at work are likely to go in the direction suggested by the above analysis (although admittedly less far). For example it does not seem absurd to expect one, two, or three steps of the cobweb tâtonnement analyzed here to be a reasonable approximation of mental activity. Naturally the general proviso affecting "out-of-equilibrium" learning studies (i.e., that they are more or less relevant according to the historical starting point of the system) applies here.

The second objection concerns the model. Clearly, extreme assumptions are adopted. However the phenomenon that I want to analyze (i.e., the fact that economic agents have to base decisions on guesses concerning variables that are ultimately influenced by these decisions) is present in many contexts of interest for economics. Although it is often the case, for example, that information concerning bounds on tomorrow's prices which are tighter than those considered in the present model can be obtained from other existing institutions (side markets, etc.), the forces at work in the present analysis are superimposed with others, but not suppressed.

1.4.3 Summary and Suggestions for Future Work

Naturally, further work is required to test the robustness of present conclusions to more general settings or to assess their relevance in somewhat different contexts. However, it is possible to summarize the present findings in terms that go somewhat beyond the formal statements proved in the chapter. Going from the less speculative to the more speculative the present analysis suggests the following conjectures.

First, the efficiency of eductive coordination in the type of situations considered here is affected by the characteristics of demand and supply. It is favored by elastic future demand; in contrast, it is negatively affected by the sensitivity of supply to price expectations, a fact that is likely to occur in some "speculative" asset markets.

Second, the efficiency of eductive coordination is affected by the characteristics of the decision process of the agents: it is improved when decisions are sequential and observable; it is favored in a multicommodity world by market independence and adversely affected by market interaction coming either from supply or demand. Increase in noise in the system may have ambiguous effects.

Third, there may be a general connection between the conditions of success for eductive and evolutive learning. In general, the failure of eductive learning may imply that evolutive convergence is fragile.

Fourth, it is possible that price-stabilization policies favor coordination of beliefs. Naturally the simplistic model under consideration in the chapter does not recognize any negative effect of price stabilization. The conjecture that such policies generally have positive effects when coordination problems alone are taken into account seems reasonable and worth investigating further.

Fifth, market structures and more generally institutions sometimes acutely affect the conditions of eductive coordination of beliefs. The present analysis suggests that changes in the market structure (e.g., the opening of a new market) or institutional changes affecting the validity (either from an eductive or an evolutive viewpoint) of the rational-expectations hypothesis should be more systematically appraised from the theoretical viewpoint.

Appendix

I assume here that (i) the supply function is continuous, (weakly) increasing, and strictly positive for $p > 0$ and (ii) the demand function is continuous and strictly decreasing, at least on some interval $(0, p_0)$, outside of which it may be zero. The following lemmas are easily proved.

LEMMA 1.A.1 The function φ has the following three properties:

i. $\varphi(\bar{p}) = \bar{p}$;

ii. φ is a well-defined, continuous function, except possibly for $p = 0$;

iii. φ is (weakly) decreasing.

LEMMA 1.A.2 The function φ has the following differentiability properties:

i. if D and S are differentiable in x and $S(x)$, then $\varphi'(x) \leq 0$;

ii. if D and S are differentiable in $\bar{p}$, then $\varphi'(\bar{p}) = S'(\bar{p})/D'(\bar{p})$;

iii. if $S''(x) < 0\ \forall x$ and $D'' > 0\ \forall x$, then $\varphi''(x) > 0\ \forall x$.

Now consider the second iterate of φ, $\varphi^2 \equiv \varphi_0\varphi$.

LEMMA 1.A.3 Properties of φ^2 are as follows:

i. $\varphi^2(\bar{p}) = \bar{p}$;

ii. φ^2 is weakly increasing.

DEFINITION The basin of attraction of $\bar{p}$ is the set of $p_0 \neq \bar{p}$ such that

$$\lim_{n\to+\infty} \varphi^n(p) = \bar{p}.$$

I call $P(\bar{p})$ the union of $\bar{p}$ and its basin of attraction; $P(\bar{p})$ is a connected open set containing $\bar{p}$.

LEMMA 1.A.4 Basins of attraction of $\bar{p}$ have the following properties:

i. If φ is differentiable almost everywhere and such that $|\varphi'(x)| < 1$, then $P(\bar{p}) = (0, +\infty)$.

ii. If $|\varphi'(\bar{p})| < 1$, then the basin of attraction is nonempty. More precisely, $P(\bar{p})$ is a connected set with nonempty interior.

iii. If $|\varphi'(\bar{p})| > 1$, then the basin of attraction is empty, that is, $P(\bar{p}) = \{\bar{p}\}$.

Proof To prove part (i), consider the sequence

$$x_n = \varphi^n(x_0),$$

$$x_{n+1} - x_n = \varphi(x_n) - \varphi(x_{n-1}) = \int_{x_{n-1}}^{x_n} \frac{d\varphi}{dx}(u)\,du.$$

Then, $|x_{n+1} - x_n| < |\varphi'| \cdot |x_n - x_{n-1}|$, and conclusion (i) follows. Conclusions (ii) and (iii) are classical properties of dynamical systems. All cases are illustrated in figure 1.4, where φ^2 (rather than φ) is drawn. ■

Proof of Proposition 1.2 The beginning of the argument of proposition 1.1 on the eductive process of elimination of strategies implies that (i) if $\underline{p}_0$ is a floor restriction then the set of possible prices at stage $2n + 1$ of the process is

$$[\varphi^{2n}(\underline{p}_0), \varphi^{2n+1}(\underline{p}_0)],$$

and (ii) if $\bar{p}_0$ is a ceiling restriction then the set of possible prices at stage $2n + 1$ of the process is

$$[\varphi^{2n+1}(\bar{p}_0), \varphi^{2n}(\bar{p}_0)].$$

Obviously these sequences converge to $\bar{p}$, if and only if $\underline{p}_0$ or $\bar{p}_0$ does belong to $P(\bar{p})$. Hence, (i) and (ii) follow from lemma 1.4.

Now, to prove (iii), note that if the graph of φ^2 transverses this bissectrix more than one time, it crosses it at least three times. Going from $\bar{p}$ to the left, call p_1 the first time when φ^2 transverses the bissectrix; p_1 and $p_2 = \varphi(p_1)$ define a cycle of φ as announced. One can then check that any $\underline{p}_0$ such that $\underline{p}_0 < p_1$ but such that $\forall p \,|\, \underline{p}_0 < p < p_1$, $\varphi^2(p) > p$, associated with a $\bar{p}_0$ such that $\bar{p}_0 > p_2 = \varphi(p_1)$ and such that $\forall p \,|\, \bar{p}_0 > p > p_2$, $\varphi^2(p) < p$, will provide credible restrictions for which the set of rationalizable prices is $[p_1, p_2]$. ■

Proof of Proposition 1.3 Part (i) has already been proved. To prove (ii), I am going to prove the following assertion:

If at some step of the iterative process of the rationalizability definition it is common knowledge that $E(p)$ must be smaller than p_0 ($\bar{p} \leq p_0 \leq \bar{p}_0$), then at the next step it is common knowledge that $E(p)$ must be greater than $\varphi(p_0)$ (where φ is the deterministic cobweb function) and smaller than $\bar{p}$.

One only has to prove this assertion when $\varphi(p_0) \geq \underline{p}_0$; in the opposite case, the property would hold trivially.

As $E(p) \leq p_0$ is common knowledge, each agent deduces that the deterministic component of supply has to be smaller than $S(p_0)$. It follows that everybody knows that the random equilibrium price will be equal to

$$\tilde{p} = \max(\underline{p}_0, \min\{D^{-1} \circ [S(p_0) - \varepsilon_0 + \varepsilon_D], \bar{p}_0\})$$

where

$$D^{-1} \circ [S(p_0) - \varepsilon_0 + \varepsilon_D] = D^{-1} \circ [S(p_0)] + \frac{\varepsilon_0 - \varepsilon_D}{B} \equiv \varphi(p_0) + \varepsilon.$$

Now compute

$$E(\tilde{p}) = \underline{p}_0 \int_{-\infty}^{\underline{p}_0 - \varphi(p_0)} dF(x) + \int_{\underline{p}_0 - \varphi(p_0)}^{\bar{p}_0 - \varphi(p_0)} [\varphi(p_0) + x]\, dF(x) + \bar{p}_0 \int_{\bar{p}_0 - \varphi(p_0)}^{+\infty} dF(x). \tag{1.A.1}$$

From (1.10) one can infer that

$$[\underline{p}_0 + \varphi(p_0) - \bar{p}] \int_{-\infty}^{\underline{p}_0 - \bar{p}} dF(x) + \int_{\underline{p}_0 - \bar{p}}^{\bar{p}_0 - \bar{p}} [\varphi(p_0) + x]\, dF(x) + [\bar{p}_0 + \varphi(p_0) - \bar{p}] \int_{\bar{p}_0 - \bar{p}}^{+\infty} dF(x) = \varphi(p_0) \tag{1.A.2}$$

[write (1.10) as $(\underline{p}_0 - \bar{p}) \int \cdots = 0$ and add $\varphi(p_0)$].

Then split (1.A.1) and (1.A.2) over a partition of $\mathbb{R}$ in five intervals limited by $-\infty$, $\underline{p}_0 - \bar{p}$, $\underline{p}_0 - \varphi(p_0)$, $\bar{p}_0 - \bar{p}$, $\bar{p}_0 - \varphi(p_0)$, and $+\infty$. One can then argue that

i. the sum of the first two terms of the right-hand side of (1.A.1) is larger than the sum of the first two terms of (1.A.2) (the inequality holds term by term);

ii. the same holds for the comparison of the last two terms of (1.A.1) and (1.A.2);

iii. the middle terms are similar.

It follows that $E(\tilde{p}) \geq \varphi(p_0)$.

Now in order to show that $E(\tilde{p}) \leq \bar{p}$, it is enough to note that the random variable $\max[p_0, \min(\bar{p} + \varepsilon, \bar{p}_0)]$ is greater than or equal to $\tilde{p}$ for each realization of ε. The above assertion follows.

A symmetric assertion can be demonstrated by replacing "$E(p)$ must be smaller than p_0, $\bar{p} \leq p_0 \leq \bar{p}_0$" in the previous assertion with "$E(p)$ must be greater than p_0, $\underline{p}_0 \leq p_0 \leq \bar{p}$" and by replacing "$E(p)$ must be greater than $\varphi(p_0)\ldots$ and smaller than $\bar{p}$" with "$E(p)$ must be smaller than $\varphi(p_0)\ldots$ and greater than $\bar{p}$."

Now, start with a credible price restriction $\bar{p}_0$ such that $\bar{p}_0 \in P(\bar{p})$. At the first step of the iteration it is common knowledge among agents that the expected equilibrium price cannot be smaller than $\bar{p}_1 = \max[\underline{p}_0, \varphi(\bar{p}_0)]$.

Then, it is common knowledge, at the second step that the expected price cannot be greater than $\bar{p}_2 = \min[\varphi(\bar{p}_1), \bar{p}_0]$ and so on. One can then show that sequences $\bar{p}_{2n+1}$ and $\bar{p}_{2n}$ both converge to $\bar{p}$. Assuming that this is not the case and that one of the sequences has an accumulation point different from $\bar{p}$ leads to a contradiction, because of the above lemma.

It remains to note that (i) the argument works when starting from $\underline{p}_0 \in P(\bar{p})$ and (ii) the argument may work for some $\bar{p}_0 \notin P(\bar{p})$ ($\underline{p}_0 \notin P(\bar{p})$). The reason is that in general the expected equilibrium price at stage 1 is strictly greater than $\bar{p}_1$ (cf. the proof of lemma 1.A.4). ■