

## THE ANALYSIS AND SYNTHESIS OF LINEAR SERVOMECHANISMS

## ABSTRACT

This paper is a formulation of a servomechanism design procedure based primarily upon an analysis of the system response to sinusoidal inputs of various frequencies. Although a knowledge of the transient performance of a servomechanism provides an excellent basis for predicting the response of the system to the conditions of a particular application, the complexity of most physical systems makes the computation of the transient performance and the translation of the resulting information into physical design criteria extremely laborious. On the other hand, knowledge of the response of the servomechanism to sinusoidal inputs is not quite as useful in enabling a prediction to be made of the servo response to the conditions of most practical applications, but the computation of the sinusoidal response and the translation of this information into useful design criteria are much simpler.

The methods developed are based upon the characteristics of the servomechanism transfer-function which is defined as the vector ratio of the servo output to the difference between the servo input and output for sinusoidal inputs of various frequencies. The characteristics of a servomechanism are completely determined once its transfer-function is specified. Moreover, physical devices that realize a prescribed transfer-function are readily synthesized if the given function is of such nature that such devices exist. This paper undertakes first to derive the interconnecting relations between particular servomechanism characteristics and the transfer-function of the servo, and second to synthesize devices that physically realize desired transfer-functions.

The study of transfer-function characteristics is aided by considering the transfer-function to be a vector and plotting the locus described by the tip of the vector as the frequency of the servo input is varied. The application of graphical analysis to this locus, termed a transfer-locus, facilitates the calculation of certain servomechanism characteristics and provides a clearer insight to many servomechanisms phenomena. It is also shown how a knowledge of the transfer loci of a servomechanism enables the system to be so adjusted that optimum performance is obtained.

The performance of a servomechanism may be unacceptable if either the steady-state error or the transient error is unsatisfactory under operating conditions. The steady-state error is defined as the difference (or error) between the servomechanism output and its input under input conditions of such nature that this difference is either constant or varying periodically. The transient error is the difference between the servomechanism output and input under input conditions that result in a nonperiodic variation in that difference. This paper derives the form the servo transfer-function must possess if the steady-state error and the transient error are to fall below allowable limits. It is shown that, if necessary, certain compensating functions can convert the servo transfer-function into the desired form. Criteria are developed that serve as guides in the analysis, application, and adjustment of the devices that physically realize the compensating functions.

Following the discussion of compensating functions, their physical realization is considered

and it is shown how several very different types of circuits may be synthesized that yield the desired function. Advantages and shortcomings of the various compensating devices are discussed.