Preface

There are two ways, distinct in principle, of mathematically describing physical systems. The first one is called the "input-output description" since it relates external variables. The mathematical model then usually takes the form of an integral equation (the Green's function approach) or more generally, of an operator equation expressing the relationship between the inputs (the variables which can be manipulated) and the outputs (the variables of interest—typically the readings of a set of sensors). Such an input-output description can usually be obtained from some representative experiments. This approach requires minimal knowledge of the physical laws governing the system and of the interconnections within the "black box."

For the "internal description" of a physical system, on the contrary, one uses these physical laws and interconnections as the basis of the mathematical model. This generally takes the form of an ordinary differential equation or a partial differential equation. In the process leading to this model one works with a set of intermediate variables, related to the concept of state. There are thus two parts to mathematical models of internally described systems: a dynamical part—which describes the evolution of the state under the influence of the inputs, and a memoryless part—which relates the output to the state (and sometimes to the instantaneous value of the input as well).

Which of the two descriptions is more convenient depends on the application and the purpose of the analysis or the synthesis that one has in mind. Modern system theory relies heavily on the state formulation

for synthesis techniques as exemplified by the highlights of modern control theory: Pontryagin's maximum principle, the regulator problem for linear systems, and the Kalman-Bucy filtering theory.

In the analysis of control systems one usually investigates questions related to stability, continuity, and sensitivity of a closed-loop system. These questions can be treated from both an input-output or a state-space point of view, but it is only very recently that successful results have been obtained and that a sufficiently general framework has been developed to treat them in an input-output setting. The pioneering work in this development has been performed by I. W. Sandberg at Bell Laboratories and G. Zames at NASA-ERC. These authors formulated the stability question (the most important design constraint in feedback control) in an input-output setting. The idea of input-output stability finds its roots in the concept of bounded-input, bounded-output stability and in the work of Nyquist. Nyquist takes the finite integrability of the impulse response as the basic requirement for stability, whereas the concept of bounded-input, bounded-output stability requires that bounded inputs produce bounded outputs. The idea of Nyquist gives an excellent type of stability but unfortunately applies only to a very restricted class of systems, the linear time-invariant systems. The concept of bounded-input, bounded-output stability never has had much success, and very few specific results have been based on it. Moreover, it has been no simple matter to analyze feedback systems in this context, in which they are described by implicit equations. The key in the generalization of these methods to feedback systems has been the introduction of extended spaces. This will be emphasized in the subsequent chapters of this monograph.

This monograph is an attempt to develop further and refine methods based on input-output descriptions for analyzing feedback systems. Contrary to previous work in this area, the treatment heavily emphasizes and exploits the causality of the operators involved. This brings the work into closer contact with the theory of dynamical systems and automata. (In fact, it can be argued that the very definitions of stability and extended spaces are ill-conceived unless the operators involved are explicitly assumed to be causal.)

The monograph is built around Chapter 4, where the relevant concepts of well-posedness, stability, continuity, and sensitivity are introduced. The mathematical foundations for this study will be found in Chapters 2 and 3. In Chapter 2 nonlinear operators are introduced and general conditions for the invertibility or the noninvertibility of nonlinear operators are derived. These conditions rely heavily on the

theory of Banach algebras and exploit causality considerations in great detail. Chapter 3 is for the most part devoted to the establishment of a series of inequalities, an unpleasant task that mathematicians usually leave to applied mathematicians and engineers. Inequalities are the workhorses of analysis—and this monograph is no exception to the rule, since these inequalities are essential ingredients for the specific stability and instability criteria described in Chapters 5 and 6. The use of linearization techniques in stability theory is then discussed in Chapter 7.

The monograph is intended primarily for researchers in system theory. The author hopes that it will also be enjoyed by control engineers who are eager to find a unified modern treatment of the analysis of feedback systems, and by mathematicians who appreciate the application of relatively advanced mathematical techniques to engineering questions.

Parts of this monograph appeared in the author's doctoral dissertation entitled "Nonlinear Harmonic Analysis" and submitted in June 1968 to the Department of Electrical Engineering of the Massachusetts Institute of Technology, Cambridge, Massachusetts.

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