III GENERAL APPENDIX

A CLG models under general assumptions on destruction shocks

The purpose of the appendix is to build the most comprehensive CLG model where all shocks affecting the firm are clearly identified. We start with continuous time. All subsequent entry equations in chapters 1, 5 and 7 can be recovered from this appendix by using the right assumptions on turnover rates. We also extend the setup to discrete time, chapters 6 and 8 being therefore the special case of this appendix.

A.1 The general entry equation in continuous time

This section introduces a new feature of the analysis: separation rates varying with the state of the firm; that is, there are several labor turnover shocks, one in stage π and one in stage g. The same is true for bankruptcy shocks, which also happen in addition in stage v.

A.1.1 Notation on turnover

Denote by $s^{Lg} > 0$ a labor turnover shock affecting the firm in stage g and by $s^{L\pi} > 0$ a labor turnover shock affecting the firm in stage π. Upon the realization of any of the two shocks, the firm returns to the vacancy stage $J_v = K$ in the entry equilibrium.

Similarly, denote by s^{Ck} a credit destruction shock in stage $k = v, g, \pi$. Upon the realization of any of the three s^{Ck} shocks $k = v, g, \pi$, the firm (the entity creditor+project) returns to the credit stage with value $J_c = 0$.

Under these conventions, we then have

$$(r + s^{Cv})J_v = -\gamma + q(\theta)\left(J_g - J_v\right) \tag{A.1}$$

$$(r + s^{Cg})J_g = -w_g + s^{Lg}\left(J_v - J_g\right) + \lambda\left(J_\pi - J_g\right) \tag{A.2}$$

$$(r + s^{C\pi})J_\pi = x\mathcal{P} - w_\pi + s^{L\pi}\left(J_v - J_\pi\right) + s^G\left(J_g - J_\pi\right) \tag{A.3}$$

Two convenient notations can be introduced: let

which allows us to rewrite equations (A.2) and (A.3) as

$$J_c = K(\phi)$$

$$J_g = \left(\frac{s^{Lg}K - w_g}{\lambda} + J_\pi\right) Q_g$$

$$J_\pi = \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} + J_g \right) Q_\pi$$

Solutions follow: one has then

$$J_g = \left[\frac{s^{Lg} K - w_g}{\lambda} + \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} + J_g \right) Q_\pi \right] Q_g \tag{A.4}$$

$$J_\pi = \left[\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} + \left(\frac{s^{Lg} K - w_g}{\lambda} + J_\pi \right) Q_g \right] Q_\pi \tag{A.5}$$

or after simplification

$$J_g = \frac{Q_g}{1 - Q_\pi Q_g} \left[\frac{s^{Lg} K - w_g}{\lambda} + \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} \right) Q_\pi \right] \tag{A.6}$$

$$J_\pi = \frac{Q_\pi}{1 - Q_\pi Q_g} \left[\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} + \left(\frac{s^{Lg} K - w_g}{\lambda} \right) Q_g \right] \tag{A.7}$$

Equation (A.1) combined with free-entry $J_v = K$ immediately delivers

$$\left(1 + \frac{r + s^{Cv}}{q} \right) K + \frac{\gamma}{q} = J_g \tag{A.8}$$

$$\left(1 + \frac{r + s^{Cv}}{q} \right) K + \frac{\gamma}{q} = \frac{Q_g}{1 - Q_\pi Q_g} \left[\frac{s^{Lg} K - w_g}{\lambda} + \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} \right) Q_\pi \right] \tag{A.9}$$

Equation (A.9) can be rewritten by putting entry costs including early wages w_g on the left-hand side and rearranging the left-hand side, so as to have:

Job creation in the most general CLG model for a given structure of wages:

$$\left(1 + \frac{r + s^{Cv}}{q} \right) K + \frac{\gamma}{q} + \frac{Q_g}{1 - Q_\pi Q_g} \left(\frac{w_g - s^{Lg} K}{\lambda} \right) = \frac{Q_g Q_\pi}{1 - Q_\pi Q_g} \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} \right) \tag{A.10}$$

or

$$\left(1 + \frac{r + s^{Cv}}{q} \right) K + \frac{\gamma}{q} + \frac{1}{1 - Q_\pi Q_g} \left(\frac{w_g - s^{Lg} K}{r + s^{Lg} + s^{Cg} + \lambda} \right) = \frac{Q_g}{1 - Q_\pi Q_g} \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{r + s^{L\pi} + s^{C\pi} + s^G} \right) \tag{A.11}$$

or finally $$K + Q_v \frac{\gamma}{q} + \frac{Q_v Q_g}{1 - Q_\pi Q_g} \left(\frac{w_g - s^{Lg} K}{\lambda} \right) = \frac{Q_v Q_g Q_\pi}{1 - Q_\pi Q_g} \left(\frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{s^G} \right) \quad \text{(A.12)}$$

with

$$Q_g = \frac{\lambda}{r + s^{Lg} + s^{Cg} + \lambda}; Q_\pi = \frac{s^G}{r + s^{L\pi} + s^{C\pi} + s^G}; Q_v = \frac{q}{r + s^{Cv} + q} \quad \text{(A.13)}$$

The left-hand side is the sum of three terms, expressed in future value of the vacancy stage. In the first two rows, the first term is the future value of financial costs, the second term is the value of hiring costs, and the third term is the value of wages net of the recovery costs K after labor turnover in stage g. The right-hand side is the expected discounted value of profits and recovery cost after labor turnover in stage π. The time perspective can also placed differently, comparing the financial costs to all present discounted values, as in the third equation.

A.1.2 Special cases

The generalized CL model: $\lambda \to +\infty$, $Q_g \to 1$ In this limit case, one obtains or using

we have the following job creation condition:

Job creation in the generalized CL model for a given wage:

$$\left(1 + \frac{r + s^{Cv}}{q} \right) K + \frac{\gamma}{q} = \frac{x\mathcal{P} - w_\pi + s^{L\pi} K}{r + s^{L\pi} + s^{C\pi}} \quad \text{(A.14)}$$

The generalized LG model: $K \to 0$ In this case, keeping for the moment positive credit shocks $s^{Ck}, k = v, g, \pi$: we have the following job creation condition:

Job creation in the generalized LG model for a given structure of wages:

$$\frac{\gamma}{q} + \frac{1}{1 - Q_\pi Q_g} \left(\frac{w_g}{r + s^{Lg} + s^{Cg} + \lambda} \right) = \frac{Q_g}{1 - Q_\pi Q_g} \left(\frac{x\mathcal{P} - w_\pi}{r + s^{L\pi} + s^{C\pi} + s^G} \right) \quad \text{(A.15)}$$

with

$$Q_g = \frac{\lambda}{r + s^{Lg} + s^{Cg} + \lambda}; Q_\pi = \frac{s^G}{r + s^{L\pi} + s^{C\pi} + s^G} \quad \text{(A.16)}$$

Assuming $s^{Ck} = 0, k = v, g, \pi$ does not simplify much.

The "generalized" L model: $K \to 0$ and $\lambda \to +\infty$, $Q_g \to 1$ In this case, still keeping positive credit shocks $s^{C\pi}$: and we obtain the following job creation condition:

Job creation in the generalized L model for a given wage:

$$\frac{\gamma}{q} = \frac{x\mathcal{P} - w_\pi}{r + s^{L\pi} + s^{C\pi}} \tag{A.17}$$

A.2 Price determination in CLG

We will assume here that $s^{L\pi} = s^{Lg}$ otherwise one would have two types of consumers from the firm's perspective, leading to additional complications.

Assume the consumer has a wage w_k that may be either $k = \pi$ or g, depending on the type of firm s(he) works at, and start from the consumption block: the asset values are given by

$$rW_{D_M} = (\Phi - \mathcal{P})x + w_k + \bar{y} + s^G(W_{D_U} - W_{D_M}) + (s^{L\pi} + s^{C\pi})(W_u - W_{D_M})$$
$$rW_{D_U} = \bar{y} + w_k + \lambda(W_{D_M} - W_{D_U}) + (s^{L\pi} + s^{C\pi})(W_u - W_{D_U})$$

The firm's block is

$$(r + s^{C\pi})J_\pi = x\mathcal{P} - w_\pi + s^{L\pi}(J_v - J_\pi) + s^G\left(J_g - J_\pi\right)$$
$$(r + s^{Cg} + s^{Lg})J_g = -w_g + s^{Lg}K + \lambda\left(J_\pi - J_g\right)$$

This leads to the firm's and consumer's consumption surpluses:

$$\begin{aligned}(r + s^{C\pi} + s^{L\pi} + s^G)(J_\pi - J_g) &= x\mathcal{P} - w_\pi + s^{L\pi}K - J_g\left(r + s^{C\pi} + s^{L\pi}\right)\\ &= x\mathcal{P} - w_\pi + s^{L\pi}K\\ &\quad - \left(-w_g + s^{Lg}K + \lambda\left(J_\pi - J_g\right)\right)\end{aligned}$$

$$\begin{aligned}(r + s^{C\pi} + s^{L\pi} + s^G)(W_{D_M} - W_{D_U}) &= (\Phi - \mathcal{P})x + w_k + \bar{y}\\ &\quad + (s^{L\pi} + s^{C\pi})(W_u - W_{D_U}) - rW_{D_U}\\ &= (\Phi - \mathcal{P})x + w_k + \bar{y} + (s^{L\pi} + s^{C\pi})\\ &\quad \times (W_u - W_{D_U}) - \big[\bar{y} + w_k + \lambda(W_{D_M} - W_{D_U})\\ &\quad + (s^{L\pi} + s^{C\pi})(W_u - W_{D_U})\big]\\ &= (\Phi - \mathcal{P})x - \lambda(W_{D_M} - W_{D_U})\end{aligned}$$

Bargaining over prices leads to

$$(W_{D_M} - W_{D_U})(1-\alpha_G) = (J_\pi - J_g)\alpha_G$$
$$\Leftrightarrow \alpha_G \left[x\mathcal{P} - w_\pi + s^{L\pi} K - \left(-w_g + s^{Lg} K + \lambda \left(J_\pi - J_g \right) \right) \right] x\mathcal{P} =$$
$$(1-\alpha_G)(\Phi - \mathcal{P})x - \lambda(W_{D_M} - W_{D_U})$$

After these few steps, one finally obtain:

Price equation in the generalized CLG model:

$$\mathcal{P}x = (1-\alpha_G)\Phi x + \alpha_G \left(w_\pi - w_g - s^{L\pi} K + s^{Lg} K \right)$$
$$= (1-\alpha_G)\Phi x + \alpha_G \left(w_\pi - w_g \right) \text{ with the above assumption that } s^{Lg} \text{- } s^{L\pi} \text{=0}$$

In the special case where wages are identical (see below), the price converges to a constant of parameters:

Price equation in the simpler CLG model (equal wages and labor turnover rates independent of the state of the firm):

$$\mathcal{P}x = (1-\alpha_G)\Phi x$$

A.3 Wage determination in stage g of CLG

Assume that the wage remains constant between stages g and π and utility is linear. The labor surpluses of workers and bargaining firms are

$$(r + s^{Lg} + s^{Cg}) \left(W_{D_U} - W_u \right) = \bar{y} + w_g + \lambda \left(W_{D_M} - W_{D_U} \right) - rW_u$$
$$(r + s^{Cg} + s^{Lg}) \left(J_g - J_v \right) = 0 - w_g + \lambda \left(J_\pi - J_g \right) - (r + s^{Cg})K$$

with

$$rW_u = \bar{y_0} + f(W_{D_U} - W_u)$$

The capital gain of the worker when (s)he will access consumption good 1 is reflected by $W_{D_M} - W_{D_U}$. It is independent of w_g. The same is true for the capital gain of the firm $J_\pi - J_g$. The maximization of the Nash product in the labor market therefore leads to $(1-\alpha_L)(W_{D_U} - W_u) = \alpha_L(J_g - J_v)$, leading to

$$(1-\alpha_L) \left[\bar{y} + w_g + \lambda \left(W_{D_M} - W_{D_U} \right) - rW_u \right] = \alpha_L \left[-w_g + \lambda \left(J_\pi - J_g \right) - (r + s^{Cg})K \right]$$
$$\Leftrightarrow w_g = \alpha_L \left[0 - (r + s^{Cg})K \right]$$
$$+ (1-\alpha_L)(rW_u - \bar{y_0}) + \lambda\alpha_L \left(J_\pi - J_g \right)$$
$$- (1-\alpha_L)\lambda \left(W_{D_M} - W_{D_U} \right)$$

Interestingly the latter term can be simplified using the price bargaining equation. In equilibrium, this delivers the wage equation:

$$w_g = (1-\alpha_L)(rW_u - \bar{y_0}) + \alpha_L\left[-(r+s^{Cg})K\right] \\ +\lambda\left(J_\pi - J_g\right)\frac{\alpha_L-\alpha_G}{1-\alpha_G}$$

One can also replace the surplus of the firm by its forward value: using again

$$J_g = \frac{Q_g}{1-Q_\pi Q_g}\left[\left(\frac{s^{Lg}K-w_g}{\lambda}\right)+\left(\frac{x\mathcal{P}-w_\pi+s^{L\pi}K}{s^G}\right)Q_\pi\right] \tag{A.18}$$

$$J_\pi = \frac{Q_\pi}{1-Q_\pi Q_g}\left[\left(\frac{x\mathcal{P}-w_\pi+s^{L\pi}K}{s^G}\right)+\left(\frac{s^{Lg}K-w_g}{\lambda}\right)Q_g\right] \tag{A.19}$$

one has

$$\begin{aligned}
J_\pi - J_g &= \frac{1}{1-Q_\pi Q_g}\left[Q_\pi\left(\frac{x\mathcal{P}-w_\pi+s^{L\pi}K}{s^G}\right)+\left(\frac{s^{Lg}K-w_g}{\lambda}\right)Q_gQ_\pi\right.\\
&\quad\left.-Q_g\left(\frac{s^{Lg}K-w_g}{\lambda}\right)-\left(\frac{x\mathcal{P}-w_\pi+s^{L\pi}K}{s^G}\right)Q_\pi Q_g\right]\\
&= \frac{1}{1-Q_\pi Q_g}\left[Q_\pi\left(1-Q_g\right)\left(\frac{x\mathcal{P}-w_\pi+s^{L\pi}K}{s^G}\right)\right.\\
&\quad\left.+\left(\frac{s^{Lg}K-w_g}{\lambda}\right)\left(Q_gQ_\pi-Q_g\right)\right]\\
&= \frac{1}{1-Q_\pi Q_g}\left(\frac{x\mathcal{P}-w_\pi+s^{L\pi}K}{r+s^{L\pi}+s^{C\pi}+s^G}\,\frac{r+s^{Lg}+s^{Cg}}{r+s^{Lg}+s^{Cg}+\lambda}\right.\\
&\quad\left.-\frac{s^{Lg}K-w_g}{r+s^{Lg}+s^{Cg}+\lambda}\,\frac{r+s^{L\pi}+s^{C\pi}}{r+s^{L\pi}+s^{C\pi}+s^G}\right)
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{1}{1-Q_\pi Q_g} &= \frac{1}{1-\frac{\lambda}{r+s^{Lg}+s^{Cg}+\lambda}\,\frac{s^G}{r+s^{L\pi}+s^{C\pi}+s^G}}\\
&= \frac{(r+s^{Lg}+s^{Cg}+\lambda)(r+s^{L\pi}+s^{C\pi}+s^G)}{(r+s^{Lg}+s^{Cg}+\lambda)(r+s^{L\pi}+s^{C\pi}+s^G)-\lambda s^G}\\
&= \frac{(r+s^{Lg}+s^{Cg}+\lambda)(r+s^{L\pi}+s^{C\pi}+s^G)}{(r+s^{Lg}+s^{Cg})(r+s^{L\pi}+s^{C\pi})+(r+s^{Lg}+s^{Cg})s^G+(r+s^{L\pi}+s^{C\pi})\lambda}
\end{aligned}$$

implying

$$J_\pi - J_g = \frac{(x\mathcal{P}-w_\pi+s^{L\pi}K)(r+s^{Lg}+s^{Cg})-(s^{Lg}K-w_g)(r+s^{L\pi}+s^{C\pi})}{(r+s^{Lg}+s^{Cg})(r+s^{L\pi}+s^{C\pi})+(r+s^{Lg}+s^{Cg})s^G+(r+s^{L\pi}+s^{C\pi})\lambda}$$

Simplifications arise when assuming identical turnover and credit shocks in each stages and finally with equal wages: one has

$$J_\pi - J_g = \frac{(x\mathcal{P} - w_\pi + s^L K) - (s^L K - w_g)}{r + s^L + s^C + s^G + \lambda}$$
$$= \frac{x\mathcal{P} - w_\pi + w_g}{r + s^L + s^C + s^G + \lambda}$$
$$= \frac{x\mathcal{P}}{r + s^L + s^C + s^G + \lambda}$$

The wage equation thus becomes, using the notation $\mu_{CLGr} = \frac{\lambda}{r+s^L+s^C+s^G+\lambda}$:

Wage equation in stage g in the general CLG model:

$$w_g = (1-\alpha_L)(rW_u - \bar{y_0}) + \frac{\alpha_L - \alpha_G}{1-\alpha_G}\mu_{CLGr} x\mathcal{P} - \alpha_L(r + s^{Cg})K$$
$$= (1-\alpha_L)(rW_u - \bar{y_0}) + (\alpha_L - \alpha_G)\mu_{CLGr} x\Phi - \alpha_L(r + s^{Cg})K$$

The price was replaced by its equilibrium expression in the last line. We also have the following simpler cases:

Wage equations in specific submodels:

Ch.7.5: CLG with $\alpha_L{=}\alpha_G \Rightarrow w_g = (1-\alpha_L)(rW_u - \bar{y_0}) - \alpha_L(r + s^{Cg})K$

Ch. 7.2: LG with $K = 0 \Rightarrow w_g = (1-\alpha_L)(rW_u - \bar{y_0}) + (\alpha_L - \alpha_G)\mu_r^{CLG} x\Phi$

Ch. 5: CL with $\lambda \to \infty$, $\alpha_G{=}0 \Rightarrow w_g = (1-\alpha_L)(rW_u - \bar{y_0}) + \alpha_L\left[x\mathcal{P} - (r + s^{Cg})K\right]$

Ch. 1: L with $K = 0, \lambda \to \infty$, $\alpha_G{=}0 \Rightarrow w_g = (1-\alpha_L)(rW_u - \bar{y_0}) + \alpha_L \mathcal{P} x$

A.4 A discrete-time extended CLG model

The equivalent discrete-time version corresponds to the following equations. Pose $\beta^{Ck} = \frac{1-s^{Cvk}}{1+r}$ for $k = c = v = \pi$:

$$J_{ct} = 0 \Leftrightarrow \frac{\kappa_B}{\phi_t p_t} + \frac{\kappa_I}{p_t} = J_{vt} = K(\phi) \tag{A.20}$$

$$J_{vt} = -\gamma + \beta^{Cv}\mathbb{E}_t\left[q_t J_{gt+1} + (1-q_t)J_{vt+1}\right] \tag{A.21}$$

$$J_{gt} = -w_t + \beta^{Cg}\mathbb{E}_t\left[\left(1 - s^{Lg}\right)\left(\lambda_t J_{\pi t+1} + J_{gt+1}\right) + s^{Lg} J_{vt+1}\right] \tag{A.22}$$

$$J_{\pi t} = \mathcal{P}_t x_t - w_t \tag{A.23}$$

$$+\beta^{C\pi}\mathbb{E}_t\left[\left(1 - s^{L\pi}\right)\left[(1-s^G)J_{\pi t+1} + s^G J_{gt+1}\right] + s^{L\pi} J_{vt+1}\right]$$

Some notations will be useful here, as the equivalent of the Q factors before. Let

$$\Theta_\pi = \frac{(1-s^{C\pi})\left(1-s^{L\pi}\right)\left(1-s^{G}\right)}{1+r}$$

$$\Theta_g = \frac{(1-s^{Cg})\left(1-s^{Lg}\right)}{1+r}$$

$$\Theta_v = \frac{(1-s^{Cv})}{1+r}$$

be the appropriate discount rates; the recursive dynamic system can be rewritten, replacing J_{vt} by its equilibrium value, as

$$J_{vt} = K(\phi) \tag{A.24}$$

$$K(\phi)\left[\frac{1-\Theta_v(1-q_t)}{q_t}\right] + \frac{\gamma}{q_t} = \Theta_v q_t \mathbb{E}_t J_{gt+1} \tag{A.25}$$

$$J_{gt} = -w_t + \Theta_g \mathbb{E}_t\left[\lambda_t J_{\pi t+1} + J_{gt+1} + \frac{s^{Lg}}{1-s^{Lg}}K(\phi)\right] \tag{A.26}$$

$$J_{\pi t} = \mathcal{P}_t x_t - w_t + \Theta_\pi \mathbb{E}_t\left[J_{\pi t+1} + \frac{s^{Lg}}{1-s^{Lg}}J_{gt+1} + \frac{s^{Lg}}{1-s^{Lg}}\frac{s^{L\pi}}{1-s^{L\pi}}K(\phi)\right] \tag{A.27}$$

B Intensive margins in the CLG model

Intensive margins have always been present in labor search models. The discussion of the search effort of endogenous job seekers and the advertising of firms has been summarized in Pissarides (2000) and earlier in work by Mortensen (1982b) and Pissarides (1984). Pissarides (2000) also recovers a Solow condition (page 131) for advertisement effort that we would obtain from perfect financial markets. Here, we show the existence of a modified Solow condition where advertising effort is increasing in the amount of entry costs due to financial frictions. This is a new result to our knowledge, as is the discussion of an equivalent Solow condition on financial markets. The properties of advertisement for goods have been derived in Hall (2012). The overall discussion of the six intensive margins and their cyclical properties is in Petrosky-Nadeau and Wasmer (2015).

B.1 Endogenous search efforts in the goods market

B.1.1 Endogenous consumer search effort and the cyclicality of "shopping time"

Thinking about consumers' effort and more generally "shopping time," a measure available in time use surveys, requires us to disentangle two types of effort. One is prematch effort. It will be denoted by e_G and may be interpreted as the effort to match to a marginal good while already consuming and purchasing other goods. The second one is a post-match effort, devoted to purchasing time. It is denoted by e_P, and transforms (lost) leisure time into actual purchasing and higher consumption. For the sake of simplicity, we assume that the relation between the quantity of a search good purchased denoted by x is linear in time spent e_P. The previous analysis where x was inelastic would be a special case where e_P has no impact on x

Workers match at a rate determined by aggregate quantities including the total number of agents on each side of the market and the average effort made by consumers $\bar{e_G}$, as well as their own individual search effort. We therefore keep the constant returns to scale matching function but augmented by average search effort: the total number of matches and the transition rates for firms and for an individual consumer are given respectively by

$$\mathcal{M}_G = \mathcal{M}_G(\bar{e_G}\mathcal{D}_U, \mathcal{N}_g) \tag{B.1}$$

$$\mathcal{M}_G/\mathcal{N}_g = \lambda(\xi, \bar{e_G}) \tag{B.2}$$

$$(e_G/\bar{e_G})\,\mathcal{M}_G/\mathcal{D}_U = (e_G/\bar{e_G})\,\lambda(\xi, \bar{e_G})/\xi = (e_G/\bar{e_G})\,\check{\lambda}(\xi, \bar{e_G}) \tag{B.3}$$

with $\partial\lambda/\partial\xi > 0$ and $\partial\lambda/\partial\bar{e_G} > 0$.The term $e_G/\bar{e_G}$ reflects the discount factor of an individual consumer facing crowding out by other consumers. In a symmetric equilibrium this ratio will be 1.

We also assume that both efforts correspond to reduced leisure and that leisure and consumption are additively separable blocks so that the utility effect of search and purchasing efforts e_G and e_P are summarized by a cost function, respectively $\sigma_G(e_G)$ and $\sigma_P(e_P)$. The values of the consumer are therefore modified as follows:

$$rW_{D_U} = v(0, \bar{y}+w) - \sigma_G(e_G) + (e_G/\bar{e}_G)\,\lambda(W_{D_M} - W_{D_U})$$
$$rW_{D_M} = v(x, \bar{y}+w-\mathcal{P}x) - \sigma_P(e_P) + s^G(W_{D_U} - W_{D_M})$$

Using the compact notation $v(\mathbf{1}) = v(x, \bar{y}+w-\mathcal{P}x) - \sigma_P(e_P)$ for the indirect utility from consuming the search good, and $v(\mathbf{0}) = v(0, \bar{y}+w) - \sigma_G(e_G)$ for the indirect utility from consuming only the quasi-numraire, and finally $\Delta v \equiv v(\mathbf{1}) - v(\mathbf{0})$ for their difference, the value of the consumer's surplus is given as

$$W_{D_M} - W_{D_U} = \frac{\Delta v}{r + s^G + (e_G/\bar{e}_G)\,\lambda} \tag{B.4}$$

Consumers incur a cost to search effort in the goods market described by the returns to leisure v_l. Given this environment, the generic optimality conditions for consumer search effort states that

$$\sigma'_G(e_G) = \frac{\lambda}{\bar{e}_G}\left(W_{D_M} - W_{D_U}\right) \tag{B.5}$$
$$\sigma'_P(e_P) = \frac{\partial x}{\partial e_P}\left[v_1(\mathbf{1}) - \mathcal{P}v_0(\mathbf{1})\right] \tag{B.6}$$

The first condition states that the marginal cost of *prospecting* effort e_G when unmatched equals the marginal benefit, that is the marginal probability of being matched to a good multiplied by the consumption surplus; the second condition states that the marginal cost of purchasing effort when matched equals the marginal gain from consumption of the non-search good net of the forgone utility of consuming the Walrasian good.

The properties of Δv and its derivatives with respect to $I = \bar{y}+w$ and x will be very important for what follows. In particular, we will assume marginal decreasing utility in both goods ($v_{11} < 0$ and $v_{00} < 0$). Under separability between the two types of goods, this leads in particular to

$$\Delta v_0 \equiv (v_0(\mathbf{1}) - v_0(\mathbf{0})) > 0 \tag{B.7}$$

which means that a marginal unit of consumption of good 0 has a greater value when the agent consumes less of it because it diverts part of its income to consume the search good as in proposition 3. Under non-separability of consumption of goods 0 and 1, the same inequality holds for utility functions in which the degree of substitutability between the two goods is not strong enough.

Proposition 1 (the cyclicality of consumer search and shopping efforts). *(i) Prematch effort e_G is pro-cyclical, in that it increases with the consumption surplus, which itself depends on income I when $\partial\Delta v/\partial I > 0$, which is true under decreasing marginal utility of the Walrasian good, when inequality B.7 is satisfied. (ii) Consumption effort e_P is pro-cyclical under fixed prices.*

The first part of the proposition is again due to the fact that a consumer's surplus is higher with higher income, because the marginal value of sacrificing income to access good 1 and get less of good 0 is lower. The second part of the proposition comes from the fact that higher income I leads to a lower marginal utility of consumption of the quasi-numraire, that is a lower $v_0(\mathbf{1})$, such that the return on consumption effort e_P is higher. Hence at a given price, purchasing effort is also pro-cyclical. With pro-cyclical prices, part of this effect is mitigated. One can safely conjecture that the mitigation effect is only a partial one, if prices reflect variations in total consumption surplus with a slope less than 1 as is the case in the bargaining setup.

B.1.2 Derivation of the model with shopping effort

From now on, since "purchasing" effort e_P is not relevant, we assume it to be fixed and revert to the previous assumption that firms supply x inelastically. The value equations for J_v and J_π are therefore mostly unchanged, with a single difference: the fact that the transition rates in the value of J_g differ from equation (7.18). We have now

$$rJ_g = -w_g + \lambda(\xi, \bar{e}_G)(J_\pi - J_g) + s^L(J_v - J_g) \tag{B.8}$$

In a symmetric equilibrium in effort, the surplus of the consumer is now

$$W_{D_M} - W_{D_U} = \frac{(\Phi - \mathcal{P})x + \sigma_G(e_G)}{r + s^L + s^G + \lambda(\xi, \bar{e}_G)/\xi} \tag{B.9}$$

and

$$J_\pi - J_g = \frac{\mathcal{P}x}{r + s^L + s^G + \lambda(\xi, \bar{e}_G)} \tag{B.10}$$

Inspection of the stock-flow equations shows that the steps leading to equation (7.14) remains valid with endogenous effort of consumers and therefore that the property that $\xi^* = 1$ remains true.

B.1.3 Price determination with endogenous consumption effort

Therefore the new price equation is simply:

Price equation in CLG with consumption effort:

$$\mathcal{P}x = (1 - \alpha_G)\left[\Phi x + \sigma_G(e_G)\right] \tag{B.11}$$

The optimal effort follows:

$$\sigma_G'(e_G) = \check{\lambda}(\xi, \bar{e}_G)(W_{D_M} - W_{D_U})\,(1/\bar{e_G}) \tag{B.12}$$

It can be solved in substituting the surplus in (B.9) into (B.5); one obtains after simple manipulations that

$$\sigma_G'(e_G) = (\lambda/\bar{e}_G)\frac{(\Phi - \mathcal{P})x + \sigma_G(e_G)}{r + s^L + s^G + \lambda(\xi, \bar{e}_G)}$$

or

$$\sigma(e_G) = \frac{\lambda(\xi, \bar{e}_G)}{(r + s^L + s^G)\eta_{\sigma G} + \lambda(\xi, \bar{e}_G)(\eta_{\sigma G} - 1)}(\Phi - \mathcal{P})x \tag{B.13}$$

where $\eta_{\sigma G} = e_G \sigma'_G(e_G)/\sigma_G(e_G) > 1$ is the elasticity of the cost of effort function. The equilibrium is now a 4-tuple $(\mathcal{P}, w, e_G, \theta)$ where the above conditions are added to the price-wage determination schedule and to the job creation condition.

B.1.4 Wage determination with endogenous consumption effort

The bargained wage is the outcome of

$$w = \text{argmax}\left(J_g - J_v\right)^{1-\alpha_L}\left(W_{D_U} - W_u\right)^{\alpha_L}$$

leading to the labor match surplus sharing rule: $(1 - \alpha_L)\left(W_{D_U} - W_u\right) = \alpha_L\left(J_g - J_v\right)$. We also have the labor surpluses of workers and bargaining firms:

$$(r + s^L + s^C)\left(W_{D_U} - W_u\right) = \bar{y_0} + w - \sigma(e) + \lambda(e)\left(W_{D_M} - W_{D_U}\right) - rW_u$$
$$(r + s^C + s^L)\left(J_g - J_v\right) = 0 - w_g + \lambda(e)\left(J_\pi - J_g\right) - k(\phi)$$

with

$$rW_u = z + \bar{y_0} + f(W_{D_U} - W_u)$$

Combining the sharing rule and the surplus equations, we obtain

$$\begin{aligned}
&(1 - \alpha_L)\left[\bar{y_0} + w - \sigma(e) + \lambda(e)\left(W_{D_M} - W_{D_U}\right) - rW_u\right] \\
&\quad = \alpha_L\left[-w + \lambda(e)\left(J_\pi - J_g\right) - k(\phi)\right] \\
&\Leftrightarrow w = \alpha_L[-k(\phi)] + (1 - \alpha_L)(\sigma(e) + rW_u - \bar{y_0}) \\
&+ \lambda(e)\alpha_L\left(J_\pi - J_g\right) - (1 - \alpha_L)\lambda\left(W_{D_M} - W_{D_U}\right)
\end{aligned}$$

Interestingly the latter term can be simplified using the price bargaining equation, delivering the wage equation:

$$\begin{aligned}
w &= (1 - \alpha_L)(rW_u + \sigma(e) - \bar{y_0}) - \alpha_L k(\phi) \\
&+ \lambda(e)\left(J_\pi - J_g\right)\frac{\alpha_L - \alpha_G}{1 - \alpha_G}
\end{aligned}$$

Using the definition of the asset values, the wage can be expressed as a function of the firm's surplus with respect to the consumer:

$$w = (1 - \alpha_L)(\sigma(e) + rW_u - \bar{y}) + \left(\frac{\alpha_L - \alpha_G}{1 - \alpha_G}\right)\lambda(e)(J_\pi - J_g) - \alpha_L k(\phi)$$

and then using the expression for the seller's surplus in the previous subsection and the discounting term, we have

$$\mu^{CLG} = \frac{\lambda(e)}{r + s^T + \lambda(e)}$$

The wage in the labor market can be linked to the price in the goods market with

$$rW_u = z + \bar{y} + \frac{\alpha_L}{1-\alpha_L} f(\theta)(J_g - J_v)$$

and

$$\frac{\gamma + k(\phi)}{q(\theta)} = J_g - J_v$$

so that

$$rW_u = z + \bar{y} + \alpha_L \theta \left[\gamma + k(\phi)\right]$$

We therefore obtain

$$w = (1-\alpha_L)\left[z + \sigma(e)\right] + \alpha_L\left[\gamma + k(\phi)\right]\theta + \left(\frac{\alpha_L - \alpha_G}{1-\alpha_G}\right)\mu^{CLG}\mathcal{P}x - \alpha_L k(\phi) \quad \text{(B.14)}$$

Replacing $\mathcal{P}x$ by its equilibrium value further simplifies the wage equation to:

Wage equation in CLG model with consumption effort:

$$\begin{aligned} w = {} & (1-\alpha_L)\left[z + \sigma(e)\right] + \alpha_L\left[\gamma + k(\phi)\right]\theta - \alpha_L k(\phi) \\ & + (\alpha_L - \alpha_G)\,\mu_{CLG}\left[\Phi x + \sigma(e)\right] \end{aligned} \quad \text{(B.15)}$$

B.1.5 Job creation condition

Replacing the wage equation and the price equation in the main job creation condition, we get:

Job creation condition in CLG, extensive form:

$$\begin{aligned} \frac{\gamma + k(\phi)}{q} &= \left(\frac{x^{CLG}\mathcal{P} - w - k(\phi)}{r + s^C + s^L}\right) \\ &= \left(\frac{\begin{array}{c}\mu^{CLG}(1-\alpha_G)\left[\gamma + k(\phi)\right] - \big((1-\alpha_L)z + \alpha_L\theta\left(\gamma + k(\phi)\right) \\ + (\alpha_L - \alpha_G)\,\mu^{CLG}\left[\Phi x + \sigma(e)\right] - \alpha_L k(\phi)\big) - k(\phi)\end{array}}{r + s^C + s^L}\right) \end{aligned}$$

B.2 Extension to bilateral effort in the goods market

The extension to search effort by firms, also called advertising, is straightforward. Denote by e_A advertising effort of an individual firm, by $\bar{e_A}$ the average firm's effort, and by $\sigma_A(e_A)$ the advertising cost function with elasticity $\eta_{\sigma_A} > 1$. The matching rates are now

$$\lambda = \frac{\mathcal{M}_G(\bar{e_A}\mathcal{N}_g, \bar{e_G}\mathcal{D}_U)}{\mathcal{N}_g} = \lambda(\xi, \bar{e_G}, \bar{e_A})$$
$$\check{\lambda} = \frac{\mathcal{M}_G(\bar{e_A}\mathcal{N}_g, \bar{e_G}\mathcal{D}_U)}{\mathcal{D}_U} = \check{\lambda}(\xi, \bar{e_G}, \bar{e_A})$$
$$\Rightarrow \check{\lambda} = \lambda/\xi$$

The Bellman equations are easily expressed as

$$rJ_g = -w_g - \sigma_A(e_A) + (e_A/\bar{e_A})\,\lambda(\xi, \bar{e}_G, \bar{e}_A)(J_\pi - J_g) + s(J_c - J_g)$$
$$rJ_\pi = x\mathcal{P} - w_\pi + s(J_c - J_\pi) + s^G(J_g - J_\pi)$$
$$rW_{D_U} = w - \sigma_g(e_g) + \left(e_g/\bar{e_g}\right)\check{\lambda}(\xi, \bar{e}_G, \bar{e}_A)(W_{D_M} - W_{D_U}) + s(W_u - W_{D_U})$$
$$rW_{D_M} = \Phi x + (w - x\mathcal{P}) + s^G(W_u - W_{D_M}) + s(W_{D_U} - W_{D_M})$$

The first-order condition says that marginal efforts are equal to the marginal expected surplus:

$$\sigma'_A(e_A) = \lambda(\xi, \bar{e}_G, \bar{e}_A)(J_\pi - J_g)\,(1/\bar{e_A}) \tag{B.16}$$

while equation (B.12) is unchanged.

Further, combining equations (B.12) and (B.16) and using Nash bargaining over prices, we have, with $\xi = 1$

$$(1 - \alpha_G)\bar{e_G}\sigma'_G(e_G) = \alpha_G\bar{e_A}\sigma'_A(e_A)$$

For instance, if the cost functions were identical and in symmetric equilibria $(e_A/\bar{e_A}) = (e_G/\bar{e_G}) = 1$ and at equilibrium goods market tightness $\xi = 1$, the effort of one side would be proportional to the effort in the other side. In the general case of convex cost functions, the effort of each side are strategic complements.

To sum up, both efforts are individually pro-cyclical; and this pro-cyclicality is therefore augmented by the strategic complementarity.

The equilibrium is now a 5-tuple $(\mathcal{P}, w, e_G, e_A, \theta)$ where the two conditions on search effort complete the system (price determination, wage determination, and job creation).

B.3 Adding intensive search margins in labor and credit markets

The CLG model allows for new potential intensive search margins. Endogenous effort can be made in all six segments of the three markets; however, many simplifications arise from the various free-entry conditions and from bargaining. A summary of the results exposed below is as follows:

- In the financial market, both search effort by new projects and by creditors are constant of parameters, determined by a Solow condition on the elasticity of search effort that is equal to 1.

- in the labor market, the effort of firms to recruit workers is also a simple function of parameters, with a modified Solow condition including the financial entry costs $K(\phi)$. The effort of workers to find jobs depends similarly on the value of unemployment W_u: in the benchmark setting, it depends positively and linearly on labor market tightness. In an extended setting with endogenous workers entry, the labor force would adjust so that W_u is equal to the opportunity cost of inactivity and would become exogenous to labor market conditions, hence the effort of workers would also be a pure function of parameters and independent of the business cycle.
- In the goods market, as covered in Chapter 7.

We will use simple notations e_K, $\sigma_K(e_K) > 0$, and $\eta_{\sigma_K}(e_K) > 1$ for, respectively, search effort, costs of search effort, and their elasticity with respect to effort; for the various markets, we will use subscripts $K = I, B, V, U, G, A$ for, respectively prospective projects, creditors, job advertisement by firms, search effort by unemployed workers, search effort by consumers, and advertising for goods by firms.

B.3.1 Endogenous effort in financial markets

The Bellman equations for projects and creditors in the financial market are now

$$rE_c(e_I) = -\sigma_I(e_I) + (e_I/\bar{e_I})\,p(\phi)(E_v - E_c) \tag{B.17}$$

$$rB_c(e_B) = -\sigma_B(e_B) + (e_B/\bar{e_B})\,\phi p(\phi)(B_v - B_c) \tag{B.18}$$

where $\sigma_I(e_I)$ replacing κ_I is still an effort cost and not a direct financial cost. Optimal effort is given by simple first-order conditions:

$$\sigma_I'(e_I) = p(\phi)(E_v - E_c)\,(1/\bar{e_I}) \tag{B.19}$$

$$\sigma_B'(e_B) = \phi p(\phi)(B_v - B_c)\,(1/\bar{e_B}) \tag{B.20}$$

At the same time, free entry leads to

$$\sigma_I(e_I) = (e_I/\bar{e_I})\;p(\phi)(E_v - E_c) \tag{B.21}$$

$$\sigma_B(e_B) = (e_B/\bar{e_B})\;\phi p(\phi)(B_v - B_c) \tag{B.22}$$

Interestingly, the combination of first-order conditions and free entry leads to a standard condition known in a totally different context as the Solow condition (Solow, 1979) applying to the effort-wage condition: the effort of each side is such that a Solow condition on the elasticity is equal to 1:

$$\sigma_I'(e_I)\;\; e_I/\sigma_I(e_I) = 1 \;\Leftrightarrow \eta_{\sigma_I}(e_I^*) = 1 \tag{B.23}$$

$$\sigma_B'(e_B)\; e_B/\sigma_B(e_B) = 1 \Leftrightarrow \eta_{\sigma_B}(e_B^*) = 1 \tag{B.24}$$

The model therefore accommodates quite easily the existence of endogenous margins in search effort in financial markets: the two associated margins simply deliver an effort level that only depends on parameters. If there is an effort level that satisfies this Solow solution, then effort is indeed fixed, independent of other aggregate conditions. There is no additional volatility arising from the response of efforts to productivity innovations. If there is no solution to these Solow conditions, then a corner solution is reached (either infinite effort or zero effort).

B.3.2 Search effort in the labor market

Here, as before in chapter 5, an issue arises as to whether the agent incurring the cost of financing the vacancy also receives its full surplus value. Indeed, if the cost was entirely born by the financier, but the owner if the investment project decided its value regardless of its cost, an infinite effort would result in equilibrium. Therefore, it is likely that part of the costs are shared between the parties, or more directly, that the joint entity (project + creditor) that we called a "firm" decides about vacancy posting effort, exactly as it decided about wages. In this case, the "firm" value of a vacancy would become

$$rJ_v(e_V) = -\sigma_V(e_V) + (e_V/\bar{e_V})\, q(J_g - J_v)$$

leading to the same first-order condition:

$$\sigma_V'(e_V) = q(J_g - J_v)\,(1/\bar{e_V})$$

Now, using the free-entry conditions in the financial market, one also obtains that

$$J_v(e_V) = K(\phi)$$

and unsurprisingly, one obtains another Solow condition on vacancy posting effort, augmented by the presence of the entry cost K:

$$\frac{\sigma_V'(e_V)e_V}{rK + \sigma_V(e_V)} = 1$$

Here again, this effort is itself a simple function of parameters and of $K(\phi)$, which is itself a function of underlying parameters in the credit market. Thus, vacancy posting effort is itself independent of other conditions. It is also important to remark that the higher K is, the higher the advertising effort. This can be seen for an isoelastic cost function, leading to

$$\sigma_V(e_V)\left[\eta_{\sigma_V} - 1\right] = rK$$

This is due to the fact that, when entry costs are higher, there is less entry thus q is higher in equilibrium, which raises the marginal value of effort, which itself increases.

On the worker side, the job search effort of job seekers is given by the maximum of

$$rW_u(e_u) = z - \sigma_U(e_U) + (e_U/\bar{e_U}) f(\theta)(W_{D_U} - W_u) \tag{B.25}$$

The first-order condition is

$$\sigma_U'(e_U) = f(\theta)(W_{D_U} - W_u)\,(1/\bar{e_u}) \tag{B.26}$$

and, substituting the above equation:

$$\frac{e_U\sigma_U'(e_U)}{rW_u - z + \sigma_U(e_U)} = 1$$

The difference now is that $rW_u - z$ is, contrary to $K(\phi)$, not a simple function of parameters but instead an increasing function of labor market tightness. We can obtain a slightly simpler expression in using the outcome of wage bargaining:

$$rW_u - z + \sigma_U(e_U) = (e_U/\bar{e_U})f(\theta)(W_{D_U} - W_u) = (e_U/\bar{e_U})f(\theta)\frac{\alpha_L}{1-\alpha_L}(J_g - J_v)$$
$$= (e_U/\bar{e_U})f(\theta)\frac{\alpha_L}{1-\alpha_L}\frac{rK + \sigma_V(e_V)}{(e_V/\bar{e_V})\,q(\theta)} = \theta\frac{\alpha_L}{1-\alpha_L}(rK + \sigma_V(e_V))$$

where the last equality uses the symmetric equilibrium condition $e_V/\bar{e_V} = 1$ and $e_U/\bar{e_U} = 1$ and therefore

$$\frac{1-\alpha_L}{\alpha_L}\left(\frac{\sigma_U'(e_U)\bar{e_U}}{rK + \sigma_V(e_V)}\right) = \theta$$

Not unsurprisingly, the effort of the unemployed is increasing in θ and this increases the volatility. In contrast, there is no additional persistence introduced by considering endogenous search effort of the unemployed.

C Specific topics of the CL model: strategic bargaining and inefficient endogenous destructions

In chapter 5, we introduced a block bargaining assumption between the worker, on one side, and the project-creditor block on the other side. This led to a convenient wage equation, comparable to the standard matching model and yielding Hosios-efficiency conditions. Relaxing this assumption however offers a rich discussion of these strategic interactions within the firm. This was first studied in section 5 of Wasmer and Weil (2004). Here we will introduce what we call "sequential bargaining" and three-party negotiation games between workers, creditors, and the owner of the project; there are complex strategic interactions, similar in spirit to the intrafirm bargaining interactions exhibited in chapter 4, for which alternative threat points matter qualitatively.

C.1 Strategic interaction within the firm

C.1.1 Sequential bargaining

There are two types of contracts in our economy: loan contracts negotiated between creditors and projects, and wage contracts bargained between projects and workers. These contracts are negotiated sequentially, reflecting the necessity for a project to find a creditor before looking for a worker. The loan contract is struck in the credit search stage, when the creditor and the project meet. The wage contract is then negotiated in the labor search stage when the project and the worker meet. In the block bargaining case this sequence had no implication for the negotiated wage or financial market tightness. This is no longer the case here.

We now assume, as in Wasmer and Weil (2004), that wage bargaining occurs between the worker and the investment project, leaving out the creditor. That is, the surplus being divided is defined as $(W_n - W_u) + (E_\pi - \iota E_v)$, and the project–worker pair take the contract arrived at in the financial market as given when they bargain. Just as in the block bargaining case, we allow for two possible outcomes for the worker's counter party if the pair fail to reach an agreement. That is, we employ the indicator function ι to indicate whether the outside option is search in the labor market, with value E_v, or dissolution of the creditor–investment project pair, with corresponding value $E_c = 0$ under free entry in the financial market. Thus the wage is the solution to

$$w = \text{argmax}(E_\pi - \iota E_v)^{1-\alpha_L}(W_n - W_u)^{\alpha_L}$$

The solution to the wage bargaining is a sharing rule

$$(1-\alpha_L)(W_n - W_u) = \alpha_L (E_\pi - \iota E_v)$$

and a wage rule

$$w = \alpha_L (x - \psi) + (1-\alpha_L) rW_u - \iota \alpha_L r E_v \tag{C.1}$$

The usual simplification where the value of unemployment is combined with free entry does not particularly simplify the analysis. We have indeed $rW_u = z + \theta q(\theta)[W_n - W_u] = z + \theta q(\theta)\frac{\alpha_L}{1-\alpha_L}[E_\pi - \iota E_v]$ such that

$$w = \alpha_L (x-\psi) + (1-\alpha_L) z + \alpha_L \frac{\kappa_I}{p(\phi)} [r(\theta - \iota) + \theta (1-\iota) q(\theta)] \tag{C.2}$$

To proceed further with the wage rule we must look at the financial market stage, and particularly at the bargaining problem between creditor and project. Creditors and projects know that the outcome of their bargaining in the financial market will affect the surplus of the labor match and, hence, the outcome of this wage bargaining. That is, they know that the asset value E_π is decreasing in ψ, while E_v is pinned down by free entry in the financial market at $E_v = \kappa_I / p(\phi)$. This resulted in the wage rule (C.2), which shows a decrease in the repayment ψ with slope α_L.

The repayment ψ is the solution to the following Nash bargaining game:

$$\psi = \text{argmax}(B_v - B_c)^{\alpha_C} (E_v - E_c)^{1-\alpha_C}$$

Contrary to the block bargaining assumption, the absolute values of the slopes of B_v and E_v are no longer identical. Indeed, we have

$$\frac{\partial B_v}{\partial \psi} = \frac{q(\theta)}{r + s^C + q(\theta)} \frac{1}{r+s^C} \quad \text{and} \quad \frac{\partial E_v}{\partial \psi} = -\frac{q(\theta)}{r+s^C+q(\theta)} \left(\frac{1-\alpha_L}{r+s^C} \right)$$

An increase in the repayment ψ has a positive effect on the surplus to the creditor of a credit match, by $\frac{q(\theta)}{r+q(\theta)}\frac{1}{r+s^C}$. However, that same increase in the flow repayment ψ reduce the surplus of the credit match to the project by a factor $(1-\alpha_L)\frac{q(\theta)}{r+s^C+q(\theta)}\frac{1}{r+s^C} < \frac{q(\theta)}{r+s^C+q(\theta)}\frac{1}{r+s^C}$. This implies that the project passes a fraction α_L of the marginal additional repayment to the worker in wage negotiation. The consequence is that the sharing rule in the financial market will be slightly different from the usual one: we will have instead

$$\alpha_C (E_v - E_c) = (1-\alpha_C)(1-\alpha_L)(B_v - B_c)$$

or

$$\tilde{\alpha}_C (E_v - E_c) = (1-\tilde{\alpha}_C)(B_v - B_c) \tag{C.3}$$

where

$$\tilde{\alpha}_C \equiv \frac{\alpha_C}{\alpha_C + (1-\alpha_C)(1-\alpha_L)} \tag{C.4}$$

The creditor receives a fraction $\tilde{\alpha}_C$ of the credit match surplus, and the project receives a fraction $(1-\tilde{\alpha}_C)$ of the credit match surplus. This fraction $\tilde{\alpha}_C$ is in the interval $[\alpha_C, 1]$

and increasing in α_L. In particular, when $\alpha_L = 0$, then $\tilde{\alpha}_C = \alpha_C$ and when $\alpha_L = 1$, then $\tilde{\alpha}_C = 1$. This is an important result obtained in Wasmer and Weil (2004). The share of the surplus accruing to the creditor is increasing the strength of the worker when it is bargaining with the project.

Recall that free entry in the financial market leads to values of the labor search stage $B_v = \kappa_B/(\phi p(\phi))$ and $E_v = \kappa_I/p(\phi)$, and that from Nash bargaining, $E_v = (1-\tilde{\alpha}_C)J_v$ and $B_v = \tilde{\alpha}_C J_v$. As a result, the combination of free entry in the financial market and Nash bargaining over the repayment ψ leads to a unique solution for financial market tightness in the case of sequential bargaining ϕ^*_α:

Equilibrium financial market tightness under sequential bargaining:

$$\phi^*_\alpha = \frac{\kappa_B}{\kappa_I}\frac{1-\tilde{\alpha}_C}{\tilde{\alpha}_C} = \frac{\kappa_B}{\kappa_I}\frac{(1-\alpha_C)(1-\alpha_L)}{\alpha_C} < \phi^* \tag{C.5}$$

$$\psi = \tilde{\alpha}_C(x-w) + (1-\tilde{\alpha}_C)\frac{\gamma(r+s^C)}{q(\theta)} \tag{C.6}$$

Equilibrium financial market tightness now depends on the outcome of bargaining in the labor market, and we distinguish equilibrium financial market tightness in the sequential bargaining case relative to the block-bargaining case with a subscript α to denote the effect of the second stage bargaining on the first-stage effective sharing weights. Equilibrium financial market tightness, as stated in equation (C.5), is strictly less than equilibrium financial market tightness under block bargaining. A greater bargaining weight to workers, α_L, leads to increasing entry of creditors relative to projects in the financial market as it effectively increases the creditor's bargaining power.

Finally, the bargained repayment to the creditor ψ under sequential bargaining is similar to that under block bargaining with the exception of the sharing weight $\tilde{\alpha}_C$: the creditor is able to extract a larger share of the profit flow $(x-w)$ than previously as the sequential nature of bargaining increases the creditor effective bargaining weight.

C.1.2 The strategic use of debt

We have just seen that increasing the repayment to the creditor can push the wage paid to labor downward. Thus agreeing on a higher repayment can increase the value of a match in the financial market by expropriating some surplus from the labor market. That is to say, creditors and firms can expropriate labor by setting the terms of their contract appropriately. Debt may thus be used strategically. Agreeing to a loan greater than the flow cost of recruiting γ will lead to large repayments ψ and a small surplus in the labor market between the worker and the project. We will now show that under full commitment of all agents, the parties in the financial market match can set a loan size such that the worker is pushed to a corner and paid his reservation wage. However, this arrangement is not privately incentive compatible. The firm will have an incentive to deviate from the terms of such a contract in the financial market. Under limited commitment, an incentive compatibility constraint places an upper bound on the size of loan that can be agreed upon in the financial market and, therefore, an upper bound on the amount of expropriation of workers.

We begin by modifying the Bellman equations of the creditor and the project in the labor search stage by introducing a flow loan L, which may be larger than the flow cost of recruiting γ, from the creditor to the firm:

$$(r+s^C)E_v = -\gamma + L + q(\theta)\,(E_\pi - E_v) \tag{C.7}$$

$$(r+s^C)B_v = -L + q(\theta)\,(B_\pi - B_v) \tag{C.8}$$

We retain the assumptions of free entry in the financial market and Nash bargaining to determine the repayment ψ. However, the financial market pair now have debt as an instrument to increase the value of their joint surplus. Indeed, by setting a larger loan L, and ensuring a greater flow repayment to the creditor in the profit stage, the pair can generate a lower equilibrium wage in the labor market. The most that can be achieved following this strategy is to push the worker down to his reservation wage, i.e., $w = z$. This is possible if the loan is sufficiently large so as to ensure that no surplus enters the project in the profit stage and bargaining. That is, the creditor and firm set the loan L such that $[E_\pi - \iota E_v] = 0$. This results in a flow repayment:

$$\psi = x - z - \iota(r+s^C)\frac{\kappa_I}{p(\phi)} \tag{C.9}$$

If the firm and creditor follow this path for setting the size of the loan, then the worker's wage is independent of the agreed repayment ψ and the credit match is divided according to the bargaining weight α_C just as in the block-bargaining environment. Given this loan and repayment schedule, it remains that equilibrium financial market tightness is given by

$$\phi^* = \frac{\kappa_B}{\kappa_I}\frac{1-\alpha_C}{\alpha_C}$$

We can now say something about the loan size and the equilibrium in the labor market. We can derive the implied value of the loan L from the value of the labor market stage for the project, $(r+s^C)E_v = -\gamma + L + q(\theta)\,(E_\pi - E_v)$. Doing so delivers a loan:

$$L = \gamma + \left[r + s^C + q(\theta)\,(1-\iota)\right]\frac{\kappa_I}{p(\phi)} \tag{C.10}$$

where we have made use of the definition of the surplus in the labor market to the project, along with free entry in the financial market.

In order to determine the equilibrium in the labor market, sum the asset values of the labor search stage for the creditor and project, B_v and E_v, and use the repayment in equation (C.9) to derive the value of the profit state for the creditor

$$B_\pi = \frac{\psi}{r+s^C} = \frac{x-z}{r+s^C} - \iota\frac{\kappa_I}{p(\phi)}$$

in order to obtain an equilibrium condition for labor market tightness:

$$\left(\frac{r+s^C}{q(\theta)} + 1\right)K(\phi) + \frac{\gamma}{q(\theta)} = \frac{x-z}{r+s^C} \tag{C.11}$$

This implies higher labor market tightness than in either the sequential or block-bargaining cases. However, the equilibrium may not be incentive compatible for the project depending on the assumption on the outside option during wage bargaining, i.e., if ι = 1 or 0.

C.2 Inefficient job destruction and endogenous firm bankruptcies

Another interesting development relates to the extension of the CL model to endogenous job destruction where a firm draws an idiosyncratic productivity x_i when meeting a worker in the labor market, and new productivities are drawn at the Poisson rate μ. We already extended the endogenous job destruction model of the labor market in chapter 1 to the presence of financial frictions in the appendix of chapter 5 (appendix 5.5.4) as studied in Petrosky-Nadeau (2013) and in section 6 of Wasmer and Weil (2004).

However, the existence of three parties (creditor, project, and worker) leads to subtle issues regarding the refinancing decision of the firm. We precisely study here how to introduce inefficient endogenous destruction of relationships in both the labor and financial market, building upon the efficient destruction model studies in the appendix of chapter 5 (appendix 5.5.4). Inefficient separations in the financial market may occur if bankruptcy is due to the inability of creditors to provide more liquidity, even when the firm is still viable. A discussion on inefficient separations is also in section 6 of Wasmer and Weil (2004).

Indeed, it was so far assumed in appendix 5.5.4 that a negative shock to productivity leads to efficient separations because projects and creditors (called the firms) were considering the separation decisions jointly with the worker. This implied notably that, when a positive total surplus exists, the creditor would agree on refinancing the firm for any value of productivity such that $J_\pi(a) > 0$ and $ax - w < 0$ since the owner of the project does not have any liquidity on its own.

However, this may not always be the case and creditors may decide to drop for negative shocks despite positive total surplus. This could arise either because they have a different assessment of the viability of the firm, or because, at the time of the decision of refinancing, their opportunity cost of funds differs from the one at the time of the agreement (in stage v). We explore what would happen in such a case. For that, we take the possibility of non-refinancing despite positive surplus seriously. To simplify, we even make the strong assumption that right away at the start of the firm, the creditor announces that there will not be any refinancing of the project beyond a fixed value $-\bar{\psi} > 0$. This would implicitly define a "creditor's threshold" productivity $A_B \geq A$ where A is still defined as $J_\pi(A) = 0$ and $A_B x - w(A_B) = -\bar{\psi}$, taking for granted that the wage is bargained as previously with the entity creditor/project and would still depends on current productivity.

Efficient separation arises when the threshold commitment by creditors is the efficient one: $A_B = A$. In contrast, the polar case of the creditor financing only the posting of a vacancy (one shot financing) but never refinancing the firm upon future negative cash flows corresponds instead to the assumption that A_B is such that $A_B x - w(A_B) = 0$, that is when $\bar{\psi} = 0$.

The new value equations of the project and of the creditor become, under these assumptions,

$$E_c = 0 \Rightarrow E_v = \kappa_I / p(\phi) \tag{C.12}$$

$$rE_v = -\gamma + \gamma + q(\theta)\left[\int_{A_B} E_\pi(a')\mathrm{d}G(a') - E_v\right] \tag{C.13}$$

$$rE_\pi(a) = \mathrm{Max}\,[0; ax - w(a) - \psi(a)] + \mu \int_{A_B} E_\pi(a')\mathrm{d}G(a') - \mu E_\pi(a) \tag{C.14}$$

$$B_c = 0 \Rightarrow B_v = \kappa_B/\check{p}(\phi) \tag{C.15}$$

$$rB_v = -\gamma + q(\theta)\int_{A_B}\left[B_\pi(a') - B_v\right]\mathrm{d}G(a') \tag{C.16}$$

$$rB_\pi(a) = \mathrm{Max}\left[\bar{\psi};\psi(a)\right] + \mu\int_{A_B} B_\pi(a')\mathrm{d}G(a') - \mu B_\pi(a). \tag{C.17}$$

where

$\psi(a) > 0$ if $ax - w(a) > 0$ and $\psi(a) = ax - w(a)$ if $ax - w(a) < 0$ and $a > A_B$

Regardless of the threshold productivity of the creditors, it could be the case that the project owner also has an incentive to drop out from the relation if in particular $E_\pi(a) \leq 0$; however, inspection of equation (C.14) shows that this is never possible since any deficit not leading to bankruptcy is paid by the creditors. Bankruptcies are therefore only driven by a being above or below the creditor threshold A_B, but not by any specific threshold of the investment project.

The asset value of the joint association of the project and creditor is:

$$rJ_v = -\gamma + q(\theta)\int_{A_B}\left[J_\pi(a') - J_v\right]\mathrm{d}G(a') \tag{C.18}$$

$$rJ_\pi(a) = ax - w(a) + \mu\int_{A_B} J_\pi(a')\mathrm{d}G(a') - \mu J_\pi(x). \tag{C.19}$$

It is straightforward to see that for any a such that $A < a < A_B$, separations are inefficient; indeed, by hypothesis, both $E_\pi(a)$ and $J_\pi(a) = E_\pi(a) + B_\pi(a)$ are positive but yet the creditor wants to step back and the separation is jointly inefficient. Finally, in this productivity range, we have that

$$(r+\mu)B_\pi(a) = \psi(a) + \mu\int_{A_B} B_\pi(a')\mathrm{d}G(a')$$

and even if $\psi(a) < 0$ it could be that creditors expect higher positive values of productivity. In that case, the creditor's induced separation would even be privately inefficient if the creditor had committed to the rule A_B.

D Specific topics of the CLG model: varying the pool of customers

The interesting property of the extensive margin CLG model in the steady state is that two of the three relevant categories of labor market tightness, namely financial market tightness and goods market tightness, are actually simple function of the parameters. This is interesting in the sense that the job creation condition determines the value of the remaining tightness (labor market) as a solution to a one-dimensional equation with the parameters of the frictions in other markets shifting this curve in interpretable ways. This is to some extent a weakness in the sense that the data analysis, and in particular the data in the introduction, revealed sometimes large swings in some of the variables proxying these market tightness: it may therefore be either that some of these fluctuations arise from cyclical fluctuations of these parameters themselves, or alternatively that the steady state assumptions and free-entry assumptions are not relevant to capture short-run movements in the data.

We addressed this question at the end of chapter 5 in introducing endogenous search effort. There is another way of delivering a link between goods and labor market tightness, namely by relaxing the assumption made regarding the total number of potential consumers for the search good. We indeed explore here the role of this specific assumption.

D.1 A general model with alternative assumptions on consumption

To vary pool of consumers and make more explicit the role of assumptions about the size of this pool, denote the total number of potential consumers by $\bar{\mathcal{D}}$; a subset of them denoted by $\bar{\mathcal{D}}_1$ is willing to search and consume search good 1. The two may differ if for instance a subset of consumers with low income does not search for the search good as in chapter 7 where only employed agents could consume the search good. We have therefore

$$\bar{\mathcal{D}}_1 = \mathcal{D}_U + \mathcal{D}_M \leq \bar{\mathcal{D}}$$

There are a number of alternative assumptions related to these two numbers $\bar{\mathcal{D}}$ and $\bar{\mathcal{D}}_1$. As regards the mass of residents $\bar{\mathcal{D}}$, we fix it to 1 but it may vary in an open economy.

1. If profits are redistributed lump sum to all workers who are also the consumers, then the total number of consumers would be $\bar{\mathcal{D}}$=1. This leads to two subcases:
   a. $\bar{\mathcal{D}}_1 = \mathcal{D}_U + \mathcal{D}_M = 1$ if we assume that all workers search for the search good. This was the assumption in chapter 8.
   b. $\bar{\mathcal{D}}_1 = \mathcal{D}_U + \mathcal{D}_M = 1 - \mathcal{U}$ if we assume that only employed workers can consume the frictional good. This was the assumption of chapter 7.
2. The total number of consumers could also be the total number of agents: thus the sum of the mass of workers (e.g. mass 1), the mass $\mathcal{N}_g + \mathcal{N}_\pi$ of firms' owners, and possibly the mass of creditors in the case of the CLG model; then $\bar{\mathcal{D}}_1 = \bar{\mathcal{D}} = 1 + \mathcal{N}_g + \mathcal{N}_\pi + \mathcal{B}_g + \mathcal{B}_\pi = 1 + 2(1 - \mathcal{U}) = 3 - 2\mathcal{U}$ or simply $1 + (1 - \mathcal{U}) = 2 - \mathcal{U}$ in an LG model with no creditors.
3. In an economy with consumer mobility, the pool of potential consumers may be endogenous and adjust freely to determine $\bar{\mathcal{D}}_1$ so as, for instance, to equalize their utility to an exogenous outside option of consumption.

D.2 Stock-flow equations

The stock-flow equations are

$$\begin{aligned}
\dot{\mathcal{D}}_m &= M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g) - (s^C + s^L + s^G)\mathcal{D}_m = 0 \\
\dot{\mathcal{N}}_\pi &= M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g) - (s^C + s^L + s^G)\mathcal{N}_\pi = 0 \\
\dot{\mathcal{N}}_g &= M_L(\mathcal{U}, \mathcal{V}) + s^G\mathcal{N}_\pi - (s^C + s^L)\mathcal{N}_g - M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g) = 0
\end{aligned}$$

or

$$\begin{aligned}
M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g) &= (s^C + s^L + s^G)\mathcal{D}_m \\
M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g) &= (s^C + s^L + s^G)\mathcal{N}_\pi \\
M_L(\mathcal{U}, \mathcal{V}) + s^G\mathcal{N}_\pi &= (s^C + s^L)\mathcal{N}_g + M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g)
\end{aligned}$$

Using $M_G(\bar{e}\mathcal{D}_u, \mathcal{N}_g) = \mathcal{N}_g\lambda(\xi, \bar{e}) = \mathcal{D}_u\lambda(\xi, \bar{e})/\xi = \mathcal{D}_u\check{\lambda}(\xi, \bar{e})$ this leads to

$$\mathcal{D}_u\lambda(\xi, \bar{e})/\xi = (s^C + s^L + s^G)\mathcal{D}_m \tag{D.1}$$

$$\mathcal{N}_g\lambda(\xi, \bar{e}) = (s^C + s^L + s^G)\mathcal{N}_\pi \tag{D.2}$$

$$M_L(\mathcal{U}, \mathcal{V}) + s^G\mathcal{N}_\pi = \left[s^C + s^L + \lambda(\xi, \bar{e})\right]\mathcal{N}_g \tag{D.3}$$

Further, the assumption on consumption implies

$$\mathcal{D}_m = \mathcal{N}_\pi$$

and

$$\mathcal{D}_u + \mathcal{D}_m = \bar{D} \tag{D.4}$$

$$\mathcal{N}_g + \mathcal{N}_\pi = 1 - u \tag{D.5}$$

where $\bar{D}$ is a pool of consumers. It can either be $1-\mathcal{U}$, or 1, or even an endogenous number. The total efficient units of search by consumers is $e\mathcal{D}_u$. From equation (D.1) and equation (D.4), one has, with $s^T = s^C + s^L + s^G$,

$$\mathcal{D}_u/\bar{D} = \frac{s^T}{s^T + \lambda(\xi,\bar{e})/\xi} \tag{D.6}$$

while from equation (D.2) and equation (D.5), one has

$$\mathcal{N}_g/(1-\mathcal{U}) = \frac{s^T}{s^T + \lambda(\xi,\bar{e})} \tag{D.7}$$

D.3 Specific case 1: equilibrium with only employed consumers

This corresponds to the case $\bar{D} = 1-\mathcal{U}$ and as shown in chapter 7, this assumption delivers $\xi = 1$. Dividing equation (D.6) by equation (D.7) one obtains a fix point equation for

$$\xi = \frac{s^T + \lambda(\xi,\bar{e})}{s^T + +\lambda(\xi,\bar{e})/\xi}$$

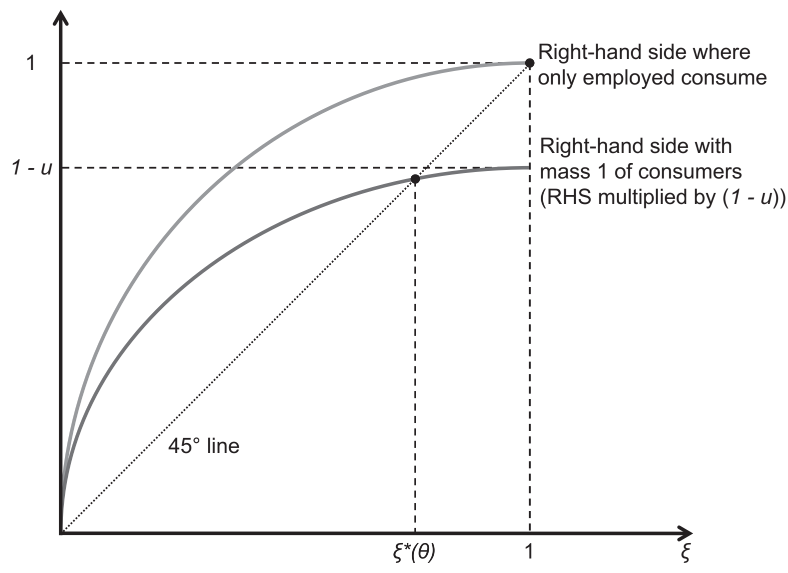

Figure D.1

The role of alternative assumptions about the consumer's pool for the steady-state relation between goods and labor market tightness where the right-hand side function is increasing in ξ. With Cobb–Douglas assumptions for instance, $\mathcal{M}_G(\mathcal{D}_U,\mathcal{N}_g) = \chi_G \mathcal{D}_U^{\eta_G} \mathcal{N}_g^{1-\eta_G}$ and so $\lambda(\xi,\bar{e}) = \chi_G \xi^{\eta_G} \bar{e}^{\eta_G}$ so that the right-hand side goes from 0 to 1 when ξ goes from 0 to infinity.

D.4 Specific case 2: all agents consume

This assumption corresponds to the case $\bar{D} = 1$ and it implies that goods market tightness depends on labor market tightness and that $\xi(\theta) < 1$. To see this, dividing equation (D.6) by equation (D.7) now delivers

$$\xi = (1-\mathcal{U}) \frac{s^T + \lambda(\xi,\bar{e})}{s^T + \lambda(\xi,\bar{e})/\xi}$$

So the fixed-point property is now dependent on the level of unemployment, itself equal to a function of θ: we have $1-\mathcal{U} = f(\theta/(s^L + s^C + f(\theta))$ is increasing in θ so that the fixed point in ξ is higher, the higher θ. Figure D.1 illustrates. The value of $\xi^*(\theta)$ is now lower than 1, and actually lower than $1-\mathcal{U}$. Note that, given the level of effort $\bar{e}$, a higher level of effort is associated with a lower RHS: this is easy to demonstrate for $\xi < 1$, and therefore a higher effort leads to a lower value of consumption tightness.

Math reminder: transitions in continuous and discrete time

In the book, workers, firms, creditors, and consumers alternate between different states. The transition probabilities per period (discrete time) or per unit of time (continuous time) are defined either by exogenous parameters (e.g., $s^L \times dt$, $s^G \times dt$) or by endogenous functions, for example, $f \times dt = \mathcal{M}_L(\mathcal{U}, \mathcal{V})/\mathcal{U} \times dt$ where dt is the time period considered. In discrete time, $dt = 1$ by normalization. In continuous time, dt is a small time interval.

In discrete time, the number of matches must be less than the stocks at the start of the period. Hence, the total number of matches for instance in the labor market is, formally, $Min\,[\mathcal{M}_L(\mathcal{U}, \mathcal{V}), \mathcal{U}, \mathcal{V}]$ with similar *Min* operators in all relevant markets.

In continuous time, the transition rate (e.g., $s \times dt$) is obtained from the assumption of underlying Poisson processes. A Poisson process of parameter s is an event that occurs k times in a time interval of T units of time with probability $P(k,T) = e^{sT}(sT)^k/k!$ where $k! = k \times (k-1)...2 \times 1$. Hence, the probability that the event has occurred once in time interval T is $P(1,T) = e^{sT}(sT)$. Applying it to an infinitely small time interval dt leads to $s \times dt + o(dt)$ where $o(dt)$ represents second- and higher-order terms from the Taylor transformation. For our purpose, rate s and all other transition rates will enter in Bellman equations.

To see this, take a transition process $f(t)$ featuring the entry rate into employment. Consider the PDV of the utility of unemployment and employment at time t, $W_U(t)$ and $W_E(t)$, and a stream of utility $z \times dt$ during time interval dt. Denote $\dot{W}_U(t)$ its partial time derivative. The PDV can be written recursively as

$$W_U(t) = z \times dt + \frac{f(t) \times dt}{1 + r \times dt} W_E(t + dt) + \frac{1 - f(t) \times dt}{1 + r \times dt} W_U(t + dt)$$

Now, using the Taylor expansion in the first order $W_U(t + dt) = W_U(t) + \dot{W}_U(t) \times dt$, the above equation is easily rewritten, eliminating second-order terms and canceling out $W_U(t)$ on the left- and right-hand side:

$$rW_U(t) = z + f\,[W_E(t) - W_U(t)] + \dot{W}_U(t)$$

The same is true for all recursive equations involving Poisson processes.