

Index

- Abbati, Pietro, 82
Abel, Niels Henrik, 1–3, 85–102,
189n–190n
Abelian Addition Theorem,
151, 200n
abelian equations, 98–100
abelian groups, 112–113
Abelian integrals, 151, 200n
Abel-Ruffini Theorem, 1–3, 89–94,
155–170, 200n
and Cauchy, 87, 93–94, 96
early life, 85–89, 189n
formulas of, 88
and Galois, 105, 108–109,
130–131, 145
and Gauss, 88–89, 95, 151
and Hamilton, 133
illness and death of, 101–102, 190n
notebooks of, 97, 152–153, 200n
and Ruffini, 87–89, 97, 190n
travels in Europe, 95–97
Académie des Sciences, 96, 104–106
Accounting. *See* Bookkeeping
Algebra
Arabic, 23–28, 45, 54
coefficient, 1, 44–45, 91–93
noncommutative, 131–143
roots, 1, 92, 98
symbolic notation, 40–45, 132
unknown, 43
variable, 1, 44
Algebraic functions, 90–91
al-Khwārizmī, Muhammed
ibn-Musa, 25–26, 30, 183n–184n
Alogon, 9
Alternating groups. *See* Groups
Analytic mathematics, 42, 59
Anrta, 9
Anticommutation, 134–136, 195n
Apollonius of Perga, 42, 51, 59
Aporia, 14, 182n
Archimedes, 32, 60
Area problem, 61–66
Aristotle, 7, 181n
Arithmos, 9, 181n
Athens, 15
Ausdehnungslehre (Grassmann),
135–136
Ayoub, Raymond, 189n, 192n–193n
Babylonian mathematics, 5, 7,
24–25, 30, 183n
Bacon, Francis, 182n–183n
Basham, A. L., 181n
Beaumarchais, Pierre Augustin
de, 200n

- Bell, Eric Temple, 191n
 Bernoulli, Daniel, 65
 Bolyai, János, 133, 196n
 Bombelli, Raphael, 54–55, 187n
 Bookkeeping, double-entry, 27–29, 184n–185n
 Boole, George, 132–133, 195n
 Boolean algebra, 132–133
 Bourbaki, Nicolas (pseudonym), 188n, 195n
 Boyer, Carl, 182n–195n
 Bring, E. S., 67, 188n
 Brioschi, Fernando, 146, 198n
 Brizio, Anna Maria, 184n
 Brown, R. G., 184n
 Bryce, R. A., 189n
 Bühler, W. K., 189n
Bulletin (Baron de Férussac), 96
 Bürgi, Jost, 48–49
 Burkert, Walter, 1981n
 Burn, R. P., 193n
 Burnside, William Snow, 190n, 192n–193n

 Cajori, Florian, 186n
 Calculators, 78, 149–150
 Calculus, 60, 62–63
 Cantor, Georg, 150, 199n
 Cardano, Girolamo, 30–40, 54, 57, 67, 69, 185n
 Cartier, Pierre, 197n
 Cauchy, Augustin, 83, 87, 96, 104–105
 Cauchy's theorem, 93, 163, 166, 175–180, 201n
 and commutativity, 132, 195n
 Causality and noncommutativity, 142, 196n
 Cayley, Arthur, 112, 126, 136–138, 195n
 Cayley numbers, 137
 Cayley tables, 112, 119, 122

 Cervantes, Miguel de, 23
 Charles X, 105
 Chatfield, Michael, 185n
 Chinese mathematics, 183n
 Christiania. *See* Oslo
 Cipher, 27–28, 33, 42
 Circle, 62
 Code. *See* Cipher
 Coleridge, Samuel Taylor, 135
 Commensurable, 7–8, 16
 Commercial arithmetic, 27–31
 Commutativity, 99, 112
 “Completing the cube,” 36–37, 120
 “Completing the square,” 25–26, 113
 Computers, 147, 197n–198n
 Conic sections, 42
 Connes, Alain, 197n
 Continuum, 11, 18
 Cooke, Roger, 200n
Cosa (coss), 27, 44
 Crelle, August, 95–97, 100, 102
 Crelle's Journal, 95, 101
 Cross product. *See* Multiplication
 Cube, 5–6
 symmetries of, 121
 Cyclic groups and symmetries, 113, 117, 122, 195n

 d'Alembert, Jean Le Rond, 68
 Dance, 111–130
 Dauben, Joseph Warren, 199n
 Dedekind, Richard, 183n, 199n
 Dehn, Edgar, 192n
 del Ferro, Scipione, 32–34
 del Ferro–Cardano–Tartaglia method, 32–35, 48–49, 54–55, 77, 174
 De Moivre, Abraham, 149, 197n
 DeMorgan, Augustus, 132, 195n
 Descartes, René, 50–59, 68, 187n
 and conic sections, 64

- Descartes's rule of signs, 53
La Geometrie, 50–58
 “relativity” of roots, 57, 140
 Dickson, Leonard E., 192n
 Dimension (algebraic), 50–51
 Dirac, Paul, 141
Disquisitiones Arithmeticae (Gauss), 79, 189n
 Dodecahedron, 5–6
 symmetries of, 124–125, 126–130
Don Quixote (Cervantes), 23
 Dörrie, Heinrich, 188n–190n
 Dunham, William, 185n, 199n
 Duplication of cube, 196n

e, 28, 150, 184n, 199n
 École Polytechnique, 105
 École Préparatoire (École Normale Supérieure), 105
 Edwards, Harold M., 191n
 Einstein, Albert, 140, 143
 Eisenstein, E. L., 184n
Elements. *See* Euclid
 Equations, algebraic
 approximate solutions, 66, 147, 198n
 cubic, 3, 28, 30–37, 90, 113–120, 148–149, 185n
 general formulation, 1–3
 quadratic, 2, 23, 25–26, 64, 90–91, 111–113, 185n
 quartic, 2, 35, 38–39, 76–78, 120–122
 quintic, 2–3, 77–78, 91–99, 122–129, 198n
 roots, 1
 Erlangen Program (Felix Klein), 138–140, 196n
 Euclid, 5, 17–23, 42, 59, 145–146, 150, 183n
 Euclidian geometry, 139
 Eudoxus, 17–18, 183m

 Euler, Leonhard, 62, 68, 90, 149, 196n
 Ewald, William B., 187n, 195n, 199n

 Fauvel, John, 184n–185n
 Fearnly-Sander, Desmond, 195n
 Fermat's Last Theorem, 87–88, 189n–190n
 Ferrari, Ludovico (Luigi), 34–35, 37–39, 57, 69, 76, 122
 Fibonacci. *See* Leonardo of Pisa
 Field, J. V., 186n, 199n
 Fields (mathematics), 139
 Fields (physics), quantum theory of, 142–143
 Fine, Benjamin, 188n
 Fine structure constant, 197n
 Fontana, Niccolò. *See* Tartaglia
 Fractions, 7
 France, 45, 96–97, 102–108, 190n
 Fundamental Theorem of Algebra, 56, 68–73, 79, 146, 188n

 Galilei, Galileo, 49–50, 187n
 Galois, Évariste, 102–109, 190n–191n
 and Abel, 105–106, 108–109, 130–131, 145
 and Cauchy, 104–105
 death of, 106–108
 education of, 102–106, 190n
 and his father, 104–105
 Galois theory, 125–130, 191n–193n
 legend of, 108, 191n
 posthumous writings of, 108
 and Société des Amis du Peuple, 106–107
 and Stéphanie Poterin-Dumotel, 106
 Gårding, Lars, 190n
 Gauge fields, nonabelian, 142–143, 196n

- Gauss, Carl Friedrich, 70–74, 97, 187n–189n
 and Abel, 89, 95, 100, 151
 and commutativity, 131–132, 195n
 and unsolvability of quintic, 79, 88
- Gazalé, Midhat, 183n
- Gel'fond, A. O., 197n
- Gentzen, Gerhard, 197n
- Geometrie, La* (Descartes), 50–54, 187n
- Geometry, 50, 60, 66
- Germain, Sophie, 104
- Gibbs, Josiah Willard, 136
- Gibbs, W. Wayt, 195n
- Gies, J. and F., 184n
- Girard, Albert, 51, 56, 68, 187n
 Girard's identities, 61, 92
- Gleason, Andrew, 187n
- God, 49, 55
- Gödel, Kurt, 197n
- "Golden ratio," 28
- Goldstine, Herman H., 198n
- González de Posada, Francisco, 198n
- Gorman, Peter, 181n
- Grafton, Anthony, 185n
- Grassmann, Hermann, 135–136, 195n
- Gray, Jeremy, 184n–185n, 197n
- Great Art* (Cardano), 30–40, 185n
- Greek mathematics, 5–21
- Greene, Brian, 196n
- Gregory, Duncan, 132, 195n
- Grossmann, Israel, 193n
- Groups, 109, 111–130, 138–140, 193n–195n
 A_3 , 118–120
 A_4 , 121–122
 A_5 , 123–129, 139
 abelian, 112–113, 129
 continuous, 140
 cosets, 176
 cyclical, 113, 117, 122, 175–180, 195n
 definition of, 125–126
 identity, 112, 119, 125
 invariant subgroups, 119, 129
 Lagrange's Theorem, 128, 175–176
 Lorentz, 196n
 monster group, 130, 195n
 nonabelian, 118, 129, 142–143
 normal subgroups, 119, 129, 193n–195n
 order, 176
 and permutations, 175–180
 philosophical aspects, 193n
 quotient, 130, 176–177, 193n–194n
 S_2 , 112–113
 S_3 , 113–120, 139
 S_4 , 120–122, 139
 S_5 , 122–124
 simple groups, 130
 solvable chains of, 130, 194n–195n
 V , 122
 visualization of, 193n
- Guérard, Albert, 190n
- Hadlock, Charles Robert, 192n
- Hamilton, William Rowan, 133–136, 196n
- Hankins, Thomas L., 196n
- Harmony of the World* (Kepler), 48, 121, 124, 186n
- Hartshorne, Robin, 181n, 191n, 194n
- Heath, Thomas, 183n
- Heisenberg uncertainty principle, 141
- Hellman, Morton J., 185n
- Henry IV, 45
- Heptagon, 48, 186n–187n
- Hermite, Charles, 146, 150, 198n
- Herrstein, I. N., 192n
- Hexagon, 48

- Hilbert, David, 197n
 Hippias of Mesopotum, 10
 Hirano, Yoichi, 193n
 Hoe, J., 183n
 Hölder, Otto, 130–131, 177, 194n
 Holmboe, Berndt Michael, 87, 97, 190n, 200n
 Holy Spirit, 55
 Huffman, C. A., 181n
 Huntley, H. E., 181n
 Hypergeometric functions, 198n–199n

 Icosahedron, 5–6
 symmetries of, 123–129
 Incommensurability, 7–14
 Indian mathematics, 9
 Indistinguishability of quanta, 142
 Infinity, 22, 146, 148, 151, 153
 Institut de France, 100, 106
 Invariance, 113, 139
 Invariant subgroups. *See* Groups
 Irrational magnitudes, 7–14, 19–21, 145–146, 183n
 Irreducible case (cubic equations), 54
 Irreversibility, 141
 Isograph, 147

 Jacobi, Carl Gustav Jacob, 100, 146
 Jacobson, Nathan, 192n
 Jerrard, George B., 67, 133, 188n, 195n
 Johnston, K. S., 184n
 Jordan, Camille, 130–131, 133, 146, 177, 194n
 “July monarchy,” 105–106

 Kabbalists, 48
 Kaku, Michio, 196n
 Kant, Immanuel, 200n
 Karl XIII, 85

 Kemp, Christine, 96, 101–102
 Kepler, Johannes, 48–49, 121, 124, 186n–187n
 Khayyām, Omar, 30, 184n
 Kiernan, B. Melvin, 193n
 King, R. Bruce, 192n
 Klein, Felix, 138–140, 143, 189n, 191n, 196n, 199n
 Klein, Jacob, 182n, 187n
 Kline, Morris, 182n, 192n
 Knorr, Wilbur Richard, 182n
 Kronecker, Leopold, 146, 190n, 198n

 Lafayette, General, 105
La Geometrie (Descartes), 50–58
 Lagrange, Joseph-Louis, 73–83, 87, 188n
 Lagrange resolvent, 74–79
 Lagrange’s Theorem, 128, 175–176, 194n
 Lalanne, Leon, 147
 La Nave, Federica, 187n
 Laplace, Pierre Simon, 51, 80
 Legendre, Adrien-Marie, 96, 100–101
 Leibniz, Gottfried Wilhelm, 55, 65–67, 183n, 187n–188n
 Le Lionnais, François, 199n
Le Mariage de Figaro (Beaumarchais), 200n
 Lemniscate, 65, 152–153, 200n
 Leonardo da Vinci, 6, 28, 184n
 Leonardo of Pisa (Fibonacci), 27–28, 30, 184n
 Lieber, Lillian R., 192n
 Lindemann, Ferdinand, 150, 199n
 Liouville, Joseph, 133
 Littlewood, D. E., 192n
 Locus problem, 57, 59
Logos, 9
 Louis XVI, 104
 Louis XVIII, 104–105

- Louis-Philippe I, 105–106
 Lycée Louis-le-Grand, 104
- Macve, Richard, 184n
 Magnitudes, 7–8, 23
 Magnus, Wilhelm, 193n
 Malfatti, Gianfrancesco, 77, 82
 Maor, Eli, 184n, 197n, 199n
 Marinoni, Augusto, 184n
*Mathematica*TM, 198n
 Matrix, 136–138
 Maxfield, John E. and Margaret W., 191n–192n
 Maxwell, James Clerk, 135–136
 Maxwellian dynamics, 141
 Mayer, Uwe F., 188n
 Mazur, Barry, 187n
 Meno, 13–14, 182n
Mercantile Arithmetic (Widman), 29
 Merzbach, Uta C., 182n–195n
 Minkowski, Hermann, 140
 Mitchell, David, 193n
 Modular functions, 198n
 Monster. *See* Groups
 Montucla, Jean Étienne, 79, 189n
 Morduhai-Boltovskoy, D., 197n
 Multiplication
 commutativity of, 131–132
 Grassmann algebra, 135–136
 matrix, 136–138
 quaternion, 134
 scalar product, 135
 vector product, 135
 Music, 7, 19–20, 48, 183n
 Mutafian, Claude, 192n
- Nahin, Paul J., 187n
 Napoleon, 104, 108
 Needham, Joseph, 183n
 Newton, Isaac, 59–66, 149–150, 187n–188n
 and Descartes, 59, 66
 lemma 28, 61–66, 148
 Newton’s identities, 60–61
 Newton’s method, 66
 Newtonian dynamics, 136, 141
 Niven, Ivan, 199n
 Nonabelian gauge fields, 142–143, 196n
 Nonabelian groups. *See* Groups
 Noncommutative geometry, 143, 197n
 Noncommutativity, 99–100, 131–143, 195n
 Normal subgroups. *See* Groups
 Norway, 85
 Numbers
 algebraic, 146, 150, 197n
 complex and imaginary, 54–56, 70, 148–149, 187n
 counting, 9
 in Greek mathematics, 9
 irrational magnitudes, 7–8, 18–19, 23, 146
 line, 51
 negative, 51–54, 187n
 octonions (Cayley numbers), 137
 place value, 24
 quaternions, 134–135, 196n
 rational, 7–8, 146
 sexagesimal, 24–25
 transcendental, 62, 66, 150, 197n, 199n
 ultraradical, 146, 150, 197n
- Octahedron, 5–6
 symmetries of, 121
 Octonions. *See* numbers
 Ore, Øystein, 185n, 189n
 Oslo, 87, 101
 Oval, 61–66
- Pacioli, Luca, 6, 28–30, 184n–185n
 Panton, Arthur William, 190n, 192n

- Pappus, 10–11, 42, 57, 182n
 Parabola, 65
 Parshall, Karen Hunger, 184n–185n
 Pascal, Blaise, 147
 Peacock, George, 132, 195n
 Pentagon, 49
 Permutations, 75–77, 82, 108–109, 111–130, 175–180
 Pestic, Peter, 142, 182n–183n, 186n, 188n, 196n–197n
 Peterson, Mark, 185n
 Pi (π), 62, 150, 199n
 Pierce, Benjamin, 138, 195n
 Pierce, C. S., 138, 195n
 Piero della Francesca, 28, 30, 185n
 Pierpont, J., 190n
 Planck, Max, 141, 196n
 Plato, 11–17, 44, 140–141, 182n
 Platonic solids, 5–6, 122, 138, 143, 193n
 Poisson, Siméon-Denis, 101
 Postnikov, M. M., 192n
 Poterin-Dumotel, Stéphanie, 106
 Pourciau, Bruce, 188n
Principia (Newton), 59–66, 187n
 Pycior, Helena M., 186n–187n
 Pythagoras, 5–11, 46, 181n
 Pythagorean theorem, 11
 Pythagoreans, 5–11, 15

 Quantum theory, 141–143, 196
 Quaternions. *See* numbers

 Radicals, 2, 35
 Ralph, Leslie, 181n
 Rashed, Roshdi, 184n
 Raspail, François-Vincent, 97, 106, 108, 191n
 Rational magnitudes, 9
 Ratios, 7
Reductio ad absurdum, 7–8, 64, 90
 Relativity
 and Galois theory, 140, 196n–197n
 general, 143, 196n
 of roots, 57, 140
 special, 140
 of space-time, 140
Republic (Plato), 15, 182n
 Resolvent, *see* Lagrange resolvent
 Richard, Louis-Paul-Émile, 104–105
 Roman law, 43
 Roots of unity, 74, 97
 Rosen, Michael, 190n, 200n
 Rosenberger, Gerhard, 188n
 Rothman, Tony, 191n
 Royal Frederick's University, Christiania (Oslo), 87
Rta, 9, 181n
 Ruffini, Paolo, 80–83

 Sacrifice, 10, 46
 Saigey, Jaques Frédéric, 96
 Scalars, 135
 Second Law of Thermodynamics, 141
 Seventeen-sided polygon, 70, 74, 189n
 Shanker, S. G., 197n
 Shurman, Jerry, 192n
 Shylock, 27, 184n
 Singh, Simon, 190n
 Skau, Christian, 190n
 Smale, Steve, 197n
 Société des Amis du Peuple, 106–107
 Socrates, 13–17, 182n
 Solomon, Ron, 195n
 Solution in radicals, 2
 Space
 four-dimensional, 135, 197n
 n-dimensional, 135–136, 138
 three-dimensional, 139–141, 143
 Spearman, Blair K., 198n
 Species, logic of, 44, 132

- Speed of light, 140, 142, 196n
 Square, 7–14
 Square roots, sound of, 20
 Squaring the circle, 150, 196n
 Stahl, Saul, 191n, 195n
 “Standard theory” (physics),
 142, 196n
 Stein, Howard, 183n
 Steiner, George, 191n
 Stewart, Ian, 192n, 197n
 Stillwell, John, 193n
 Stubhaug, Arild, 189n–191n, 200n
 Subgroups. *See* Groups
 Suleiman II, 153
Summary of Arithmetic (Pacioli),
 28–29
 Swetz, Frank, 185n
 Sylvester, James Joseph,
 136–138, 195n
 Symmetric groups. *See* Groups
 Symmetry, 113
 in algebraic expressions, 60
 of fundamental particles,
 142–143
 of polyhedra (*see* Triangle;
 Tetrahedron; Cube;
 Dodecahedron; Icosahedron;
 Octahedron)
 of three-dimensional space, 124
 Synthetic mathematics, 42–43
- Tartaglia, 32–34, 185n
 Taton, René, 191n
 Taylor, R. Emmett, 184n
 Tetractys, 10
 Tetrahedron, 5–6
 symmetries of, 120–121
 Theaetetus, 15–17, 20, 182n
 Theodorus, 17, 182n
 Theology, Christian, 55
 Thermodynamics, 141
 Theta functions, 146, 198n
- Third Law of Planetary Motion
 (Kepler), 49
 Tignol, Jean–Pierre, 192n
 Time, irreversibility of, 141
 Topology, 72
 Torres Quevedo, Leonardo,
 147, 198n
 Torture, 16–17, 182n–183n
 Toti Rigatelli, Laura, 190n–192n
 Transcendental. *See* Number
 “Treviso arithmetic,” 29
 Trial, 18
 Triangle, symmetries of, 113–120
 Trigonometry, 47, 149–150, 186n
 Trisection of angle, 187n, 196n
 Tschebotaröw, N., 192n
- Universal Arithmetic* (Newton), 60
 Uspensky, J. V., 185n, 188n
- Vandermonde, Alexandre-
 Théophile, 75, 188n
 van der Waerden, B. L., 183n–184n,
 188n, 193n
 van Roomen, Adriaan, 45–47, 186n
 Vectors, 135
 Vernier, Hippolyte Jean, 104
 Verriest, G., 192n
 Vetter, Guido, 186n
 Viète, François, 41–47, 56–58, 73,
 132, 151, 186n
 Voltaire, François Marie
 Arouet de, 85
 von Fritz, Kurt, 182n
 von Tschirnhaus, Count Ehrenfried
 Walter, 66–68, 188n
 Tschirnhaus transformation, 68
- Walker, D. P., 186n
 Wallis, John, 186n
 Weil, Simone, 183n
 West, M. L., 183n

Weyl, Hermann, 140, 193n, 196n
Widman, Johann, 29
Wiles, Andrew, 189n
Williams, Kenneth S., 198n
Wilson, Curtis, 193n
Winternitz, Emmanuel, 184n
Wussing, Hans, 188n–189n, 193n

Xenophon, 182n

Yaglom, I. M., 192n, 195n–196n
Yamey, B. S., 184n

Zammattio, Carlo, 184n
Zero, 9, 43