Q16.1-Q16.3 Friberg: Managing risk and uncertainty

Q16.1 A little thought notes that programs 1 and 3 are equivalent and that 2 and 4 are equivalent. When put before subjects a majority chooses 1 over 2 but 4 over 3: thus different choices depending on how choices are framed (the original experiments posed the choice between 1 and 2 to different group than those that were asked to select between 3 and 4).

Q16.2 From the question we know that the probability that a cab is green P(G)=0.85, that a cab is blue is P(B)=0.15 and that the conditional probability that the subject says that the cab is blue given that it is in fact blue p(SB|B)=0.8 (and since there are only two alternatives here we know then that the probability that say green given that the cab is blue is p(SG|B)=0.2).

We want to find P(B|SB).

Using Bayes rule (Appendix 16.A) we know that

$$P(B|SB) = \frac{P(SB|B) \times P(B)}{P(SB)}.$$

Using the information in the question we know that

$$P(B|SB) = \frac{0.8 \times 0.15}{P(SB)}.$$

We now only need to find the unconditional probability that the witness says that the cab is blue.

$$P(SB) = P(B) \times P(SB|B) + P(G) \times P(SB|G).$$

$$P(SB) = 0.15 \times 0.8 + 0.15 \times 0.2 = 0.29$$

And hence

$$P(B|SB) = \frac{0.8 \times 0.15}{0.29} \approx 0.41.$$

So the probability that the cab was blue is only 41 percent. Despite the relatively high reliability of the witness (0.8) it is thus more likely than not that the he is wrong when he states that the cab was blue. Intuition for many of us leads us wrong and we overestimate the likelihood that the cab is indeed the same color that the witness says it is. This is sometimes known as the base rate fallacy - we tend to give to little weight to the base rate, the unconditional probability that only 15% of cabs are blue.

Q16.3 In the eyes of many Bill is portrayed as a stereotypical accountant but not as a jazz player. Denote by A the alternative that Bill is an accountant, as J that he plays jazz and by AJ that he is an accountant that plays jazz. Many subjects give the ranking A>AJ>J where the last ranking breaks the rules of probability. The likelihood that he is a jazz player must be greater than the likelihood that he is not only a jazz player but also an accountant (unless the probability that he’s an accountant is 1 in which the probabilities coincide).