Friberg: Managing risk and uncertainty. Q12.1

1 First consider the case with a constraint:

$$L = (p - c)(100 - p) + (sP - c)(100 - P) - \lambda(p - sP)$$

foc (answer to a))

$$\begin{aligned}
\frac{\partial L}{\partial p} &= c - 2p - \lambda + 100 \\
\frac{\partial L}{\partial P} &= c + 100s + \lambda s - 2Ps \\
\frac{\partial L}{\partial \lambda} &= -p + sP = 0
\end{aligned}$$

$$
\begin{aligned}
0 &= c - 2p - \lambda + 100 \\
0 &= c + 100s + \lambda s - 2Ps \\
0 &= -p + sP
\end{aligned}
$$

which yields equilibrium values (note if you do this in Scientific Workplace you can use "evaluate" to derive first order conditions and "solve" to find equilibrium values).

$$
\begin{aligned}
P &= \frac{1}{2s + 2s^2}(c + 200s + cs) \\
p &= \frac{1}{2s + 2}(c + 200s + cs) \\
\lambda &= -\frac{100s - 100}{s + 1}
\end{aligned}
$$

2 Now look at segmented markets.

For clarity use other notation for prices now

$$
\begin{aligned}
\Pi_S &= \max_{x,X} (x-c)(100-x) + (sX-c)(100-X) \\
\Pi_S &= (x-c)(100-x) + (sX-c)(100-X)
\end{aligned}
$$

foc

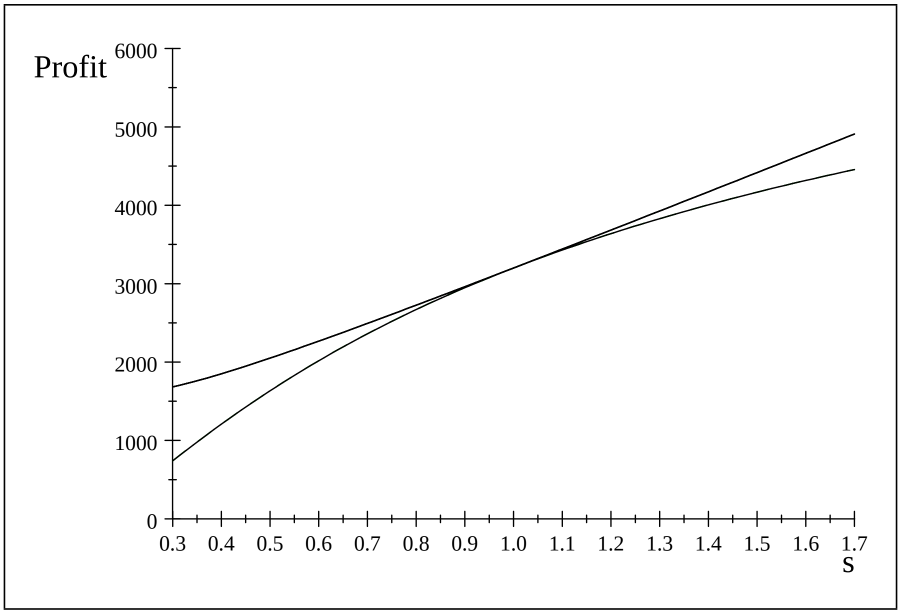
$$
\begin{aligned}
\frac{\partial \Pi_S}{\partial x} &= 120 - 2x \\
\frac{\partial \Pi_S}{\partial X} &= 100s - 2Xs + 20
\end{aligned}
$$

$$
\begin{aligned}
0 &= 120 - 2x \\
0 &= 100s - 2Xs + 20
\end{aligned}
$$

$$
\begin{aligned}
X &= \frac{1}{s}(50s + 10) \\
x &= 60
\end{aligned}
$$

Now, plot both

$$
\Pi_S = 1600 - (50s - 10)\left(\tfrac{1}{s}(50s + 10) - 100\right)
$$



Now look at expected values

let $s = 0.7$ with pr=0.5 and $s = 1.3$ also with pr=0.5

Then profits under integrated markets are $\Pi = 0.5 * 3829.1 + 0.5 * 2360.5 = 3094.8$

and under segmented markets $\Pi_S = 0.5 * 3926.9 + 0.5 * 2492.9 = 3209.9$