Q13.1 Friberg: Managing risk and uncertainty.

1 a) Linear demand

$q = 10 - p \Leftrightarrow p = 10 - q$

$$\max_q \left(10 - q\right) q - cq$$

first order condition:

$$\begin{aligned} \frac{d\Pi}{dq} &= 10 - 2q - 2 = 0 \\ q &= \frac{10 - c}{2} \end{aligned}$$

$$\begin{aligned} p &= 10 - q \\ &= 10 - \frac{10-c}{2} \\ &= 5 + \frac{c}{2} \Rightarrow \\ \frac{dp}{dc} &= \frac{1}{2} \end{aligned}$$

if $c = 2$:

$p = 5 - \frac{2}{2} = 4$

$q = 10 - 4 = 6$

$$\frac{dp}{dc}\frac{c}{p} = \frac{1}{2}\frac{2}{4} = \frac{1}{8}$$

if $c = 4$:

$p = 5 - \frac{4}{2} = 3$

$q = 10 - 3 = 7$

$$\frac{dp}{dc}\frac{c}{p} = \frac{1}{2}\frac{4}{7} = \frac{4}{14}$$

if cost increases from 2 to 3 price increases by 0.5 and if cost increases from 3 to 4 price likewise increases by 0.5. Thus pass-through equals 0.5 but is less than 0.5 in elasticity terms.

2 b) Constant elastic demand

Note that an omission in the question might invite confusion. In keeping with much of economics we are not very clear on whether we consider the elasticity of demand (negative) or its absolute value (positive). In this question $\varepsilon > 1$ whereas we also used ε to denote the elasticity of demand in the general case in equation 13.1. In that equation ε is negative.

$$q = p^{-\varepsilon} \Leftrightarrow p = q^{-\left(\frac{1}{\varepsilon}\right)}$$

$$\max_q q^{-\left(\frac{1}{\varepsilon}\right)} q - cq \quad \Leftrightarrow$$

$$\max_q q^{1-\left(\frac{1}{\varepsilon}\right)} - cq$$

first order condition:

$$\begin{aligned}
\frac{d\Pi}{dq} &= \left(1 - \left(\frac{1}{\varepsilon}\right)\right) q^{-\left(\frac{1}{\varepsilon}\right)} - c = 0 \\
q &= \left(c\frac{\varepsilon}{(\varepsilon - 1)}\right)^{-\varepsilon} \Rightarrow \\
p &= c\frac{\varepsilon}{(\varepsilon - 1)}
\end{aligned}$$

so pass-through is given by

$$\frac{dp}{dc} = \frac{\varepsilon}{(\varepsilon - 1)}$$

if $c = 2$:

$p = 2\frac{\varepsilon}{(\varepsilon-1)}$

$q = \left(2\frac{\varepsilon}{(\varepsilon-1)}\right)^{-\varepsilon}$

$$\frac{dp}{dc}\frac{c}{p} = \frac{\varepsilon}{(\varepsilon-1)}\frac{2}{2\frac{\varepsilon}{(\varepsilon-1)}} = 1.$$

The pass-through elasticity equals 1 also for $c = 4$. That the pass-through elasticity equals unity is a general result for a firm that faces a given constant-elastic demand function. When cost increase by one unit (from 2 to 3 or 3 to 4) price increases by $\frac{\varepsilon}{(\varepsilon-1)} > 1$.Thus pass-through is greater than 1 but equal to 1 in elasticity terms.