Chapter 8
Sticky prices
Answer Key to Exercises[1]

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1. Temporary reduction in money growth rate

The purpose of this exercise is to show that the sticky-prices model developed in the text is capable of explaining situations of "stagflation" (i.e., the co-existence of high inflation, relative to the rate of money growth, and output below the full-employment level).

In the context of the model developed in Section 2:

(a) Analyze the effects of a temporary reduction in the money growth rate.

(b) Explain the intuition behind the results.

Answer

Clearly, from (5), c^T is not affected. Consider then the unstable differential in m given by equation (22). A temporary reduction in μ implies the path depicted in Figure A1, Panel B. From the money demand (10), it then follows that i follows the path depicted in Figure A1, Panel C.

The phase diagram remains, of course, given by Figure 4. Imagine the pre-shock saddle path (i.e., the one corresponding to a high μ) going through point A. The saddle path corresponding to the low value of μ goes through point B, as depicted in the figure. The key to solving the temporary case graphically is to establish whether n and π jump at T or not. Clearly, n does not jump at T because M_T is given and so is

[1] This answer key is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries", currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 8.

P^N. Inflation of non-tradable goods cannot jump either because it is continuous along any perfect foresight path, as follows from equation (79). Since the dynamic system does not jump at T, we conclude that at time T the system must be somewhere along the saddle path going through point B. If this were not the case, the system would diverge over time.

The second important piece of information is that, as time goes by, the locus $\dot{\pi} = 0$ shifts leftward because m falls over time. This means that at time T the system cannot be to the right of point A; it must be to the left of point A.

In light of the above information, we infer that two dynamics paths are possible:

(a) For large values of T, the system will jump at time 0 from point A to a point close to C (more specifically, to a point on the vertical segment AC below μ^L and above C). The system will then travel in a northeast direction until it hits the $\dot{n} = 0$ locus and then travel in a northwest direction hitting the saddle path going through the point A at time T (at some point to the left of point A). The system then travels along the saddle path towards point A. (Figure A2, which was produced with the the MATLAB program developed in Chapter 14 and available online, illustrates this case.)

(b) For small values of T, the system will jump at time 0 from point A to some point that will lie along the segment AC but above μ^L. The system will then travel in a northwest direction and hit the saddle path going through point A at time T.The system then travels along the saddle path towards point A. The corresponding paths of π and n are depicted in Figure A1, Panel D and E, respectively. (Figure A3, which was produced with the the MATLAB program developed in Chapter 14 and available online, illustrates this case.)

    What will be the path of c^N (and hence of output of non-tradables)? From equation (96), we know that since $\dot{\pi}_t > 0$ then $c_t^N < \overline{y^N}$. We infer that, as depicted in Panel F, c_t^N falls on impact and then gradually return to its unchanged steady-state.

In sum, we can see how a temporary fall in μ leads to a situation of stagflation (during the period $[0, T)$) in the sense that even though output of non-tradables is below its full employment level, inflation is above the rate of money growth.

2. Fiscal policy in a sticky prices model

This exercise incorporates fiscal policy into the flexible exchange rates, sticky prices model analyzed in this chapter and studies the effects of a permanent increase in government spending on non-tradable goods.

Specifically, suppose preferences are given by

$$\int_0^\infty \left[\log(c_t^T) + \log(c_t^N) + \log(m_t)\right] e^{-\beta t} dt, \tag{86}$$

The consumer's intertemporal constraint is given by

$$a_0 + \int_0^\infty \left(y_t^T + \frac{y_t^N}{e_t} + \tau_t \right) e^{-rt} dt = \int_0^\infty \left(c_t^T + \frac{c_t^N}{e_t} + i_t m_t \right) e^{-rt} dt.$$

The government's flow budget constraint is given by

$$\dot{h}_t = r h_t + \dot{m}_t + \varepsilon_t m_t - \tau_t - \frac{g_t^N}{e_t}. \tag{87}$$

The corresponding intertemporal constraint is given by

$$h_0 + \int_0^\infty (\dot{m}_t + \tau_t + \varepsilon_t m_t) dt = \int_0^\infty \left(\frac{g_t^N}{e_t} \right) e^{-rt} dt. \tag{88}$$

The rest of the model remains the same as in the text.

Equilibrium in the non-tradable goods market dictates that

$$y_t^N = c_t^N + g_t^N.$$

In the context of this model, analyze the effects of a permanent and unanticipated increase in government spending on non-tradable goods.

Answer

First-order conditions are given by

$$\frac{1}{c_t^T} = \lambda, \tag{89}$$

$$\frac{1}{c_t^N} = \frac{\lambda}{e_t}, \tag{90}$$

$$\frac{1}{m_t} = \lambda i_t. \tag{91}$$

Combining (5) and (6), we obtain the familiar condition :

$$\frac{c_t^N}{c_t^T} = e_t. \tag{92}$$

Combining (5) and (7), we obtain a standard real money demand (in terms of m)

$$m_t = \frac{c_t^T}{i_t}. \tag{93}$$

As in the text, the system is block recursive. We can thus derive the unstable differential equation for m and show that it will be constant along a PFEP and equal to:

$$\bar{m} = \frac{\overline{c^T}}{(r + \bar{\mu})}. \tag{94}$$

Hence, shocks to full employment output of g^N will have no effect whatsoever on m, c^T, and hence on i and ε.

We now derive a two-differential-equation dynamic system in n_t and π_t. Since, by definition, $n = M/P^N$, then

$$\dot{n}_t = n_t(\bar{\mu} - \pi_t), \tag{95}$$

where, by definition, π_t ($\equiv \dot{P}^N/P^N$) is the rate of inflation of non-tradable goods.

To derive our second dynamic equation, first use (16) to rewrite (12) as

$$\dot{\pi}_t = \theta\left(\bar{y}^N - \bar{g}^N - c^N\right). \tag{96}$$

Taking into account (92) and noting that $e_t = n_t/m_t$, equation (96) can be rewritten as as:

$$\dot{\pi}_t = \theta\left(\bar{y}^N - \bar{g}^N - \frac{\overline{c^T}}{\bar{m}}n_t\right). \tag{97}$$

The system's steady-state is given by

$$\begin{aligned}\pi_{ss} &= \bar{\mu},\\ n_{ss} &= \frac{m}{c^T}(\bar{y}^N - \bar{g}^N).\end{aligned}$$

It is easy to check that, as in the text, the dynamic system is saddle-path stable. Notice that the phase diagram shown in Figure 4 remains valid for this dynamics sytem.

Unanticipated and permanent increase in g^N Suppose that prior to time 0 the economy is in a stationary equilibrium, At time 0, there is an unanticipated and permanent increase in g^N (Figure A2, Panel A). Clearly, c^T, m, i, and ε are not affected by this shock. Further, the fact that m remains unchanged at time 0 implies that the nominal exchange rate does not change either at time 0.

In terms of the dynamic system, we see that n_{ss} will fall. The phase diagram (not shown) then tells us that π jumps up on impact and then falls gradually over time (Panel B). Real money balances in terms of non-tradable goods (n) fall gradually over time (Panel C). Intuitively, the increase in g^N fully crowds out private consumption (c^N) in the new steady-state. The lower consumption leads to a fall in real money balances (n). For real money balances to fall over time, inflation of non-tradable goods must be higher than the rate of monetary growth.

To derive the path of the real exchange rate, recall that $e_t = n_t/m_t$ and that m_t is not affected by the change in g^N. It follows that e_t will mimic the behavior of n and fall gradually towards its lower steady-state (Panel D). Intuitively, in the new steady-state there is a lower net supply of non-tradable goods ($\bar{y}^N - g^N$) and, hence, there is an excess demand for non-tradable goods at the pre-shock relative price. The relative price must increase (i.e., e must fall) to clear the market. This real appreciation is effected over time by inflation of non-tradable goods being higher than the rate of depreciation (which does not change).

What about c^N? Recall that $c^N = ec^T$. Hence, c^N does not change on impact and then falls over time towards its lower steady-state value (Panel E).

What about r^d? Recall that $r_t^d = i_t - \pi_t$. Hence, r_t^d falls on impact and then gradually increases towards its unchanged steady-state value. The domestic real interest rate must be lower on impact to induce a declining path of c^N over time.

What about y^N? Recall that $y^N = c^N + g_t^N$. Hence, on impact y^N increases and then falls over time to its full-employment level (Panel F).

In sum, the permanent increase in g^N is expansionary and inflationary. In the steady-state, it fully crowds out c^N. (Interestingly enough, you should check that in this particular case, the same results would obtain if you solved this same experiment under predetermined exchange rates.)

Figure A1. Sticky prices model: Temporary reduction in money growth rate

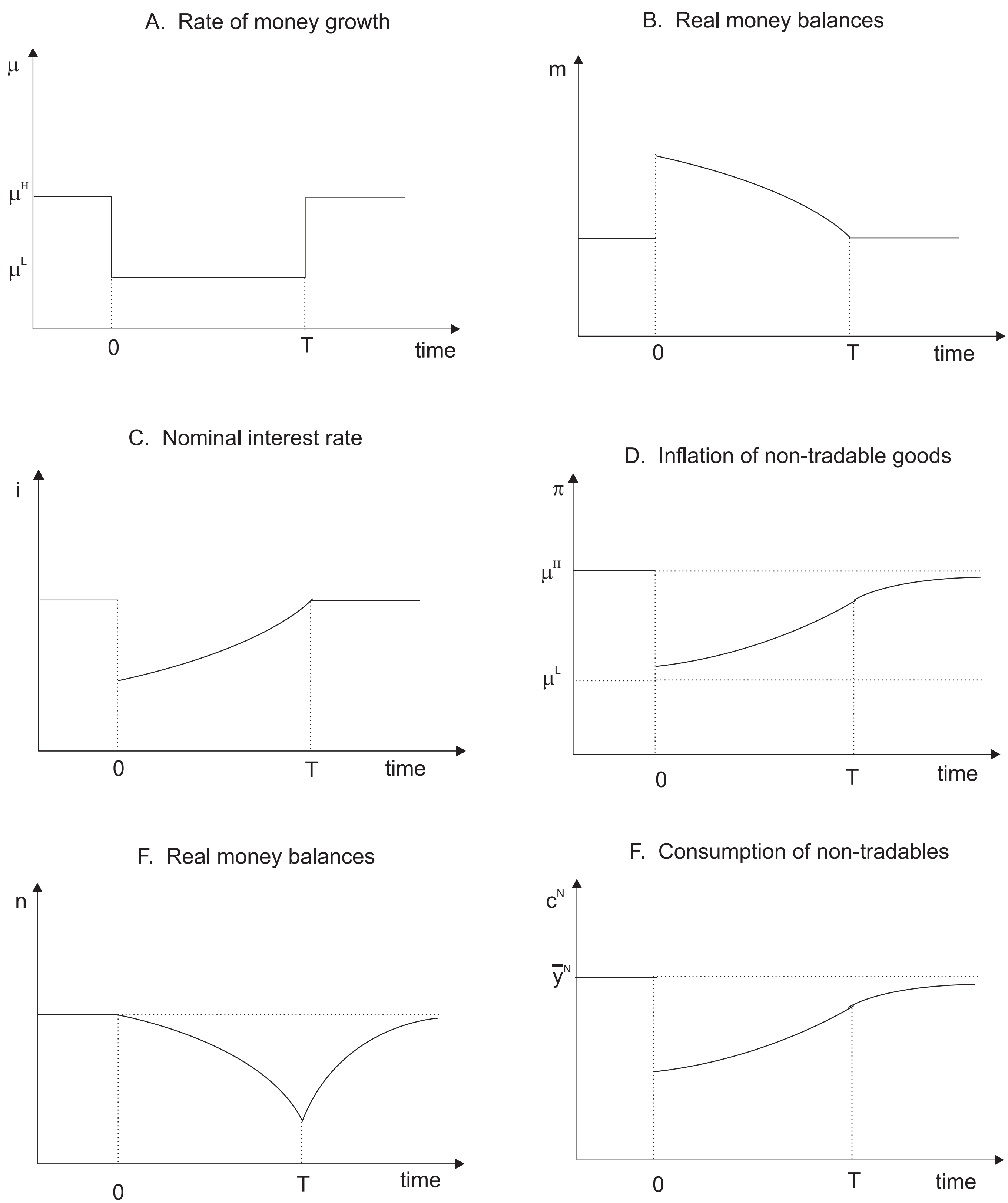
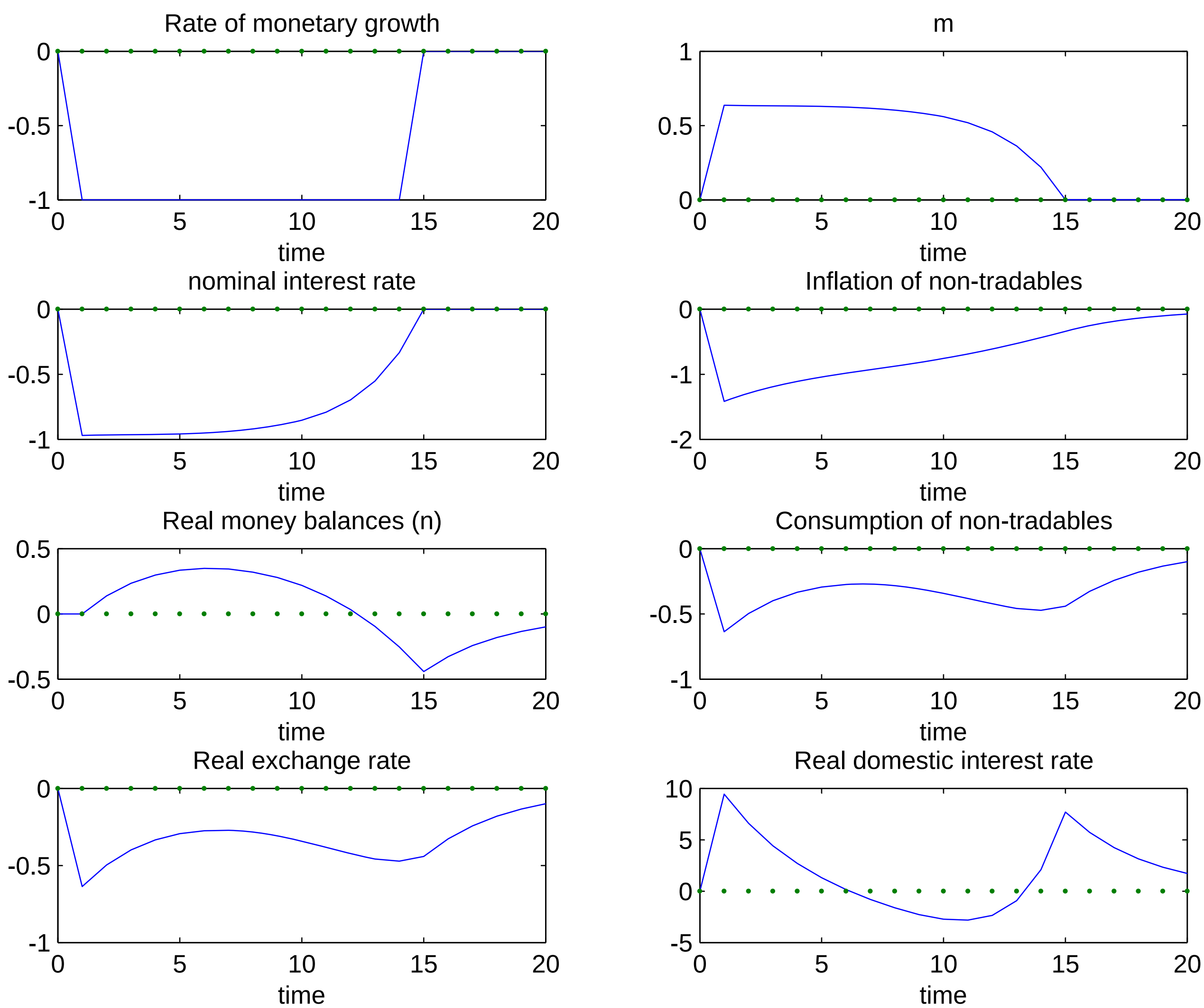


Figure A2. Temporary reduction in rate of monetary growth (large T)



A3
Figure ~~A1~~. Temporary reduction in rate of monetary growth (small T)

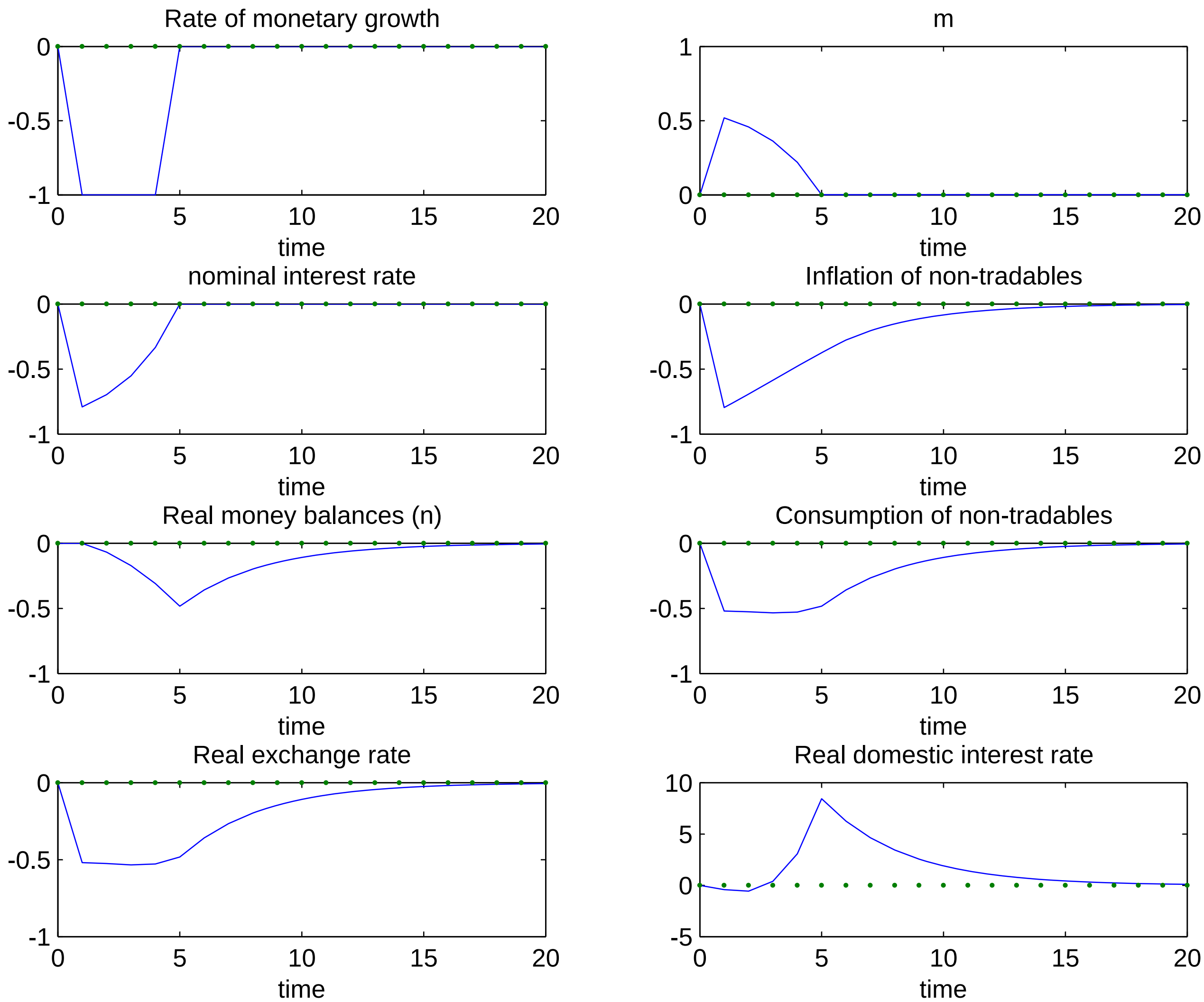


Figure A4. Permanent increase in government spending on non-tradable goods