Chapter 6
The basic monetary model
Answer Key to Exercises[1]

Carlos A. Végh
UCLA and NBER
E-mail: cvegh@ucla.edu

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1. The monetary approach model with a cash-in-advance

Consider the monetary model with no bonds developed in Section 2 of this chapter with the following modification. Instead of introducing money into the utility function, suppose that preferences are given by

$$\int_0^\infty u(c_t)e^{-\beta t}dt,$$

and that consumption is subject to a cash-in-advance constraint of the form:

$$m_t = \alpha c_t. \tag{100}$$

The rest of the model remains unchanged. In this context:

(a) Analyze the effects of an unanticipated and permanent devaluation. Explain the intuition behind the results. Do the results differ from those in the text? Why or why not?

(b) Analyze the effects of an unanticipated and permanent increase in the stock of domestic credit, D. Explain the intuition behind the results. Do the results differ from those in the text? Why or why not?

[1] This answer key is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries", currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 4. I am extremely grateful to Pablo Lopez Murphy for his invaluable help in the preparation of this manuscript.

(c) Analyze the effects of an unanticipated and permanent increase in the rate of devaluation (ε). Explain the intuition behind the results. Do the results differ from those in the text? Why or why not?

Answer

(a) The key to the solution is to realize that the consumer's problem becomes a completely mechanical one and that the consumer has no choice variable.[33] Substituting the cash-in-advance constraint into the equilibrium condition (15), we obtain:

$$\dot{m}_t = y - \frac{m_t}{\alpha},$$

which is a stable differential equation in m_t. In the initial steady-state, $m_{ss} = \alpha y$.When the unanticipated and permanent devaluation takes place at time 0, m falls on impact and then increases gradually towards its unchange steady-state value. Through the cash-in-advance, we know that consumption will also fall on impact and then rise gradually towards its unchanged steady-state. Hence, during the adjustment period the economy runs a trade surplus. The results are thus the same as in the text.

(b) When the unanticipated and permanent increase in D takes place, real money balances increase on impact and then fall gradually towards the unchanged steady-state. Consumption follows the same path. The economy gets rid of the unwanted money balances by running trade deficits throughout the transition. The results are thus the same as in the text.

(c) Since the cash-in-advance assumption implies that money demand is interest-rate inelastic, the increase in the rate of devaluation has no effect on real money demand (and, of course, it does not have any effect on money supply either). There are thus no real effects whatsover from an increase in the devaluation rate.

[33] Notice that we are assuming that the cash-in-advance binds.

2. Neutrality and superneutrality under flexible exchange rates

Consider once again the monetary model with no bonds developed in Section 2 of this chapter with the following two modifications. First, suppose that there are non-tradable goods. Preferences then become:

$$\int_0^\infty [u(c_t^T) + z(c_t^N) + v(m_t)]e^{-\beta t}dt,$$

where c_t^T and c_t^N denote consumption of tradable and non-tradable goods, respectively, and all functions are strictly increasing and strictly concave. The endowments of tradable (y^T) and non-tradable goods (y^T) are constant over time. Second, assume that the economy is operating under a flexible exchange rate regime.

(a) Show that an unanticipated and permanent increase in the level of the money supply has no real effect (i.e., only leads to an equiproportional change in the exchange rate). Discuss the intuition behind the results.

(b) Show that an unanticipated and permanent increase in the rate of growth of the money supply has no real effects. Discuss the intuition behind the results.

Answer

(a) Let us first solve the model for a constant value of the money growth rate, $\bar{\mu}$. Under flexible exchange rates, the change in international reserves is, by definition, zero. Hence, equation (14) implies that $c_t = y$ for all t. In this light, equation (9) reduces to

$$v'(m_t) = (\beta + \varepsilon_t)u'(y)]. \tag{101}$$

This just an equilibrium condition because both m_t and ε_t are endogenous variables under flexible exchange rates. From the definition of real money balances,

$$\frac{\dot{m}_t}{m_t} = \bar{\mu} - \varepsilon_t. \tag{102}$$

Solving for ε_t from (101) and substituting into (102), we obtain the following differential equation in m_t:

$$\dot{m}_t = m_t \left[\bar{\mu} + \beta - \frac{v'(m_t)}{u'(y)} \right]. \tag{103}$$

It's easy to check that this an unstable differential equation. Hence, a convergent equilibrium path requires that m_t be constant over time and implicitly defined by:

$$\frac{v'(m_t)}{u''(y)} = \bar{\mu} + \beta, \tag{104}$$

which is of course a money demand-type equation.

Suppose now that there is now an unanticipated and permanent increase in the level of the money supply. Clearly, the initial equilibrium just described is not affected. We thus conclude that the nominal exchange rate changes in the same proportion as M so as to leave real money balances unchanged. In other words, monetary policy is neutral.

(b) Suppose now that there is an unanticipated and permanent increase in the rate of growth of the money supply. From (104),.we infer that real money demand must fall. Intuitively, since the opportunity cost of holding money has increased, money demand falls. But since m_t is governed by the unstable differential equation (103), m needs to jump immediately to its new and lower value. This will be effected through a rise in the nominal exchange rate. There are thus no real effects; that is, monetary policy is also superneutral.

3. The two-good model with linear production

Consider the model of Handout # 3 with a linear production function for tradable goods. In other words, production of tradable goods is given by:

$$y_t^T = Z^T n_t^T.$$

In the context of this model, analyze the economy's response to a permanent devaluation. Explain the intuition behind the results.

Answer

Hamiltonian (controls are c^T, c^N, and n^T; state is: m):

$$H = \gamma \log(c_t^T) + (1-\gamma)\log(c_t^N) + \log(m_t) + \lambda_t \left[Z^T n + \frac{Z^N \left(\bar{n} - n_t^T\right)}{e_t} + \tau_t - c_t^T - \frac{c_t^N}{e_t} - \varepsilon_t m_t \right]$$

Optimality conditions are given by:

$$\begin{aligned}
\frac{\gamma}{c_t^T} &= \lambda_t \\
\frac{1-\gamma}{c_t^N} &= \frac{\lambda_t}{e_t} \\
Z^T &= \frac{Z^N}{e_t} \Rightarrow e_t = \frac{Z^N}{Z^T} \\
\dot{\lambda}_t &= (\beta + \varepsilon_t)\lambda - \frac{1}{m_t}
\end{aligned}$$

The new feature introduced by the assumption of linear production in both sectors is that the real exchange rate is fully determined by the technology (i.e., $e_t = Z^N/Z^T$). [It is already obvious, therefore, that a devaluation will not affect the real exchange rate and hence that the price of non-tradable goods (P^N) will increase by the same proportion as the nominal exchange rate.]

Proceeding as in the text, we can derive the following dynamic system in c^T and m:

$$\begin{aligned}
\dot{c}_t^T &= c^T \left(\frac{c^T}{\gamma m_t} - \beta - \varepsilon_t \right), \\
\dot{m}_t &= Z^T \bar{n} - \frac{c^T}{\gamma}.
\end{aligned}$$

Notice that this is, qualitatively, the same system that we had for the one-good model. It is easy to check that the system is saddle-path stable.

The initial steady-state of the economy is characterized by:

$$
\begin{aligned}
e_{ss} &= \frac{Z^N}{Z^T}, \\
m_{ss} &= \frac{Z^T \bar{n}}{\beta + \varepsilon}, \\
c_{ss}^T &= \gamma Z^T \bar{n}, \\
c_{ss}^N &= (1-\gamma) Z^N \bar{n}, \\
y_{ss}^T &= \gamma Z^T \bar{n}, \\
y_{ss}^N &= (1-\gamma) Z^N \bar{n}, \\
P_{ss}^N &= E \frac{Z^N}{Z^T}.
\end{aligned}
$$

To compute the effects of an unanticipated and permanent devaluation, we proceed as in the text. You should find that, on impact c^T falls and then gradually increases back to its unchanged steady-state. On impact, n^T goes up, and hence n^N goes down. Intuitively, consumers reduce c^T to rebuild real money balances. At an unchanged relative price, they would also like to reduce c^N. Hence, there would be an excess supply of non-tradable goods. Since the relative price is fixed by the technology, all the adjustment must be effected through a change in quantities (i.e., by a shift of labor from the non-tradable sector to the tradable sector). Since the real exchange rate remains unchanged, P^N rises by the same proportion as the nominal exchange rate.

Notice that total production is given by:

$$
\begin{aligned}
y^T + \frac{y^N}{e} &= Z^T n^T + \frac{Z^N n^N}{e_t} \\
&= Z^T n^T + Z^T n^N \\
&= Z^T \bar{n}.
\end{aligned}
$$

Total production is thus constant (i.e., production in dollars is constant). Production in "pesos" is of course also constant.