Chapter 7
Temporary stabilization
Answer Key to Exercises[1]

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Current draft: February 22, 2008

1. Exchange rate-based inflation stabilization with fiscal implications

This exercise analyzes the case in which, instead of rebating the proceeds from the inflation tax and interest on reserves to the consumer, the government spends those proceeds (wasteful spending). We will see how in this case there is a wealth effect associated with either a permanent or temporary reduction in the devaluation rate.

The model is a one-good version of the cash-in-advance model of Section 2. Unless otherwise noticed, the notation remains the same. Preferences are given by

$$\int_0^\infty \log(c_t)e^{-\beta t}dt,$$

where c is consumption of the tradable good. Consumers' flow constraint takes the form:

$$\dot{a}_t = ra_t + y - c_t - i_t m_t, \tag{88}$$

while the intertemporal constraint reads as

$$a_0 + \frac{y}{r} = \int_0^\infty (c_t + i_t m_t)e^{-rt}dt.$$

The cash-in-advance constraint is given by

$$m_t = \alpha c_t.$$

The government's flow constraint is given by

$$\dot{h}_t = rh_t + \dot{m}_t + \varepsilon_t m_t - g_t,$$

where g denotes government spending.

[1] This answer key is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries", currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 7.

The economy's flow constraint is given by

$$\dot{k}_t = rk_t + y - c_t - g_t,$$

while the economy's resource constraint is given by

$$k_0 + \frac{y}{r} = \int_0^\infty (c_t + g_t)e^{-rt}dt. \tag{89}$$

In the context of this model:

(a) Characterize a perfect foresight equilibrium path for a constant path of the rate of devaluation.

(b) Suppose that, starting from the perfect foresight equilibrium path characterized above, there is an unanticipated and permanent reduction in the rate of devaluation. Derive (and plot) the paths of all endogenous variables. What are the welfare implications? Explain the intuition behind your results.

(c) Suppose that, starting from the perfect foresight equilibrium path characterized above, there is an unanticipated and temporary reduction in the rate of devaluation. Derive (and plot) the paths of all endogenous variables. What are the welfare implications? Explain the intuition behind your results

Answer

(a) Set up the Lagrangean to obtain the following first-order condition:

$$\frac{1}{c_t} = \lambda(1 + \alpha i_t). \tag{90}$$

Let us now characterize a PFEP with a constant rate of devaluation, $\bar{\varepsilon}$. If ε_t is constant over time, so is the nominal interest rate (from the interest parity condition). Then, from (90), so is consumption. (Notice also, for further reference, that $c_t(1 + \alpha i_t)$ will *always* be constant along a PFEP regardless of the path of i_t.)

To find out the level of consumption, rewrite the consumers' intertemporal constraint, using the cash-in-advance constraint, as

$$a_0 + \frac{y}{r} = \int_0^\infty c_t(1 + \alpha i_t)e^{-rt}dt. \tag{91}$$

Hence,

$$c_t = \frac{ra_0 + y}{1 + \alpha(r + \bar{\varepsilon})}. \tag{92}$$

Since c_t is constant over time, so will be m_t (from the cash-in-advance constraint). From the central bank's balance sheet, we know that

$h_t + d_t = m_t$. Since both m_t and d_t are constant over time, so will h. From the government's flow constraint, if follows that

$$g_t = rh_t + \varepsilon_t m_t.$$

Since h_t and $\varepsilon_t m_t$ are constant over time, so is g_t. The level of g_t follows from the economy's resource constraint, (89):

$$g_t = rk_0 + y - c.$$

(b) An unanticipated and permanent reduction in $\bar{\varepsilon}$ will lead to the same PFEP characterized above. We can thus use the equations derived above to assess the outcome. From (92), it follows that c will be higher in the new PFEP. And, hence, g_t will be lower. From the CIA, it follows that m will also be higher. Hence, h will also be higher. Intuitively, the permanent reduction in the rate of devaluation reduces the proceeds from the inflation tax and hence wasteful government spending, which leads to higher private consumption due to the resulting wealth effect.

(c) To solve the temporary case, the critical point is to observe that $c_t(1 + \alpha i_t)$ will always be constant over time, regardless of the path of i_t. We can then use (91) to write

$$c_t(1 + \alpha i_t) = ra_0 + y$$

Clearly, the unanticipated and temporary fall in i_t at time 0 does not change the level of $c_t(1 + \alpha i_t)$. Hence, when i_t falls at time 0, c_t increases. When i_t rises back to its $t < 0$ level at time T, c falls back to its $t < 0$ level as well. Clearly, the PDV of the new path of c is higher than before the shock. By the CIA constraint, m goes up at time 0 and falls back to its pre-shock level at time T.

What happens to the path of g_t? Since we already know that the PDV of c_t goes up, we infer from the resource constraint that the PDV of g_t goes down. Further,since we can write the consumers' flow constraint as

$$\dot{a}_t = ra_t + y - c_t(1 + \alpha i_t),$$

and $c_t(1 + \alpha i_t)$ is constant over time, we infer that $\dot{a}_t = 0$ and hence $a_t = a_0$ for all $t \geq 0$. Since $\dot{m}_t = 0$ (which implies also that $\dot{h}_t = 0$), it then follows that $\dot{b}_t = 0$. Hence, since $\dot{b}_t = \dot{h} = 0$, it follows that $\dot{k} = 0$. Hence, from the economy's flow constraint:

$$g_t = rk_0 + y - c_t.$$

In other words, g_t falls at time 0 and then increases back at time T to its pre-shock level.

Intuitively, the temporary fall in the rate of devaluation leads to (i) an intertemporal substitution effect and (ii) a wealth effect resulting from the fact that the lower proceeds from the inflation tax during time 0 and time T get reflected in lower government spending. But the wealth effect prevails and welfare increases.

2. Exchange rate-based inflation stabilization with MIUF

This exercise shows that if money is introduced in the utility function and the cross derivative between money and consumption is positive, the same results of a temporary exchange rate-based stabilization with a cash-in-advanced derived in the text go through.

Consider the same economy analyzed in Section 2 with the only difference that preferences are given by

$$\int_0^\infty u(c_t, m_t) \exp(-\beta t) dt, \tag{93}$$

where c is consumption of a tradable good, m denotes real money balances, and $\beta(> 0)$ is the rate of time preference. The utility function $u(c_t, m_t)$ is increasing in both arguments and strictly concave. Specifically, it satisfies:

$$u_c > 0,\ u_m > 0,\ u_{cc} < 0,\ u_{mm} < 0,\ u_{cc}u_{mm} - u_{cm}^2 > 0. \tag{94}$$

(Notice that we have not assume any particular sign for u_{cm}, so $u_{cm} \gtreqless 0$.) Naturally, the cash-in-advance constraint is dropped since we have introduced money in the utility function.

In addition, assume that both good are normal, which implies that

$$\begin{aligned} u_m u_{cc} - u_c u_{mc} &< 0, \\ u_c u_{mm} - u_m u_{cm} &< 0. \end{aligned}$$

The rest of the model remains unchanged.

In the context of this model:

(a) Derive the first-order conditions. Show that they yield a standard money demand function.

(b) Characterize a perfect foresight equilibrium path for a constant rate of devaluation.

(c) Analyze the effects of an unanticipated and temporary exchange rate-based stabilization. In particular, show how the results critically depend on the sign of u_{cm}. Discuss the intuition behind the results.

(d) Suppose that preferences are given by

$$u(c_t, m_t) = (c^{\alpha} + m^{\beta})^{\frac{1}{\sigma}}.$$

Show that u_{cm} is zero if $\sigma = 1$, positive if $\sigma < 1$ and negative if $\sigma > 1$.

Answer

(a) First-order conditions (assuming $\beta = r$):

$$u_c(c_t, m_t) = \lambda \tag{95}$$

$$u_m(c_t, m_t) = \lambda i_t \tag{96}$$

Combining these two first-order conditions, we obtain:

$$u_c(c_t, m_t) i_t = u_m(c_t, m_t), \tag{97}$$

which implicitly defines the real money demand

$$m = L(c, i), \tag{98}$$

where

$$\begin{aligned} \frac{\partial L(c,i)}{\partial i} &= \frac{u_c}{u_{mm} - i u_{cm}} = \frac{u_c^2}{u_c u_{mm} - u_m u_{cm}} < 0, \\ \frac{\partial L(c,i)}{\partial c} &= \frac{i u_{cc} - u_{mc}}{u_{mm} - u_{cm}} = \frac{u_m u_{cc} - u_c u_{mc}}{u_c u_{mm} - u_m u_{cm}} > 0, \end{aligned}$$

where we have used the normality conditions (21) and (22). Notice how, as long as goods are normal, we get a standard money demand regardless of the sign of u_{cm}.

(b) We now characterize a PFEP for a constant rate of devaluation, $\bar{\varepsilon}$. We will show that all endogenous variables are constant along such a PFEP.

From the interest parity condition, we immediately infer that the nominal interest rate will also be constant along a PFEP and equal to $r + \bar{\varepsilon}$. We also know, of course, that λ is constant along a PFEP. The fact that λ and i are constant along a PFEP, however, does not necessarily imply that c and m are also constant since, from (95) and (96), c and m could, in principle, be varying over time. To show that this cannot be the case, totally differentiate (95) and (96) along a PFEP to obtain

$$\begin{bmatrix} u_{cc} & u_{cm} \\ u_{mc} & u_{mm} \end{bmatrix} \begin{bmatrix} dc \\ dm \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

which implies that $dc = dm = 0$ since $u_{cc}u_{mm-}u_{cm}^2 > 0$. Hence, if i_t is constant over time, both c_t and m_t are also constant over time. From the economy's resource constraint, $c_t = rk_0 + y$. Real money balances follow from (98).

(c) Suppose that, starting from the stationary equilibrium just characterized, there is an unanticipated and temporary fall in the devaluation rate at time 0. At time $T > 0$, the devaluation rate goes back to its pre-shock level. Clearly, from the interest parity condition, the nominal interest rate will mimic the path of ε_t. Consider now three cases:

i. $u_{cm} = 0$. In that case, it follows from (95) that c will also be constant along the new PFEP. Since the resource constraint has not changed, c will not change either. From (98) and the path of the nominal interest rate, it follows that real money demand will increase at 0 and fall back to its pre-shock level at time T.

ii. $u_{cm} > 0$. The first step is to ascertain the possible changes of endogenous variables at time T. We know that at time T, i rises but the Lagrange multiplier (denote it by $\tilde{\lambda}$) does not. Totally differentiating (95) and (96) at time T, we obtain:

$$\begin{bmatrix} u_{cc} & u_{cm} \\ u_{mc} & u_{mm} \end{bmatrix} \begin{bmatrix} dc \\ dm \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{\lambda} \end{bmatrix} di.$$

Applying Cramer's rule,

$$\left.\frac{dc}{di}\right|_{t=T} = \frac{-\tilde{\lambda} u_{cm}}{u_{cc}u_{mm} - u_{cm}^2} < 0, \tag{99}$$

$$\left.\frac{dm}{di}\right|_{t=T} = \frac{\tilde{\lambda} u_{cc}}{u_{cc}u_{mm} - u_{cm}^2} < 0, \tag{100}$$

where we have used the fact that, by strict concavity of the utility function, $u_{cc}u_{mm} - u_{cm}^2 > 0$, and $u_{cm} > 0$.
Hence, at T, the rise in i implies that both consumption and real money balances fall. The economy's resource constraint then implies that consumption increases at time 0 and falls at T below its initial level. From (98), we know that m increases at $t = 0$ because consumption increases and i falls. At T, real money balances fall below its initial level (because the nominal interest rate has returned to its initial level but consumption is lower than initially). This case (of Edgeworth complementarity) thus replicates the results that we obtained in the text for the cash-in-advance case.

iii. $u_{cm} < 0$. If $u_{cm} < 0$, it follows from (99) that

$$\left.\frac{dc}{di}\right|_{t=T} = \frac{-\tilde{\lambda} u_{cm}}{u_{cc}u_{mm} - u_{cm}^2} > 0, \tag{101}$$

while the sign of (100) does not change. Hence, at time T, the rise in i leads to an increase in c and a fall in m. Since the resource constraint has not changed, this implies that c will fall at $t = 0$ and then increase at T over and above its pre-shock level. The change of real money balances at time 0 is ambiguous because c falls (which tends to reduce real money demand) while i falls (which tends to increase real money demand). The net effect will depend on the specific preferences.

(d) Straightforward derivations lead to:

$$u_{cm} = c^{\alpha-1}m^{\beta-1}(c^{\alpha} + m^{\beta})^{\frac{1}{\sigma}-2}\frac{\left(\frac{1}{\sigma} - 1\right)}{\sigma}.$$

The stated results follow immediately.

Temporary reduction in devaluation rate under alternative assumptions about cross-derivative

Consider the model with labor supply analyzed in the text. When it comes to a temporary reduction in the devaluation rate, we solved for the case in which $u_{c\ell} < 0$. You are asked to:

(a) i. Solve for the separable case (i.e., $u_{c\ell} = 0$). In particular, show that labor rises permanently at $t = 0$ and that consumption rises at $t = 0$ and falls at time T below its pre-shock value. Discuss the intuition behind he results.

ii. Solve for the case in which $u_{c\ell} > 0$. In particular, show that consumption always rises at $t = 0$ whereas labor could either increase, remain the same, or fall.

Answer

(a) Case in which $u_{\ell c} = 0$.

In this case leisure does not change at T (as follows from expression (37) in the text). Consumption falls at T.

We now show that leisure falls at $t = 0$. We proceed by contradiction. Suppose it did not. We first contradict the possibility that leisure may remain constant and then the possibility that leisure increases at $t = 0$.

i. Suppose that leisure remains constant at 0. An important piece of information is that the condition

$$\frac{u_c(c_t, \ell_t)}{u_\ell(c_t, \ell_t)} = 1 + \alpha\bar{\imath} \tag{102}$$

is the same before $t = 0$ and for $t \geq T$. So if leisure is the same, consumption would need to be the same. To see this, totally differentiate this condition taking into account that ℓ and $\bar{\imath}$ are the same.

$$dc\frac{\overbrace{(u_{cc}u_\ell - u_c u_{\ell c})}^{(-)\text{ by normality}}}{u_\ell^2} = 0 \Rightarrow dc = 0. \tag{103}$$

But if c is the same before $t = 0$ and for $t \geq T$, then we infer that it must increase at 0 to be able to fall at T to its $t < 0$ level and hence the PDV of consumption exceeds the PDV of output (which does not change), which is a contradiction.

ii. Suppose that leisure goes up. Then, differentiating (102), keeping constant i (signs follow from normality conditions)

$$dc\underbrace{(u_{cc}u_\ell - u_c u_{\ell c})}_{-} + \ d\ell\ \underbrace{(u_{c\ell}u_\ell - u_c u_{\ell\ell})}_{+} = 0$$

$$\frac{dc}{d\ell} = -\frac{u_{c\ell}u_\ell - u_c u_{\ell\ell}}{u_{cc}u_\ell - u_c u_{\ell c}} = -\frac{+}{-} > 0 \tag{104}$$

which implies that consumption would go up. But this would violate the intertemporal constraint: labor has gone down permanently, and consumption goes up at $t = 0$ and when it comes down at T it does at a higher value than initially. QED.

We thus conclude that on impact leisure goes down (i.e., labor goes up permanently). This implies that in the new stationary state, consumption is *below* its initial level. But of course it must go up at $t = 0$ (otherwise the intertemporal constraint would be violated).

In this case, there is clearly a trade deficit because labor goes up permanently at $t = 0$. To satisfy intertemporal constraint, consumption must go up by more than output at $t = 0$.

(b) Case in which $u_{\ell c} > 0$.

When $u_{\ell c} > 0$, it follows from expression (37) in the text that $d\ell/di$ at $t = T$ is negative. Hence, since i increases at time T, leisure goes down, which implies that output increases at T. Consumption goes down at T.

We now show that while leisure could stay constant, go up, or go down at $t = 0$, consumption always increases at time T.

(a) Suppose leisure does not change at $t = 0$. Then leisure falls at T, which implies that PDV value of output increases.

What about consumption? For T onwards, since leisure is below its pre-shock level, consumption must also be below its pre-shock level. What happens to consumption at $t = 0$? Since leisure does not change at 0, the fall in i implies that c goes up at $t = 0$. Indeed, differentiating (102) with respect to c and i, keeping ℓ constant, we obtain

$$\frac{dc}{di} = \frac{\alpha u_\ell^2}{u_{cc}u_\ell - u_c u_{\ell c}} < 0.$$

So consumption increases at 0 and then falls at T below its pre-shock level. Hence, the impact on the PDV of consumption is ambiguous and we cannot contradict the fact that leisure may stay constant at $t = 0$.

i. Suppose leisure goes up at $t = 0$. Then it falls at T. The impact on the PDV of labor is thus not clear. What about consumption? Differentiating (102) at time 0, we obtain (taking into account the normality conditions)

$$dc\underbrace{(u_{cc}u_{\ell} - u_c u_{\ell c})}_{-} + \underbrace{d\ell}_{+}\underbrace{(u_{c\ell}u_{\ell} - u_c u_{\ell\ell})}_{+} = u_{\ell}^2 \alpha \underbrace{di}_{-} \Rightarrow dc > 0. \tag{105}$$

In other words, consumption goes up at $t = 0$.

ii. Suppose leisure goes down at $t = 0$. Then it falls further at T. So PDV of output increases. Consumption falls at T below its pre-shock level (we know this because leisure is below its pre-shock level at $t = 0$). What happens with consumption at $t = 0$? It cannot fall because, if it did, PDV of consumption will fall which is a contradiction. So we know that consumption increases at $t = 0$ and then falls at T below its pre-shock level.

We thus conclude that consumption always increases at $t = 0$ but leisure could go up, down, or stay the same. (It would be instructive to show numerically examples of all three cases.)

3. Numerical solution of MIUF flexible exchange rates model

Consider the discrete-time MIUF model developed in Appendix 7.5. Perform the following numerical exercises using the corresponding MATLAB programs (MATLAB programs are available at the book's website).

(a) A temporary fall in the money growth rate for a value of $\sigma = 1$.(This is the separable case analyzed in Chapter 5.)

(b) A temporary fall in the money growth rate for a value of $\sigma = 0.5$. (Notice how the response in consumption of tradables is the opposite to the case of $\sigma = 1.5$ depicted in Figure 6).

(c) Compute analytically U_{cz} and show how it depends on the value of σ. Explain intuitively why the sign of U_{cz} critically affects the response of c^T. What is the case that replicates the cash-in-advance results obtained in the text?

(d) A temporary fall in y^T (for $\sigma = 1$). Explain the intuition behind the results.

(e) A temporary fall in y^N. (for $\sigma = 1$). Explain the intuition behind the results.

1. Answer

   (a) See Figure A1.

   (b) See Figure A2.

   (c) Utility is given by

$$U(c,z) = \frac{\left[(c_t)^{\eta}(z_t)^{1-\eta}\right]^{1-1/\sigma} - 1}{1-1/\sigma}.$$

   Then:

$$U_{cz}(c,z) = \eta(1-\eta)\left[(c_t)^{\eta}(z_t)^{1-\eta}\right]^{-1/\sigma}(z_t)^{-\eta}(c_t)^{1-\eta}\left(\frac{\sigma-1}{\sigma}\right).$$

   Hence:

$$\begin{aligned}\sigma &= 1 \Longrightarrow U_{cz} = 0,\\ \sigma &> 1 \Longrightarrow U_{cz} > 0,\\ \sigma &< 1 \Longrightarrow U_{cz} < 0.\end{aligned}$$

   Intuitively, the sign of U_{cz} critically affects the effect of higher real money balances on c^T because of the following. If $U_{cz} = 0$, then the behavior of z does not affect consumption and hence the path of c^T is flat (Figure A.1). If $U_{cz} > 0$, then an increase in z raises the marginal utility of consumption and hence c^T increases (Figure 6 in the text). If $U_{cz} < 0$, then an increase in z decreases the marginal utility of consumption and hence c^T falls (Figure A2).

   (d) See Figure A3. Intuitively, the consumer reduces consumption of tradables goods on impact by the amount of the fall in permanent income but otherwise smooths consumption of tradable goods over time (as in Chapter 1). The trade balances goes into deficit to absorb the shock.

   Since c^T falls by the permanent income component, at the pre-shock real exchange rate, there is an excess supply of non-tradable goods. This requires a fall in the relative price of non-tradables goods (i.e., an increase in e), which is effected through a fall in the nominal price of non-tradables, P_t^N (which explains the negative inflation rate of non-tradables on impact).

   (e) See Figure A4.

Figures for Answer Key Chapter 7

A1. Temporary fall in money growth rate ($\sigma = 1$)

A2. Temporary fall in money growth rate ($\sigma = 0.5$)

A3. Temporary fall in endowment of tradables ($\sigma = 1$)

A4. Temporary fall in endowment of non-tradables ($\sigma = 1$)

Figure A1. Temporary fall in money growth (σ = 1)

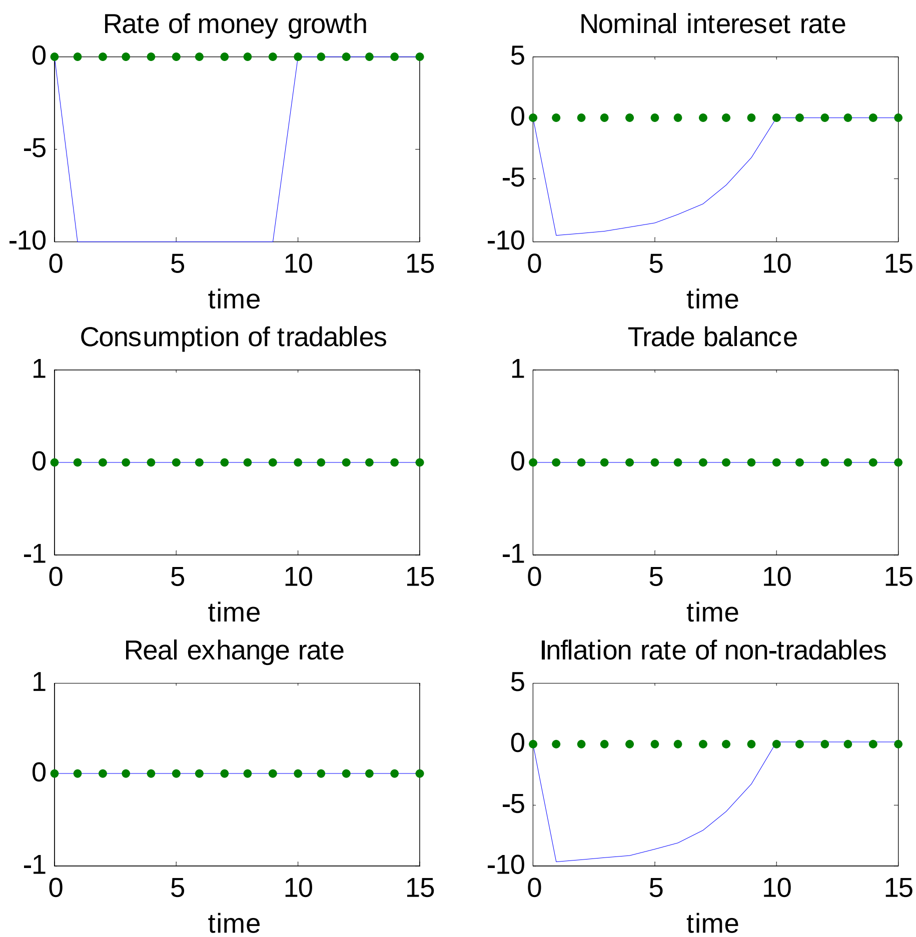

Figure A2. Temporary fall in money growth ($\sigma = 0.5$)

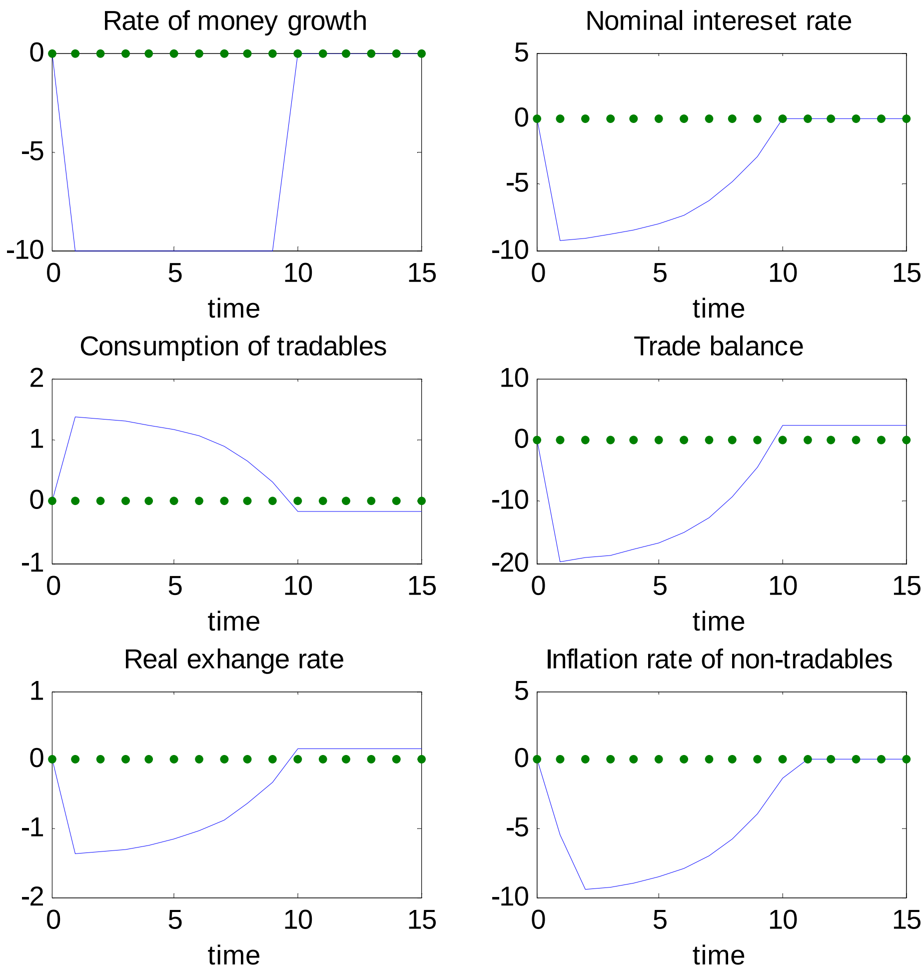

Figure A3. Temporary fall in endowment of tradables (σ = 1)

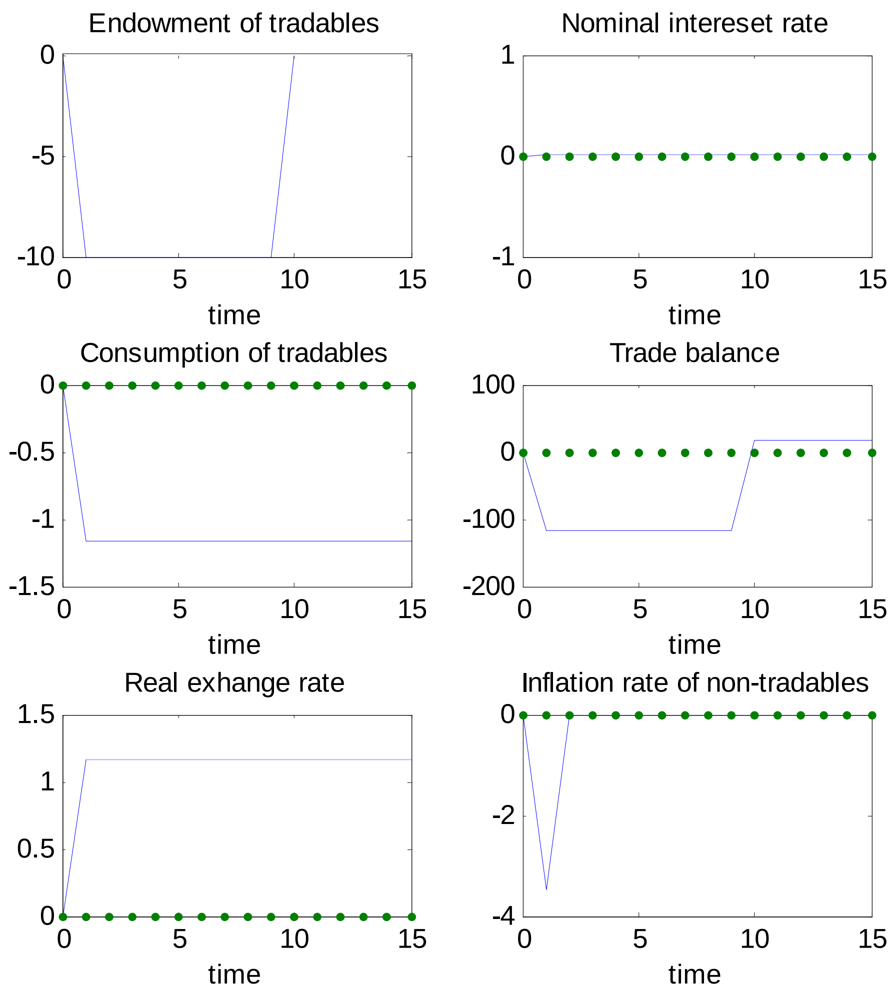

Figure A4. Temporary fall in endowment of non-tradables (σ = 1)