Chapter 5
The basic monetary model
Answer Key to Exercises[1]

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1. Demand shocks

This exercise shows that, as one would expect, the dichotomy between the real and the monetary sectors is still valid when the path of consumption is not constant over time. To this effect, consider the following variation of the model in the text. Preferences are given by:

$$\int_0^\infty [\alpha_t u(c_t) + v(m_t)]e^{-\beta t}dt, \tag{95}$$

where α_t is a preference shock. The rest of the model is unchanged. The parameter α_t can be viewed as a demand shock. Suppose that the path of α_t is as follows:

$$\alpha_t = \begin{cases} \alpha^H, & 0 \leq t < T, \\ \alpha^L & t \geq T, \end{cases}$$

where $\alpha^H > \alpha^L$. In this context:

(a) Solve for the perfect foresight equilibrium path corresponding to predetermined exchange rates.

(b) Solve for the perfect foresight equilibrium path corresponding to flexible exchange rates and show that it coincides with the one you just derived for predetermined exchange rates.

Answer

(a) Consumers maximize (95) subject to the lifetime budget constraint given by (10). The first-order conditions are given by

$$\alpha_t u'(c_t) = \lambda, \tag{96}$$
$$v'(m_t) = \lambda i_t. \tag{97}$$

[1] This answer key is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries", currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 5. I am extremely grateful to Pablo Lopez Murphy for his invaluable help in the preparation of this manuscript.

Since λ is constant along a perfect foresight path, it follows from (96) that so will $\alpha_t u'(c_t)$. Hence when α_t is high (low), c will also be high (low). In other words, consumption will be high until T, at which it points it falls to a lower level and remains there afterwards. Assuming – just to fix ideas – that $k_0 = 0$, the economy will run a trade deficit between 0 and T (which it finances by borrowing from the rest of the world) and a surplus after T (which it uses to repay its debt).

Further, it follows from (97) that real money demand is not affected by changes in consumption. The path of real money balances will therefore be constant over time. Intuitively, notice that combining first-order conditions (96) and (97) implicitly defines a money demand of the form:

$$m_t = L(\underset{+}{c_t}, \underset{-}{i_t}, \underset{-}{\alpha_t}).$$

An increase in the parameter α_t in and of itself reduces real money demand because it makes consumption more attractive *relative* to real money balances. Hence, an change in α_t affects real money demand through two distinct channels: a scale effect (by increasing consumption and hence money demand) and a substitution effect by decreasing the relative attractiveness of real money balances and hence decreasing real money demand. In this exercise, these two effects exactly cancel each other out.

Since real money balances are constant over time, the path of the nominal variables will be exactly the same as that derived in the text in the subsubsection 2.5.1

(b) Since we just solved for the consumption path without having to specify the monetary/exchange rate regime, the path for consumption just derived continues to be valid. To solve for the path of real money balances, we would proceed exactly as in subsubsection 2.5.2 by deriving an unstable differential equation for m. It follows that the same path of nominal variables obtains. Hence, in response to these demand shocks, the economy's equilibrium will be exactly the same under either predetermined or flexible exchange rates. In other words, the the dichotomy between the real and monetary sector remains of course valid even if the path of consumption is not flat over time and the economy is thus running trade/current account imbalances.

2. Increase in domestic credit

Consider the economy analyzed in the text operating under predetermined exchange rates. Analyze the effects of an unanticipated and permanent increase in the stock of domestic credit at time 0.

Answer

Clearly, this once-and-for all increase in the level of domestic credit at the central bank has no effect on consumption, which is given by (27) irrespective of the path of the nominal exchange rate or the path of domestic credit. Clearly, it has no effect either on the nominal interest rate, which implies that real money demand remains unchanged. The interesting action takes place at the central bank. The central bank's balance sheet at time 0 reads:

$$h_0 + d_0 = m_0.$$

We already know that m_0 does not change. But, by assumption, d_0 does increase. Hence, h_0 falls (and, in fact, it falls precisely by the amount that real domestic credit increases). We thus conclude that *a permanent increase in domestic credit leads to a loss in international reserves.* Conversely, a reduction in domestic credit would lead to a gain in international reserves (which explains the typical IMF recommendation to countries to lower/reduce the rate of growth of domestic credit). (We will have more to say about IMF policy prescriptions in Chapter 6.)

3. Equivalence between predetermined and flexible rates illustrated with an anticipated increase in the level of the money supply

Consider the economy analyzed in the text operating under flexible exchange rates. Suppose that the rate of money growth is zero (i.e., $\mu = 0$) and that the level of the money supply follows the path given by:

$$M_t = \begin{cases} M^L, & 0 \leq t < T, \\ M^H & t \geq T, \end{cases}$$

where $M^L < M^H$.

In this context:

(a) Solve for the perfect foresight path of all relevant variables.

(b) Show that if the economy were operating under predetermined exchange rates and the central bank set the path of the nominal exchange rate

Answer

(a) We proceed as in the text by resorting to the unstable different equation (38), repeated here for convenience (with $\mu = 0$):

$$\dot{m}_t = m_t \left[r - \frac{v'(m_t)}{\lambda} \right].$$

The important observation here is that m will jump up at time T, because, by assumption, M increases and E cannot jump along a perfect foresight path (for, if it did, there would be infinite arbitrage opportunities). To accommodate this upward jump in m at time T, a moment's reflection reveals that m_0 must lie just below its long-run equilibrium value, fall over time until just before T and at T jump up to its stationary equilibrium (see Figure E1, Panel B). Using the fact that $\dot{m}_t/m_t = -\varepsilon_t$, we can derive the path for ε depicted in Figure E1, Panel C). From interest parity, the corresponding path for the nominal interest rate is illustrated in Panel D. Hence, the path of the nominal exchange will be given by Panel E.

(b) Assuming that, in a regime of predetermined exchange rates, the central bank sets the path of the nominal exchange rate depicted in Figure E1, Panel E (with the corresponding rate of depreciation in Panel C), it is straightforward to show that the same paths depicted in Figure E1 would follows.

4. Inflationary consequences of anticipated changes in policy

This exercise explores yet another important distinction between a predetermined and flexible exchange rate systems: the behavior of the inflation rate in response to an anticipated changes in policy.

Consider the model of Section 2. Characterize the perfect foresight equilibrium paths corresponding to the following cases:

(a) Under predetermined exchange rates, suppose that the rate of devaluation is zero between 0 and T and increases to $\bar{\varepsilon} > 0$ at $t = T$. Solve for the path of all relevant variables.

(b) Under flexible exchange rates, suppose that the rate of money growth is zero between 0 and T and increases to $\bar{\mu} > 0$ at time T. Solve for the path of all relevant variables.

(c) How does the behavior of inflation differ? What is the intuition behind the results?

Answer

(a) Figure E2 shows the path of the more relevant variables. Given the path of the rate of devaluation, the path of the nominal interest rate follows directly from the interest parity condition. Since consumption is flat and independent of the rate of devaluation, real money demand falls at time T in response to the increase in the nominal interest rate. This is reflected in a fall in international reserves at time T.

(b) Figure E3 shows the path of the more relevant variables. To derive the path of real money balances, notice that (i) the stationary value of m falls at time T due to the increase in μ and (ii) m cannot jump at T because M is a policy parameter that does not change at T

and E cannot change at T (for, if it did, there would be infinite arbitrage opportunities). Given these two pieces of information, the path illustrated in Panel B is the only consistent with equilibrium. Given this path of m, the path of ε follows from the fact that $\dot{m}_t/m_t = \mu - \varepsilon$ and that $\ddot{m} < 0$ (given the instability of the differential equation in m). The path of i then follows from the interest parity condition.

(c) The different behavior of inflation is illustrated in Figure E3, Panel C (where the dotted line illustrated the path of the rate of depreciation under predetermined exchange rates). (Remember, of course, that due to the law of one price, the rate of inflation, π, is equal to the rate of depreciation/devaluation.) Under predetermined exchange rates, the rate of inflation is fully controlled by the monetary authority and does not respond to the anticipated change in the devaluation rate at time T. In sharp contrast, under flexible exchange rates, the rate of inflation begins to rise at $t = 0$ in anticipation of the rise in the rate of money growth at T. Intuitively, notice that under either regime, real money balances will need to be lower at T due to the increase in the nominal interest rate starting at T. Under predetermined exchange rates, the change in real money balances can take place instantaneously at time T through a fall in international reserves. Under flexible rates, however, real money balances cannot change discretely at time T. Hence, this adjustment must take place gradually between 0 and T, which requires the rate of inflation/depreciation to be higher than the rate of money growth between 0 and T.

5. Dirty floating

This exercise illustrates how one would think about "dirty floating" in the monetary model analyzed in the main section. Specifically, we analyze how the economy would respond to positive monetary shock that would lead to an appreciation of the domestic currency and how the monetary authority (MA) might intervene to partly offset such an appreciation (perhaps because, for reasons left out of the model, the MA fears that a large appreciation might worsen the trade balance).

Consider the model of Section 2 with the only modification that preferences are now given by:

$$\int_0^\infty [u(c_t) + \alpha_t v(m_t)]e^{-\beta t}dt, \tag{98}$$

where α should be thought of as a money demand shock. In the context of this model:

(a) Consider the case of flexible exchange rates (with $\mu = 0$). Suppose that just before $t = 0$, the economy is in a stationary equilibrium with a constant α. At $t = 0$, there is unanticipated and permanent increase in α. Solve for the non-intervention case (i.e., a "pure floating"). Explain the intuition behind the results.

(b) Solve for the extreme case of "full intervention" (i.e., the MA reacts in such a way that it does not let the nominal exchange rate change). Explain intuitively how this policy works.

(c) Consider an "intermediate case" in which the MA chooses to intervene in the foreign exchange market (but allows some of the adjustment to take place through the nominal exchange rate). In particular, derive a "policy reaction function" that would tell the MA how much to intervene as a function of the change in real money demand (which the MA must take as given) and the targeted change in the nominal exchange rate. [HINT: a) Think of small changes so that you can use differentiation to compute changes at $t = 0$. b) Think of the MA as having an initial positive stock of international reserves and that capital gains/losses reserves are not monetized; that is, there is some non-monetary liability (call it NM) that is adjusted.]

Answer

(a) Under flexible exchange rates, the first-order condition for real money balances will now read:

$$\alpha v'(m_t) = \lambda i_t.$$

Hence, the unstable differential equation for m (for $\mu = \pi^* = 0$) becomes:

$$\dot{m}_t = m_t \left(r - \frac{\alpha v'(m)}{\lambda} \right).$$

The stationary value for real money balances is thus implicitely given by:

$$r = \frac{\alpha v'(m)}{\lambda}.$$

Suppose that starting from an initial stationary equilibrium, there is an unanticipated and permanent increase in α at $t = 0$. This increases the real money balances that consumers will want to hold in the long run. The only convergent equilibrium path is for real money balances to jump right away to the new and higher value. If that were not the case, the path of m would diverge over time. This implies that E falls at t. Intuitively, under a pure floating and at the pre-shock value of the nominal exchange rate (which is identical to the price level), there would be an excess demand for money. To equilibrate the money market, the nominal exchange rate must fall. Notice that the nominal exchange rate will fall by roughly the same proportion as the real money demand increases. In other words, if real money demand increases by 10 percent, the nominal exchange rate will fall by roughly 10 percent.

(b) Full intervention means that the monetary authority does not let E change and hence corresponds to what would happen in a predetermined exchange rate regime. Hence, all the adjustment takes place through an increase in h (international reserves). Intuitively, since there an incipient appreciation of the domestic currency (i.e., an excess demand for the domestic currency), the monetary authority steps in the foreign exchange market and buys foreign assets from the consumer in exchange for domestic money. In this case, the, say, 10 percent increase in real money demand will lead to no change in the nominal exchange rate.

(c) To address "intermediate situations," consider the central balance sheet given by

$$Eh + D = M + NM,$$

where NM stands for a "non-monetary liability" which, for accounting purposes, will be adjusted to reflect capital gains/losses on reserves. Assuming small changes, we can totally differentiate to obtain:

$$hdE + Edh + dD = dM + dNM.$$

By assumption, $dD = 0$. Also, by the assumption that capital gains/losses on reserves are not reflected in the nominal money supply, $hdE = dNM$. Imposing these two conditions, we can rewrite the last equation as

$$Edh = dM \tag{99}$$

Let us now derive something akin to a "policy rule" that will tell the central bank how much to intervene as a function of the desired change in the nominal exchange rate (with pure flexible rates and full intervention as particular cases).

The change in real money balances is taken as given by the monetary authority

$$\frac{dm}{m} = \frac{dM}{M} - \frac{dE}{E}.$$

Substituting (99) into this last equation and rearranging terms:

$$\frac{dh}{m} = \hat{m} + \hat{E}, \tag{100}$$

where a "hat" over a variable indicates proportional changes. This rule is telling the monetary authority how much to intervene as a function of $\hat{E}$. The less they wish the exchange rate to fall,the more they will have to intervene. In the extreme case ($\hat{E} = 0$), they will need to fully intervene (meaning they will buy all the foreign bonds offered by the private sector). In the other extreme case ($\hat{E} = -\hat{m}$), there will be no intervention whatsoever (pure floating).

Figure E1. Equivalence between predetermined and flexible exchange rates

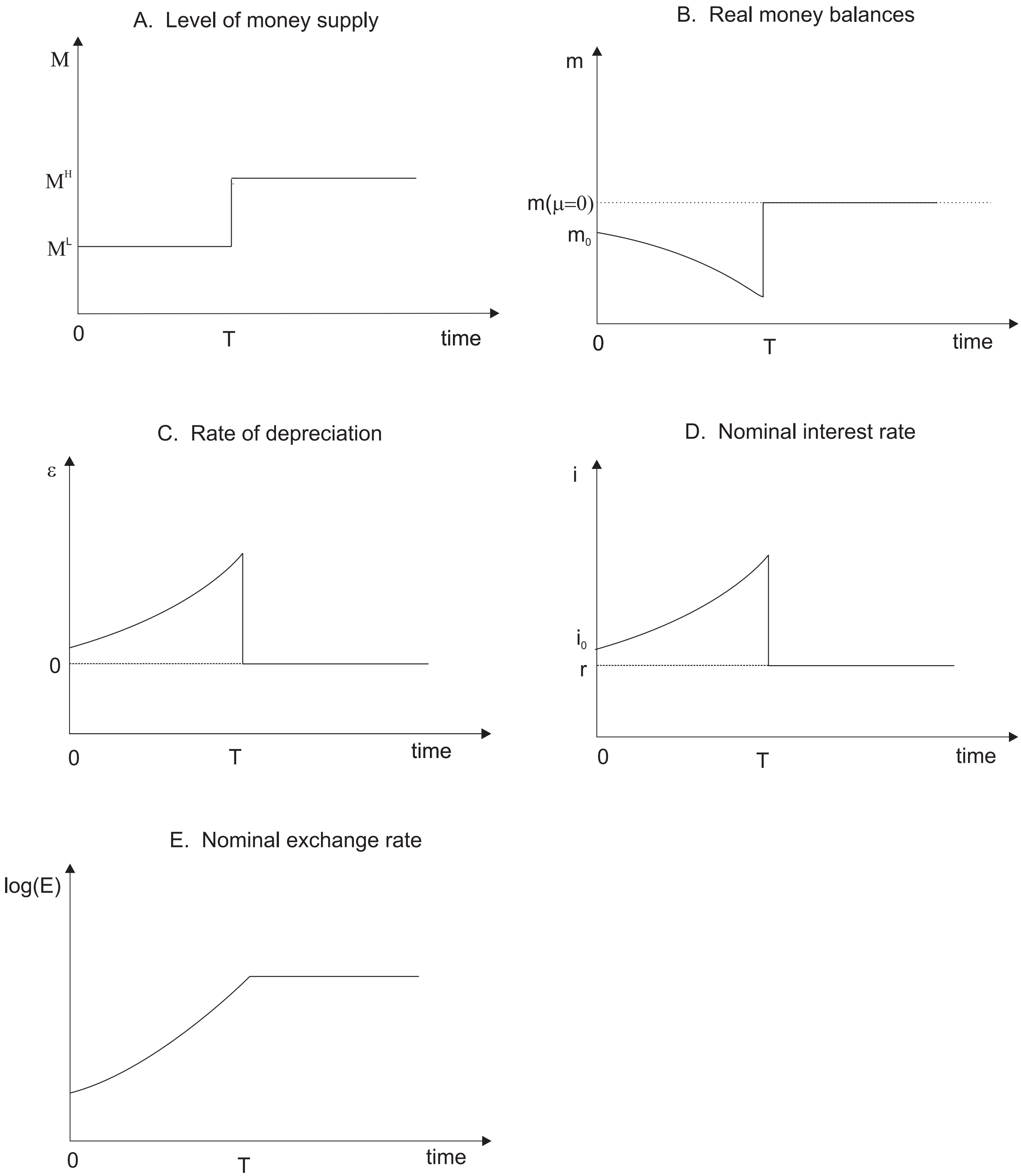

Figure E2. Anticipated increase in devaluation rate

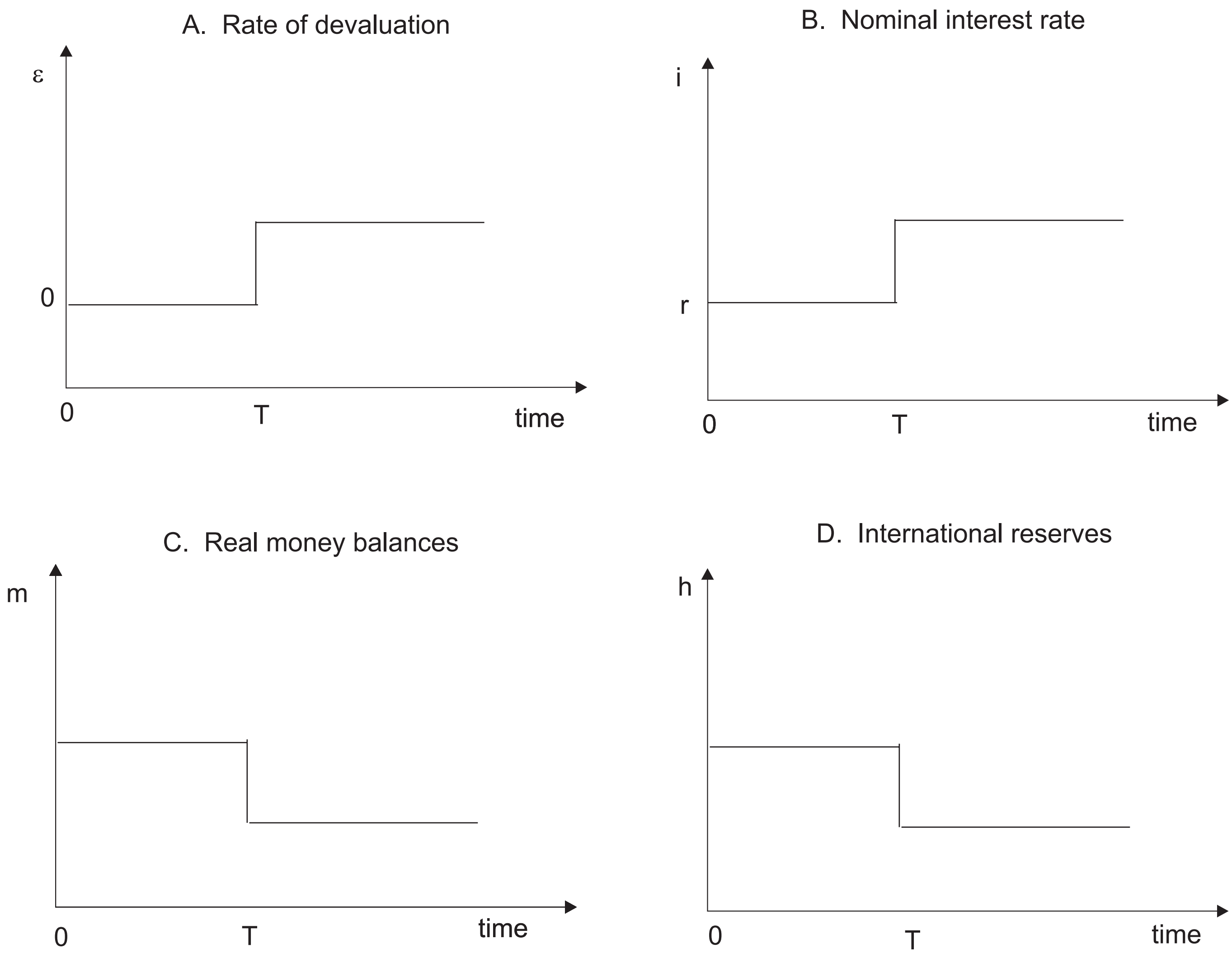

Figure E3. Anticipated increase in money growth rate