

# Chapter 3

## Intertemporal Distortions

### Answer Key to Exercises<sup>1</sup>

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1. Effects of changes in the liberalization period and the intertemporal elasticity of substitution

Consider again the temporary liberalization discussed in Subsection 3.2 with  $q \equiv p^L/p^H < 1$ . Assume that preferences take the iso-elastic form given by equation (17) where  $\sigma$  is the intertemporal elasticity of substitution.

In this context:

- (a) Obtain a reduced form for  $c^1$  and  $c^2$  as a function of  $q$ ,  $T$ , and  $\sigma$ .
- (b) Obtain a reduced form for the Lagrange multiplier,  $\lambda$ .
- (c) Show how  $c^1$  and  $c^2$  change as the liberalization period is shortened (i.e., as  $T$  becomes smaller).
- (d) Show how  $c^1$  and  $c^2$  change as the intertemporal elasticity of substitution ( $\sigma$ ) becomes larger.
- (e) Consider the logarithmic case. Show first that logarithmic preferences are the limiting case of (17), when  $\sigma \rightarrow 1$ . Then derive the indirect lifetime utility as a function of  $q$  and  $T$ . Show how it varies with  $T$ .
- (f) Show how welfare changes in the case of (a) a one instant liberalization (i.e.,  $T \rightarrow 0$ ) and (b) an arbitrarily long liberalization ( $T \rightarrow \infty$ ).
- (g) Plot welfare as a function of  $T$  (for reasonable parameter values) and verify that welfare has a U-shaped form.

### Answer

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<sup>1</sup>This answer key is part of a graduate textbook on “Open Economy Macroeconomics in Developing Countries”, currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 1. I am extremely grateful to Pablo Lopez Murphy for his invaluable help in the preparation of this manuscript. I thank Carolina Mejía Mantilla for helpful comments on this answer key.

(a) Carrying out the required maximization, we obtain:

$$c^1 = \frac{rb_0 + y}{1 - e^{-rT}(1 - q^\sigma)}, \quad (48)$$

$$c^2 = \frac{q^\sigma(rb_0 + y)}{1 - e^{-rT}(1 - q^\sigma)}, \quad (49)$$

where  $q \equiv p^L/p^H < 1$ .

(b) To obtain a reduced form for the Lagrange multiplier, substitute expression (48) into the first-order condition  $c_1^{-1/\sigma} = \tilde{\lambda}p^L$  to obtain

$$\tilde{\lambda} = \frac{1}{p^L} \left\{ \frac{1 - e^{-rT} \left[ 1 - \left( \frac{p^L}{p^H} \right)^\sigma \right]}{rb_0 + y} \right\}^{1/\sigma}.$$

We can see how the Lagrange multiplier depends on the intertemporal distortion.

(c) To find out how  $c^1$  and  $c^2$  change with  $T$ , differentiate (48) and (49) with respect to  $T$  to obtain:

$$\begin{aligned} \frac{\partial c^1}{\partial T} &= -\frac{rb_0 + y}{[1 - e^{-rT}(1 - q^\sigma)]^2} [re^{-rT}(1 - q^\sigma)] < 0. \\ \frac{\partial c^2}{\partial T} &= -\frac{q^\sigma(rb_0 + y)}{[1 - e^{-rT}(1 - q^\sigma)]^2} [re^{-rT}(1 - q^\sigma)] < 0 \end{aligned}$$

Both  $c^1$  and  $c^2$  are a decreasing function of  $T$ . Intuitively, as  $T$  becomes smaller, the period during which consumers will be able to take advantage of a lower intertemporal price shrinks, which implies that they will consume even more. In addition, since the intertemporal relative price does not depend on  $T$ , the ratio of  $c^1$  to  $c^2$  does not depend on  $T$  either. Hence, as  $c^1$  increases as a result of a lower  $T$ , so will  $c^2$ .

(d) To find out how  $c^1$  and  $c^2$  change with  $\sigma$ , differentiate with respect to  $\sigma$  to obtain:

$$\begin{aligned} \frac{\partial c^1}{\partial \sigma} &= -\frac{rb_0 + y}{[1 - e^{-rT}(1 - q^\sigma)]^2} e^{-rT} q^\sigma \log(q) > 0, \\ \frac{\partial c^2}{\partial \sigma} &= \frac{rb_0 + y}{[1 - e^{-rT}(1 - q^\sigma)]^2} (1 - e^{-rT}) q^\sigma \log(q) < 0. \end{aligned}$$

As we should have expected,  $c^1$  is an increasing function of  $\sigma$  whereas  $c^2$  is a decreasing function of  $\sigma$ . Intuitively, the higher is the intertemporal elasticity of substitution, the more consumers will substitute away from  $c^2$  (the relatively expensive good) and towards  $c^1$  (the relatively cheaper good).

(e) Consider the logarithmic case. We first show that

$$\lim_{\sigma \rightarrow 1} \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma} = \log(c), \quad (50)$$

Since when taking the limit, we obtain the expression  $0/0$ , we need to apply L'Hôpital rule. L'Hôpital rule implies that

$$\lim_{\sigma \rightarrow 1} \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma} = \lim_{\sigma \rightarrow 1} \frac{\frac{d}{d\sigma} (c^{1-1/\sigma} - 1)}{\frac{d}{d\sigma} (1 - 1/\sigma)}$$

Since<sup>2</sup>

$$\frac{d}{d\sigma} (c^{1-1/\sigma} - 1) = \frac{\log(c)c^{1-1/\sigma}}{\sigma^2},$$

it follows that

$$\lim_{\sigma \rightarrow 1} \frac{\frac{d}{d\sigma} (c^{1-1/\sigma} - 1)}{\frac{d}{d\sigma} (1 - 1/\sigma)} = \log(c)$$

To derive the indirect lifetime utility, substitute the expressions for  $c^1$  and  $c^2$  into the utility function to obtain:

$$W(T, q) = \frac{\log c_1}{r} + e^{-rT} \frac{\log q}{r}, \quad (51)$$

where  $c_1$  is given by (48).

Differentiating with respect to  $T$ , we obtain

$$\frac{\partial W(T, q)}{\partial T} = - \underbrace{\frac{e^{-rT}(1-q)}{1 - e^{-rT}(1-q)}}_{-} - \underbrace{\log(q)e^{-rT}}_{+} \leq 0,$$

which says that welfare may, in principle, be a non-monotonic function of  $T$ .

(f) Note that welfare just prior to the liberalization is given by

$$W|_{t=0^-} = \frac{\log(rb_0 + y)}{r}. \quad (52)$$

To compute welfare for the cases that  $T \rightarrow 0$  and  $T \rightarrow \infty$ , notice that from (48) (for  $\sigma = 1$ ), it follows that

$$\begin{aligned} \lim_{T \rightarrow 0} c^1 &= \frac{rb_0 + y}{q}, \\ \lim_{T \rightarrow \infty} c^1 &= rb_0 + y. \end{aligned}$$

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<sup>2</sup>Recall that  $d(a^{f(x)})/dx = \log(a)a^{f(x)}f'(x)$ .

Using these last two expressions and (51), it follows that

$$\begin{aligned}\lim_{T \rightarrow 0} W &= \frac{\log\left(\frac{rb_0+y}{q}\right)}{r} + \frac{\log q}{r} = \frac{\log(rb_0+y)}{r}, \\ \lim_{T \rightarrow \infty} W &= \frac{\log(rb_0+y)}{r}.\end{aligned}$$

Comparing these last two expressions with welfare just before the liberalization (given by (52)), we conclude that in both cases the change in welfare is zero.

- (g) Figure 3 in the text shows that the change in welfare is a U-shaped function of  $T$ .

## 2. Durable goods and intertemporal price speculation.

This exercise follows Calvo (1988). Consider the case of an individual who has Leontief preferences over time and therefore chooses a flat path of consumption ( $c$ ) independently of the path of  $p$ . Suppose also that the importable good can be stored at no cost and that there is no depreciation. Based on arbitrage considerations, it should be clear that, if  $p$  is constant over time, there are no incentives to accumulate stocks of the importable good because the good is dominated in rate of return by the foreign bond. Suppose instead that there is a “one-instant liberalization”:

$$\begin{aligned}p_0 &= 1, \\ p_t &= p > 1, \quad t > 0.\end{aligned}$$

Assume that the proceeds from the tariff are given back to consumers in a lump-sum fashion.

The consumer’s intertemporal budget constraint can be written as

$$Z + pc \int_{\phi}^{\infty} e^{-rt} dt = b_0 + \frac{y}{r} + T, \quad (53)$$

where  $Z$  denotes the stock of importables accumulated at  $t = 0$ ,  $y$  is the constant endowment of the exportable good,  $T$  denotes the present discounted value of government transfers, and  $\phi$  denotes the time at which the stock of importables,  $Z$ , is depleted. Since there is no depreciation, it follows that

$$\phi c = Z. \quad (54)$$

In this context:

- (a) Show that the intertemporal budget constraint can be written as

$$c(r\phi + pe^{-r\phi}) = r\left(b_0 + \frac{y}{r} + T\right).$$

- (b) Find the optimal  $\phi$ . Discuss the intuition behind the results.  
(c) Find a reduced form for  $c$ . (Hint: take into account that, in equilibrium,  $T = (p - 1)c \int_{\phi}^{\infty} e^{-rt} dt$ .)  
(d) Discuss the welfare implications of a one-instant liberalization.

Answer

- (a) Integrating the LHS of equation (53) and using (54), we obtain:

$$c(r\phi + pe^{-r\phi}) = r\left(b_0 + \frac{y}{r} + T\right). \quad (55)$$

- (b) To find the optimal  $\phi$ , notice that the consumer takes as given the present discounted value of transfers. Hence, maximizing  $c$  is equivalent to minimizing  $r\phi + pe^{-r\phi}$ . The first-order condition is then given by:

$$1 - pe^{-r\phi} = 0 \quad (56)$$

The second-order condition is given by

$$rpe^{-r\phi} > 0,$$

which confirms that we have found a local minimum.

Solving for  $\phi$  from (56) yields:

$$\phi = \frac{\log(p)}{r}. \quad (57)$$

Three observations are in order regarding the optimal choice of  $\phi$ . First, if  $p > 1$ , then  $\phi > 0$ . In other words, if the price of the durable good is temporarily lower, consumers will choose to store durable goods (i.e., engage in intertemporal *price* speculation). (Of course, if  $p = 1$ , then  $\phi = 0$ , which means that there is no accumulation of durable goods.) Second, the optimal  $\phi$  is an increasing function of  $p$ . Intuitively, the higher is  $p$ , the more pronounced is the one-instant liberalization at time 0. As a result, the consumer will choose to store a larger stock of durable goods. Third, the optimal  $\phi$  is a decreasing function of  $r$ . Intuitively, a higher  $r$  increases the opportunity cost of storing durable goods (since bonds now yield a higher return). Consumers will respond by storing less of the durable good.

- (c) To find a reduced form for  $c$ , notice that, in equilibrium,  $T = (p - 1)c \int_{\phi}^{\infty} e^{-rt} dt$ . Substituting this and (57) into (55), we obtain:

$$c = \frac{rb_0 + y}{\ln p + 1/p}. \quad (58)$$

- (d) First, notice that a benevolent planner would set  $p = 1$ . To show this, notice that  $c$  (given by 58) is a strictly decreasing function of  $p$  and hence has a maximum at  $p = 1$ . In other words, the planner's solution would be not to have a liberalization at all. Intuitively, a one-instant liberalization induces consumers to store durable goods, which is socially inefficient because the social return on storing durables (which is zero) is lower than the return on foreign bonds ( $r$ ). Hence, a one-instant liberalization (i.e.,  $p > 1$ ) is welfare-reducing.

### 3. Lack of credibility

This exercise, which follows Engel and Kletzer (1991), deals with a formalization of the idea of lack of credibility that we discussed in Section 4. Consider a two-period endowment economy. The economy is endowed with a constant endowment,  $y$ , of an exportable good. It consumes an importable good,  $c$ . The international terms of trade are equal to one. The domestic price of importables, however, may be greater than one if a tariff is imposed. The economy can borrow/lend at a fixed rate,  $r$ . In the first period there is no tariff (i.e., the domestic relative price of importables is equal to one). In the second period a tariff ( $p - 1$ ) may be imposed with probability  $\pi$ .

The consumer's problem is to maximize expected utility

$$E\{U\} = \log c_1 + \beta(1 - \pi) \log c_2 + \beta\pi \log c_2^*,$$

where  $\beta(1 + r) = 1$ ,  $c_1$  is consumption in period 1,  $c_2$  is consumption in period 2 if the tariff is not imposed and  $c_2^*$  is consumption in period 2 if a tariff is imposed.

Using the exportable good as the numeraire, the consumer's flow constraints are given by (initial net assets are assumed to be zero; that is,  $b_0 = 0$ ):

$$\begin{aligned} c_1 &= y - b_1, \\ c_2 &= y + (1 + r)b_1, \\ pc_2^* &= y + (1 + r)b_1 + \tau^*, \end{aligned}$$

where  $b_1$  denotes end of period 1 (beginning of period 2) net foreign assets and  $\tau^*$  denotes lump-sum transfers in case the tariff is imposed.<sup>3</sup> (In general equilibrium,  $\tau^* = (p - 1)c_2^*$ .)

In this context:

- (a) Compute reduced forms for  $c_1$ ,  $c_2$ , and  $c_2^*$ . Discuss the intuition behind the results.
- (b) Compute consumer's welfare as a function of  $\pi$ . Interpret the results.
- (c) Compare consumer's welfare when the tariff is  $(p - 1)$  in both periods with certainty.
- (d) What would you conclude about the desirability of trade reforms if you took the model at face value?
- (e) How could you modify the model to yield a more sensible policy prescription?

Answer

- (a) The consumer's problem can be formulated as:

$$\begin{aligned} \text{Max}_{\{c_1, c_2, c_2^*\}} \log c_1 + \beta(1 - \pi) \log c_2 + \beta\pi \log c_2^* + \lambda \left[ \left( \frac{2 + r}{1 + r} \right) y - c_1 - \frac{c_2}{1 + r} \right] \\ + \lambda^* \left[ \left( \frac{2 + r}{1 + r} \right) y + \frac{\tau^*}{1 + r} - c_1 - \frac{pc_2^*}{1 + r} \right] \end{aligned}$$

The first-order conditions are given by (assuming  $\beta(1 + r) = 1$ ):

$$\begin{aligned} \frac{1}{c_1} &= \lambda + \lambda^*, \\ \frac{1 - \pi}{c_2} &= \lambda, \\ \frac{\pi}{c_2^*} &= p\lambda^*. \end{aligned}$$

Combining these equations, we get the stochastic Euler equation:

$$\frac{1}{c_1} = \frac{1 - \pi}{c_2} + \frac{\pi}{pc_2^*}.$$

Using the fact that  $\tau^* = (p - 1)c_2^*$ , it is easy to check that, in equilibrium,  $c_2 = c_2^*$ . Using this piece of information together with the

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<sup>3</sup>Note that we are implicitly assuming that consumers cannot insure against uncertain trade policy in period 2 (which is, of course, the natural assumption). In other words, there are incomplete markets (as defined in Chapter 2).

intertemporal budget constraint, we obtain:

$$c_1 = \frac{(2+r)p}{(2+r)p - \pi(p-1)}y > y, \quad (59)$$

$$c_2 = c_2^* = \frac{(2+r)[p - \pi(p-1)]}{(2+r)p - \pi(p-1)} < y. \quad (60)$$

If  $\pi = 0$  (i.e., no tariff is imposed in period 2), then  $c_1 = c_2 = y$  and full consumption smoothing obtains. If  $\pi > 0$ , the probability that a tariff may be imposed in period 2 induces consumers to engage in intertemporal consumption substitution and consume more in period 1 and less in period 2. Further, as one should expect, the higher is  $\pi$ , the higher is  $c_1$  and the lower is  $c_2$ .

- (b) To compute welfare as a function of  $\pi$ , substitute (59) and (60) into the utility function to obtain:

$$EU(\pi) = \log \left( \frac{(2+r)p}{(2+r)p - \pi(p-1)}y \right) + \beta \log \left( \frac{(2+r)[p - \pi(p-1)]}{(2+r)p - \pi(p-1)}y \right). \quad (61)$$

It follows that:

$$\frac{dEU(\pi)}{d\pi} = \frac{\pi(p-1)^2}{[(2+r)p - \pi(p-1)][p - \pi(p-1)]} < 0.$$

As expected, by introducing a larger intertemporal distortion, a higher  $\pi$  reduces welfare.

- (c) If the tariff is imposed in both periods with certainty, it is straightforward to check that

$$c_1 = c_2 = y.$$

Since there is no intertemporal distortion, consumers fully smooth consumption over time.

Welfare is therefore given by:

$$W = (1 + \beta) \log(y).$$

Notice that from (61),

$$EU(0) = (1 + \beta) \log(y)$$

Hence, for any  $\pi > 0$ ,

$$W > EU(\pi).$$

As we already know, for given resources, a constant path of consumption will always welfare-dominate a non-constant path.

- (d) If you took the model at face value, you would need to conclude that a small open economy would be better off having an import tariff in both periods than engaging in a liberalization in period 1 if there is



some probability that there will be a reversal of the liberalization in period 2 (perhaps because there is no full credibility in the reforms). Hence, you would never advice a government to engage in a trade liberalization if there were the slightest probability of a reversal.

- (e) The more relevant modification of the model would be to take into account that, in practice, even a temporary trade liberalization may lead to higher output through higher productivity and/or a more efficient use of resources. We could capture this by, say, assuming that a liberalization in period 1 will raise output in that period. In that case, we would be introducing a wealth effect and the desirability of a trade reform with a possibility of future reversal would depend on the relative strength of the intertemporal distortion versus the wealth effect.

#### 4. Welfare in the no rebate case

Consider the model of Section 5 with no rebates. In this context:

- (a) Compute a reduced form for welfare for the CES case and show that welfare increases for  $\sigma > 1$ .
- (b) Compute a reduced form for welfare for the logarithmic case.
- (c) Show that welfare increases if  $T$  becomes larger.

Answer

- (a) For the CES case, it can be shown that

$$W = \frac{1}{(1 - 1/\sigma)r} \left[ \tilde{\lambda} (rb_0 + y) - 1 \right], \quad (62)$$

where

$$\tilde{\lambda} = (rb_0 + y)^{-1/\sigma} \left[ (p^L)^{1-\sigma} (1 - e^{-rT}) + (p^H)^{1-\sigma} e^{-rT} \right]^{1/\sigma}.$$

If  $\sigma > 1$ , then  $1 - 1/\sigma > 0$  and a lower  $p^L$  leads to a higher  $\tilde{\lambda}$ . Welfare is therefore higher. (As a check, we can also see that if  $\sigma < 1$ , welfare also increases.)

- (b) For the logarithmic case, it can be shown that

$$W = \frac{1}{r} \left[ \log (rb_0 + y) - (1 - e^{-rT}) \log p^L - e^{-rT} \log p^H \right]. \quad (63)$$

A lower  $p^L$  also leads to higher welfare (this is just a check because we already knew this from the discussion in the text).

(c) Differentiating  $\tilde{\lambda}$  with respect to  $T$ :

$$\frac{d\tilde{\lambda}}{dT} = (rb_0 + y)^{-1/\sigma} \frac{1}{\sigma} \left[ (p^L)^{1-\sigma} (1 - e^{-rT}) + (p^H)^{1-\sigma} e^{-rT} \right]^{1/\sigma-1} r e^{-rT} \left[ (p^L)^{1-\sigma} - (p^H)^{1-\sigma} \right]$$

If  $\sigma < 1$ , then  $(p^L)^{1-\sigma} - (p^H)^{1-\sigma} < 0$  and  $(d\tilde{\lambda}/dT) < 0$ . But since  $\frac{1}{(1-1/\sigma)r} < 0$  then  $\frac{dW}{dT} > 0$ .

If  $\sigma > 1$ , then  $(p^L)^{1-\sigma} - (p^H)^{1-\sigma} > 0$  and  $(d\tilde{\lambda}/dT) > 0$ . Since  $\frac{1}{(1-1/\sigma)r} > 0$ , then  $\frac{dW}{dT} > 0$ .

For the logarithmic case, it follows from (63) that

$$\frac{dW}{dT} = e^{-rT} (\log p^H - \log p^L) > 0.$$

## 5. Effects of changes in the liberalization period with wealth effect

Let preferences be given by

$$u(c_t) = \log c_t.$$

Assume that a fraction  $\phi$  of tariff revenues is spent on unproductive government spending,  $g$ , while a fraction  $1 - \phi$  is returned to consumers as lump-sum transfers:

$$\begin{aligned} g_t &= \phi(p_t - 1)c_t, \\ \tau_t &= (1 - \phi)(p_t - 1)c_t. \end{aligned}$$

Notice that the two cases analyzed in the text are particular cases of this more general formulation:  $\phi = 0$  corresponds to the full rebate case (Section 2) while  $\phi = 1$  corresponds to the no rebate case (Section 5).

In this context:

- Compute the reduced form for  $c^1$  and  $c^2$  as a function of  $q$ ,  $T$ , and  $\phi$ .
- Derive the indirect utility function as a function of  $q$ ,  $T$ , and  $\phi$ .
- Plot the consumer's indirect utility function as a function of  $T$  for different values of  $\phi$ . In particular, show that for low values of  $\phi$  there is a welfare loss for values of  $T$  below some critical value and a welfare gain for higher values (as illustrated in Figure 6), whereas for higher values of  $\phi$  the temporary liberalization will always be welfare improving.

Answer

- (a) Carrying out the maximization and using the economy's resource constraint, we obtain:

$$\begin{aligned} c_1 &= \frac{rb_0 + y}{1 - \phi(1 - qp^H) - (1 - \phi)e^{-rT}(1 - q)}, \\ c_2 &= q \frac{rb_0 + y}{1 - \phi(1 - qp^H) - (1 - \phi)e^{-rT}(1 - q)}. \end{aligned} \quad (64)$$

- (b) Substituting the expressions for  $c_1$  and  $c_2$  just derived into the utility function, we obtain welfare as a function of  $q$ ,  $T$ , and  $\phi$

$$W(T, q, \phi) = \frac{\log c_1}{r} + e^{-rT} \frac{\log q}{r},$$

where  $c_1$  is given by (64). Figure 1 shows a plot of  $W$  as a function of  $T$  for different values  $\phi$  (for the same parameterization as Figure 6 in the text). The bottom curve corresponds to a value of  $\phi = 0.04$ , the intermediate curve to a value of  $\phi = 0.1$  and the top curve to a value of  $\phi = 0.2$ . We can see how for  $\phi = 0.04$  and  $\phi = 0.1$ , the temporary liberalization is welfare-reducing for low values of  $T$  (as the intertemporal distortion effect dominates the wealth effect) whereas for larger values of  $T$  the temporary liberalization becomes welfare improving (as the wealth effect becomes the dominant force). For  $\phi = 0.2$ , the welfare effect dominates regardless of  $T$ .

## 6. Increases in government spending with lump-sum taxation

Solve for the two experiments carried out in Section 6 – a permanent and a temporary increase in government spending – assuming that the government can resort to lump-sum taxation. Explain the intuition behind the differences that may arise.

Answer

The consumer's flow constraint now reads:

$$\dot{b}_t = rb_t + y - c_t - s_t, \quad (65)$$

where  $s_t$  denotes lump-sum taxes. The consumer's first-order condition is now given by

$$\frac{1}{c_t} = \lambda. \quad (66)$$

The government's flow constraint is given by

$$g_t = s_t.$$

It is easy to check that an unanticipated and permanent increase in  $g$  will lead to a one-to-one fall in consumption. The outcome is exactly the same as in the distortionary taxation case because the government did not impose an intertemporal distortion in that case either.

An unanticipated and temporary increase in  $g$  reduces consumption permanently by the permanent income component of the increase in  $g_t$ . Indeed, since first-order condition 66 indicates that consumption will remain constant in the new PFEP, we infer from the economy's resource constraint that the new (and constant) value of  $c_t$  is given by

$$c_t = rb_0 + y - r \int_0^\infty g_t e^{-rt} dt.$$

Although it falls relative to the pre-shock equilibrium, consumer's welfare is higher than in the distortionary taxation case. The reason is that, in the lump-sum taxation case, consumption falls only by the negative wealth effect imposed by the temporary increase in  $g_t$ . In addition, in the distortionary case, welfare falls further because the path of consumption is not flat even though the present-discounted of consumption is, of course, the same as in the lump-sum taxation case.

## 7. The HLM effect with debt in terms of importable.

As a result of using exportables as the numeraire, the model developed in the text assumes that external debt is denominated in terms of the exportable good. Perhaps a more natural assumption is that debt is denominated in terms of importables since, after all, in the real world importables and foreign debt of the typical emerging country are denominated in U.S. dollars.

To examine this alternative scenario, set-up the model of Section 7 in terms of importables (and with a general utility function) and study the current account response's to both a permanent and a temporary improvement in the terms of trade. In particular, does the HLM effect hold for temporary shocks?

**Answer**

The flow budget constraint takes the form:

$$\dot{b}_t = rb_t + \frac{y}{p_t} - c_t,$$

where  $p_t$  continues to denote the relative price of importables in terms of exportables. Notice that we are now using importables as the numeraire and hence the real debt is assumed to be denominated in terms of importables.

The Lagrangian takes the form:

$$\mathcal{L} = \int_0^\infty u(c_t)e^{-\beta t} dt + \lambda \left[ b_0 + \int_0^\infty \left( \frac{y}{p_t} \right) e^{-rt} dt - \int_0^\infty c_t e^{-rt} dt \right]$$

The first-order condition is given by

$$u'(c_t) = \lambda.$$

This condition makes clear that changes in  $p_t$  will not alter the path of consumption.

The rest of the solution follows exactly the basic endowment model of Chapter 1 since the effects of  $p_t$  come only through their effect on the value of the endowment in terms of importables ( $y/p$ ). Proceeding along those lines, it is easy to show that an unanticipated and permanent fall in  $p_t$  (i.e., a permanent improvement in the terms of trade) will lead to a permanent increase in consumption and no change in the trade balance or the current account, whereas an unanticipated and temporary fall in  $p_t$  (i.e., a temporary improvement in the terms of trade) will lead to a permanent increase in consumption as well (but by a smaller amount) and an increase in the trade balance and current account on impact. Hence, the HLM effects holds. Intuitively, a temporary improvement in the terms of trade acts exactly like a temporary increase in the endowment. To smooth consumption over time, consumers must run a trade surplus.

**Figure 1. Welfare change as a function of T for different values of  $\phi$**

