Chapter 4
Non-Traded Goods and Relative Prices
Answer Key to Exercises[1]

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1. Comovements between output and trade balance

This exercise uses a specific class of preferences to illustrate the effect that fluctuations in the endowment of non-tradable goods have on tradable-goods consumption and hence on the trade balance.

Consider the same economy analyzed in Section 2. Let preferences be given by [43]

$$u(c_t^T, c_t^N) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \tag{105}$$

where c is a CES consumption composite defined as:

$$c = z^{\frac{\rho}{\rho-1}}, \tag{106}$$

where

$$z = q(c^T)^{\frac{\rho-1}{\rho}} + (1-q)(c^N)^{\frac{\rho-1}{\rho}}. \tag{107}$$

The parameter $\sigma > 0$ is the *intertemporal* elasticity of consumption substitution. The parameter $\rho > 0$ captures the *intratemporal* elasticity of substitution between tradables and non-tradables goods.

[1]This answer key is part of a graduate textbook on "Open Economy Macroeconomics in Developing Countries", currently under preparation by the author (to be published by MIT Press) and should be cited accordingly. The equation numbering of this answer key continues that of Chapter 4. I am extremely grateful to Pablo Lopez Murphy for his invaluable help in the preparation of this manuscript.

[43]In many instances, these CES preferences are simply written as $c^{1-\frac{1}{\sigma}}/(1-\frac{1}{\sigma})$. The reason for adding the term minus one on the numerator is to have the Cobb-Douglas formulation as a particular case. To see this, take the limit of (111) when $\sigma = 1$ and apply L'Hopital rule to obtain $\log(c)$ as the result.

(a) Suppose that the endowment of tradable goods is flat over time and that the endowment of non-tradable goods fluctuates over time. Show that whether the economy runs trade surpluses or deficits during periods of high non-tradable endowment depends on the relation between σ and ρ. [Hint: Differentiate the marginal condition for tradable goods along a perfect foresight path.]

(b) Available empirical evidence for developing countries suggests that $\rho > \sigma$ (see Ostry and Reinhart (1992)).[44] What does the model predict in terms of the relation between good times (i.e., high non-tradable endowment) and the trade balance? Does the sign of this comovement match the stylized facts described in Box 1 in Chapter 1?

(c) Compute the cross derivative for the CES preferences in (111) and show that it depends on the relation between σ and ρ. Relate this finding to the expression that you derived in item a) above.

Answer

(a) At an optimum, the marginal utility of tradable goods satisfies

$$c^{-\frac{1}{\sigma}} z^{\frac{1}{\rho-1}} q(c^T)^{-\frac{1}{\rho}} = \lambda. \tag{118}$$

Totally differentiating with respect to c^T and c^N and rearranging terms, we obtain:

$$\frac{dc^T}{dc^N} = \frac{\frac{\partial c}{\partial c^N}(\rho-1)z\rho c^T - \frac{\partial z}{\partial c^N}\sigma c \rho c^T}{-\frac{\partial c}{\partial c^T}(\rho-1)z\rho c^T + \frac{\partial z}{\partial c^T}\sigma c\rho c^T - \sigma c(\rho-1)z}.$$

Taking into account (118), we can express the last expression as:

$$\frac{dc^T}{dc^N} = (\sigma-\rho)\frac{(1-q)(c^N)^{-\frac{1}{\rho}}c^T}{q(c^T)^{1-\frac{1}{\rho}}\rho + \sigma(1-q)(c^N)^{\frac{\rho-1}{\rho}}} \begin{cases} = 0 \text{ if } \sigma = \rho. \\ > 0 \text{ if } \sigma > \rho. \\ < 0 \text{ if } \sigma < \rho. \end{cases} \tag{119}$$

[44] Based on data for 13 developing countries, Ostry and Reinhart (1992) estimate the intratemporal elasticity of substitution between tradable and non-tradable goods to be in the range 1.22-1.27, whereas the intratemporal elasticity is estimated to be in the range 0.38-0.50 (which is consistent with the evidence provided in Chapter 3).

Since, by assumption, the path of y^T is flat over time, we can distinguish among three cases:

i. $\sigma = \rho$. In this case, fluctuations in y^N (or c^N) do not affect c^T and the trade balance will always be zero.

ii. $\sigma > \rho$. In this case, when y^N is, say, high, c^T is also high (and hence there will be a trade deficit). Here the economy runs trade deficits in good times (i.e., when the endowment of tradable goods is high).

iii. $\sigma < \rho$. In this case, when y^N is, say, high, c^T is low (and hence there will be a trade surplus). Here the economy runs trade surpluses in good times.

(b) According to the empirical evidence (see Ostry and Reinhart (1992)), $\sigma < \rho$. In terms of our model, this implies that the trade balance is procyclical (i.e., trade surpluses in good times). As discussed in Chapter 1, however, the evidence indicates that the trade balance is countercyclical. We thus conclude that, in the context of our model, fluctuations in the supply of non-tradable goods cannot provide an explanation for the countercyclicality of the trade balance.

(c) From (111), it follows (after some algebra):

$$u_{c^T c^N}\left(c^T, c^N\right) = \frac{(\sigma-\rho)q(1-q)}{\sigma\rho(c^T c^N)^{\frac{1}{\rho}}}\left[q(c^T)^{\frac{\rho-1}{\rho}} + (1-q)(c^N)^{\frac{\rho-1}{\rho}}\right]^{\frac{2\sigma-\rho-\sigma\rho}{\sigma(\rho-1)}} \gtreqless 0 \tag{120}$$

Since $\rho > 0$ and $\sigma > 0$, the sign of the cross derivative depends on the interaction between σ and ρ. Specifically:

- If $\sigma = \rho$ then c^T and c^N are Edgeworth independent
- If $\sigma > \rho$ then $u_{c^T c^N}\left(c^T, c^N\right) > 0$ and c^T and c^N are Edgeworth complements
- If $\sigma < \rho$ then $u_{c^T c^N}\left(c^T, c^N\right) < 0$ and c^T and c^N are Edgeworth substitutes

How is this related to expression (119)? Notice that, along a perfect foresight equilibrium path,

$$\frac{dc^T}{dc^N} = -\frac{u_{c^T c^N}\left(c^T, c^N\right)}{u_{c^T c^T}\left(c^T, c^N\right)}. \tag{121}$$

You should check that if we compute $u_{c^T c^T}\left(c^T, c^N\right)$ from (111) and then plug this expression and (120) into (121), we obtain (119).

2. Consumption-based real interest rate (based on Dornbusch (1983))

This exercise derives the so-called "consumption-based real interest rate", an influential concept popularized by Dornbusch (1983).

(a) Write a discrete-time version of the model analyzed in Section 2 with the discount factor given by δ and preferences given by (111) with z given by (113) and c, the consumption composite, given by

$$c \equiv \left(c^T\right)^{\alpha}\left(c^N\right)^{1-\alpha}. \tag{108}$$

(b) Show that the domestic real interest rate, r^d, is given by $r^d \equiv (1+r)\frac{p_t}{p_{t+1}} - 1$. Interpret intuitively this real interest rate.

(c) Show that you can combine the first-order conditions to obtain the following Euler equation for the consumption aggregate:

$$\frac{c_{t+1}}{c_t} = \left[\delta(1+r_t^c)\right]^{\sigma},$$

where

$$r_t^c \equiv (1+r)\left(\frac{p_t}{p_{t+1}}\right)^{1-\alpha} - 1$$

is the consumption-based real interest rate. Notice how in determining the time profile for the consumption aggregate the consumer compares the discount factor, δ, to r_t^c. Further, if the relative price of non-tradable goods does not vary over time, then $r_t^c = r$. In contrast, if the relative price of non-tradable goods is not constant over time, then aggregate consumption will not be constant over time even if $\delta(1+r) = 1$.

Answer

(a) The consumer's problem is to

$$\max_{\{c^T, c^N\}} U = \sum_{t=0}^{\infty} \delta^t \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}},$$

where c is given by (114), subject to:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(c_t^T + p_t c_t^N\right) = b_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(y_t^T + p_t y_t^N\right).$$

(b) The first order conditions are

$$\delta^t c_t^{-\frac{1}{\sigma}} \alpha (c_t^T)^{\alpha-1} (c_t^N)^{1-\alpha} = \lambda \left(\frac{1}{1+r}\right)^t, \tag{122}$$

$$\delta^t c_t^{-\frac{1}{\sigma}} (1-\alpha) (c_t^T)^{\alpha} (c_t^N)^{-\alpha} = \lambda \left(\frac{1}{1+r}\right)^t p_t. \tag{123}$$

By iterating forward each first-order condition and dividing, we obtain the corresponding Euler equations:

$$\begin{aligned} \frac{c_t^{-\frac{1}{\sigma}} (c_t^T)^{\alpha-1} (c_t^N)^{1-\alpha}}{c_{t+1}^{-\frac{1}{\sigma}} (c_{t+1}^T)^{\alpha-1} (c_{t+1}^N)^{1-\alpha}} &= \delta(1+r), \\ \frac{c_t^{-\frac{1}{\sigma}} (c_t^T)^{\alpha} (c_t^N)^{-\alpha}}{c_{t+1}^{-\frac{1}{\sigma}} (c_{t+1}^T)^{\alpha} (c_{t+1}^N)^{-\alpha}} &= \delta(1+r) \frac{p_t}{p_{t+1}}. \end{aligned} \tag{124}$$

From the first Euler equation (the one corresponding to consumption of tradable goods), we see that the relevant interest rate for choices at the margin is r. In contrast, from the second Euler equation (the one corresponding to consumption of non-tradable goods) we see that the relevant interest rate is what we call the domestic real interest rate:

$$r^d = (1+r)\frac{p_t}{p_{t+1}} - 1.$$

Intuitively, suppose that you forego one unit of non-tradable consumption today. The market value of that unit (in terms of the numeraire) is p_t. Hence, you can buy tradable bonds for a value of p_t. Since the return on these bonds is r, next period you will have $p_t(1+r)$. This would allow you to buy $p_t(1+r)/p_{t+1}$ units of non-tradable goods next period. If the relative price of non-tradable goods is increasing over time (i.e., $p_{t+1} > p_t$), the domestic real interest rate is lower than the world real interest rate (i.e., $r^d < r$) because non-tradable become more expensive over time.

(c) Dividing (122) by (123), we obtain:

$$\frac{c_t^T}{c_t^N} = \frac{\alpha}{1-\alpha} p_t$$

Substituting this last equation into (124),

$$\frac{c_t^{-\frac{1}{\sigma}}}{c_{t+1}^{-\frac{1}{\sigma}}} \left(\frac{p_t}{p_{t+1}} \right)^{\alpha} = \delta(1+r) \frac{p_t}{p_{t+1}},$$

which boils down to

$$\frac{c_{t+1}}{c_t} = \left[\delta(1+r) \left(\frac{p_t}{p_{t+1}} \right)^{1-\alpha} \right]^{\sigma}.$$

3. External deficits and the real exchange rate

This exercise illustrates the fact that Edgeworth substitutability is a sufficient (though not necessary) condition for the association between trade deficits (trade surpluses) and real appreciation (real depreciation) derived in Section 3 to hold for the general preferences given by equation (1).

Consider the model of Section 3 with preferences given by (1). Assume that goods are Edgeworth substitutes. In this context, analyze the effects of a temporary and equiproportional fall in the endowment of both goods. In particular, show that trade deficits go hand in hand with real appreciation.

Answer

The first-order conditions are given by

$$u_{c^T}(c^T, c^N) = \lambda, \tag{125}$$
$$u_{c^N}(c_t^T, c_t^N) = \lambda p_t. \tag{126}$$

Combining these two equations and imposing equilibrium in the non-tradable goods market, we obtain:

$$p_t = \frac{u_{c^N}(c_t^T, y_t^N)}{u_{c^T}(c^T, y^N)}. \tag{127}$$

As usual, the first question is to ask what happens to the endogenous variables at time T. We know that at time T, y^N increases back to its initial level. What will happen to c^T at T? Noting that $c_t^N = y_t^N$ for all t, totally differentiate first-order condition (125) to obtain:

$$\left.\frac{dc^T}{dy^N}\right|_{t=T} = -\frac{u_{c^T c^N}(c^T, y^N)}{u_{c^T c^T}(c^T, y^N)} < 0.$$

Hence, c^T will fall at T. Totally differentiating (126), we get:

$$\frac{dp}{dy^N} = \frac{1}{\lambda u_{c^T c^T}}\left[u_{c^T c^T} u_{c^N c^N} - (u_{c^T c^N}^2)\right] < 0$$

where the sign follows from the strict concavity of the utility function which implies that $u_{c^T c^T} < 0$ and $u_{c^T c^T} u_{c^N c^N} - u_{c^T c^N}^2 > 0$. Hence, p will fall at time T in response to the increase in y^N (and this occurs regardless of whether the two goods are Edgeworth substitutes or complements).

What happens to the endogenous variables at time 0?

From the equilibrium condition in the market for non-tradable goods we know that c^N falls.

Since c^T falls at time T and the present discounted value of c^T also falls, c^T could in principle increase, remain unchanged, or fall at time T. The important point, however, is that if c^T were to fall it would do so by less than the fall in y^T. If it did fall by more, the present discounted value of consumption would fall short of the present discounted value of the endowment. We thus conclude that at time 0, the trade balance will jump into deficit (assuming that $b_0 = 0$). At time 0, it will increase and become a surplus.

What happens with p at time 0? Totally differentiating (127) with respect to c^T and y^N, we get:

$$\left.\frac{dp}{dy^N}\right|_{t=0} = \frac{1}{u_{c^T}^2}\left[\underbrace{-\left(u_{c^N}u_{c^Tc^N} - u_{c^T}u_{c^Nc^N}\right)}_{-} + \underbrace{\left(u_{c^T}u_{c^Nc^T} - u_{c^N}u_{c^Tc^T}\right)}_{+}\frac{dc^T}{dy^N}\right]$$
$$\begin{cases} < 0 & \text{if } \dfrac{dc^T}{dy^N} \leq 0 \\ \gtrless 0 & \text{if } \dfrac{dc^T}{dy^N} > 0 \end{cases},$$

where the signs indicated on the RHS follow from the normality conditions (recall (2) and (3)). Hence, if c^T either increases or remains unchanged at $t = 0$ in response to the fall in y^N, p will increase. This the result that one would have expected: the fall in y^N leads, other things being equal, to an excess demand for non-tradables which requires an increase in their relative price. There is a second effect, however: if c^T increases (or remains unchanged), this in and of itself would also call for a higher relative price of non-tradable goods. If c^T falls, however, then this would call for a lower p and the net effect on p is ambiguous (and likely to depend on the size of the shock and on T). (Ideally, one would check in either MATLAB or Mathematica that either case can occur; I have not done it, please let me know if you do!)

In summary, we conclude that there will be trade deficits during the period $[0, T)$ and trade surpluses afterwards. We can also conclude that the relative price of non-tradable goods will be higher during the interval $[0, T)$ in comparison with its level during the interval $[T, \infty)$. Thus, trade deficits go hand in hand with real appreciation while trade surpluses go hand in hand with real depreciation.

Finally, notice that Edgeworth substitutability is a sufficient condition to ensure that trade deficits (surpluses) go hand in hand with real appreciation (depreciation). In other words, even if goods were Edgeworth complements, it could still be the case that the same results go through. (It would be interesting to solve this model numerically – as a system of non-linear equations using, for instance, Mathematica or

Matlab, and find examples where the results go through and do not go through for the case of Edgeworth complementarity. If you do, let me know!).

4. Demand shocks in an endowment economy

This exercise analyzes the effects of demand shocks in the endowment model developed in Section 2. Suppose that preferences are given by

$$\int_0^{\infty} \left[\alpha_t^T \log(c_t^T) + \alpha_t^N \log(c_t^N)\right] e^{-\beta t} dt,$$

where $\alpha_t^T > 0$ and $\alpha_t^N > 0$ are parameters meant to capture "demand shocks" to consumption of tradables and non-tradables, respectively.

(a) Suppose that $\alpha_t^N = 1 - \alpha_t^T$.
   i. Characterize the perfect foresight equilibrium of this economy for a constant path of all exogenous variables. Does the relative price of non-tradable goods depend on α^T? Explain the intuition behind this result.
   ii. Analyze the effects of (i) an unanticipated and permanent increase in α^T and (ii) an unanticipated and permanent fall in α^T. Show the results both analytically and graphically (i.e., in terms of Figure 2). Explain the intuition behind the results.

(b) Suppose $\alpha_t^T = \alpha_t^N = \alpha_t$.
   i. Characterize the perfect foresight equilibrium of this economy for a constant path of all exogenous variables. Does the relative price of non-tradable goods depend on α? Explain the intuition behind this result.
   ii. Show that an unanticipated and permanent increase in α_t does not affect the economy's equilibrium.
   iii. Analyze the effects of an unanticipated and temporary increase in α_t. In particular, show that high demand periods will be characterized by a consumption boom, real appreciation, and trade deficits while low demand periods will be associated with low consumption, real depreciation, and trade surpluses.

Answer

(a) Suppose $\alpha_t^N = 1 - \alpha_t^T$.

i. The consumer's first order conditions are then given by:

$$\begin{aligned} \frac{\alpha^T}{c_t^T} &= \lambda \qquad (128) \\ \frac{1-\alpha^T}{c_t^N} &= \lambda p_t \qquad (129) \end{aligned}$$

The equilibrium conditions are:

$$\begin{aligned} c_t^N &= y^N \qquad (130) \\ b_0 + \frac{y^T}{r} &= \int_0^\infty c_t^T e^{-rt} dt. \qquad (131) \end{aligned}$$

From (128), it follows that $c_t^T = c^T$. Hence, from (37), it follows that:

$$c_t^T = rb_0 + y^T. \qquad (132)$$

Combining the two first order conditions, equations (128) and (129), taking into account (130) and (37), we obtain a reduced form for the relative price:

$$p_t = \frac{1-\alpha^T}{\alpha^T} \frac{rb_0 + y^T}{y^N}.$$

We see that the equilibrium relative price depends on α^T because α^T affects the marginal rate of substitution between the two goods.

ii. A permanent increase in α^T implies an increase in the relative valuation of tradable goods, which leads to an increase in the demand for tradable goods relative to non-tradable goods. The excess demand for tradable goods must lead to a fall in p to clear the market. An analogous reasoning leads to the conclusion that a fall in α^T will lead to an increase in p. In terms of Figure 2, an increase in α^T would show up as a leftward shift in the $D(p)$ curve because the demand for tradable goods falls for a given p. At the initial p, there is an excess supply of non-tradable goods. Hence, p will be lower in the new equilibrium. Conversely, a fall in α^T would result in a rightward shift of the $D(p)$ curve and p will be higher in the new equilibrium.

(b) Suppose $\alpha_t^T = \alpha_t^N = \alpha_t$.

i. The consumer's first order conditions are then given by:

$$\frac{\alpha}{c_t^T} = \lambda \tag{133}$$

$$\frac{\alpha}{c_t^N} = \lambda p_t \tag{134}$$

By combining these two first-order conditions, we obtain:

$$p_t = \frac{c_t^T}{c_t^N}. \tag{135}$$

The equilibrium conditions continue to be given by (130) and (131). Hence, since (133) tells us that c^T will be constant along a perfect foresight equilibrium path, its value will still be given by (42).

Substituting (**??**) and (42).into (23), we obtain a reduced form for the relative price:

$$p_t = \frac{rb_0 + y^T}{y^N}$$

We see that the equilibrium relative price is not affected by the value of α because α does not affect the marginal rate of substitution between the two goods.

ii. It should be clear from the previous point that none of the endogenous variables depends on α. Hence, a permanent change in α does not alter the equilibrium. Intuitively, since α affects equally the valuation of both tradables and non-tradables, it does not lead to any change in the demand for non-tradable goods relative to tradable goods. α.

iii. To analyze an unanticipated and temporary increase in α, notice that from (133), we can infer that c^T falls at $t = T$. On the other hand, equation (**??**) tells us that c^N remains constant at $t = T$. Hence, from (135), we can tell that p falls at $t = T$.

Given that c^T falls at $t = T$ and that wealth has not changed, we conclude that c^T jumps up at $t = 0$. Since c^N does not change at $t = 0$, condition (135) tells us that p jumps up at $t = 0$

Assume, for simplicity, that $b_0 = 0$. Given that c^T is relatively high during the time interval $[0, T)$ while y^T is constant $\forall t$, we conclude that there will be trade deficits during the time interval $[0, T)$ and trade surpluses during the time interval $[T, \infty)$.

In sum, we have shown that high demand periods (i.e., the period $[0, T)$) will be characterized by a consumption boom, real appreciation, and trade deficits while low demand periods (i..e., the period $[T, \infty)$) will be associated with low consumption, real depreciation, and trade surpluses.

5. Demand shocks in a production economy

Suppose that households consume only non-tradable goods and that non-tradable goods are produced using tradable goods as an input. (If it helps you, think of an economy importing candies (which are not directly consumed), wrapping them domestically, and selling them domestically for consumption.)

Specifically, consider a small open economy fully integrated into the world economy. Preferences are given by

$$\int_0^\infty \gamma_t u(c_t^N) e^{-\beta t} dt,$$

where $u' > 0$, $u'' < 0$, c_t^N is consumption of non-tradable goods and $\gamma_t > 0$ is a parameter that captures demand shocks..

Non-tradables goods are produced using tradables goods as an input:

$$y_t^N = \frac{(c_t^T)^\alpha}{\alpha}, \qquad \alpha < 1.$$

There is an exogenous and constant endowment of tradable goods, y^T.

(a) State the household's flow and intertemporal budget constraints. (Assume that households also carry productive activities.)

(b) Derive the first-order conditions. Explain the intuition.

(c) Suppose that, starting from an initial stationary equilibrium, there is an unanticipated and temporary increase in γ_t. Derive the time path for all endogenous variables (and plot them against time). Explain the intuition behind your results.

Answer

(a) Flow constraint is given by

$$\dot{b}_t = rb_t + y^T + p_t y_t^N - c^T - p_t c_t^N.$$

The corresponding intertemporal constraint is given by:

$$b_0 + \int_0^\infty \left(y^T + p_t y_t^N - c_t^T - p_t c_t^N\right) e^{-rt} dt = 0.$$

Imposing equilibriun in the non-tradable goods market ($y^N = c^N$), we obtain the current account equation:

$$\dot{b}_t = rb_t + y^T - c^T$$

(b) Maximization:

$$L = \int_0^\infty \gamma_t u(c_t^N) e^{-\beta t} dt + \lambda \left[b_0 + \int_0^\infty \left(y^T + p_t \frac{(c_t^T)^\alpha}{\alpha} - c^T - p_t c_t^N\right) e^{-rt} dt\right]$$

Focs with respecto to c^T, and c^N:

$$\begin{aligned} \lambda\left[p_t (c_t^T)^{\alpha-1} - 1\right] &= 0 \Rightarrow p_t = (c_t^T)^{1-\alpha}, \\ \gamma_t u'(c_t^N) &= \lambda p_t. \end{aligned}$$

(c) Temporary shock. As always, we first find out what happens at T and then go back and, using the resource constraint, find out what happens at time 0.

To find out what happens at T, totally differentiate the following system:

$$\begin{aligned} -(c_t^T)^{1-\alpha} + p_t &= 0, \\ \gamma_t u' \left(\frac{(c_t^T)^\alpha}{\alpha} \right) - \lambda p_t &= 0, \end{aligned}$$

to obtain:

$$\begin{aligned} -(1-\alpha)(c_t^T)^{-\alpha} dc^T + dp_t &= 0, \\ \gamma_t u''(c_t^T)^{\alpha-1} dc_t^T - \lambda dp_t &= -u' d\gamma. \end{aligned}$$

Applying Cramer's rule:

$$\begin{aligned} \frac{dc^T}{d\gamma} &= \frac{1}{\Delta} \begin{vmatrix} 0 & 1 \\ -u' & -\lambda \end{vmatrix} = \frac{u'}{\Delta} > 0, \\ \frac{dp}{d\gamma} &= \frac{1}{\Delta} \begin{vmatrix} -(1-\alpha)(c_t^T)^{-\alpha} & 0 \\ \gamma_t u''(c_t^T)^{\alpha-1} & -u' \end{vmatrix} = \frac{(1-\alpha)(c_t^T)^{-\alpha} u'}{\Delta} > 0, \end{aligned}$$

where

$$\Delta = \begin{vmatrix} -(1-\alpha)(c_t^T)^{-\alpha} & 1 \\ \gamma_t u''(c_t^T)^{\alpha-1} & -\lambda \end{vmatrix} = (1-\alpha)(c_t^T)^{-\alpha}\lambda - \gamma_t (c_t^T)^{(\alpha-1)} u'' > 0.$$

(a) So at T, when γ falls, both c^T and p fall. Intuitively, the fall in γ reduces demand for non-tradables. The resulting excess supply for NT causes the relative price, p, to fall. The fall in p reduces production of non-tradables, and hence c^T falls.

Paths. Since tradable resources did not change, c^T must increase at 0 and then fall at T below its initial value. TB and CA follow accordingly.

Since $p_t = (c_t^T)^{1-\alpha}$, the path of p is the same (i.e., p increases at $t = 0$ and decreases at T *below* its pre-shock value.)

6. The twin deficits revisited

The purpose of this exercise is to show that once we allow the government to borrow/lend over time, we can establish a link between fiscal deficits and trade deficits. In other words, we will see how a temporary increase in g^T leads to both a primary fiscal deficit and a trade deficit.

Consider the same model analyzed in subsection 4.1 but, for simplicity, assume that the government spends only on tradable goods. More importantly, relax the assumption that the government must balance its budget period by period (reflected in equation (34)). Instead, assume that the government's flow constraint takes the form

$$\dot{b}_t^G = rb_t^G + pb_t,$$

where

$$pb_t \equiv \bar{\tau} - g_t^T. \tag{109}$$

denotes the primary balance, $\bar{\tau}$ is a constant tax rate (whose level will be endogenously determined) and b^G denotes net foreign assets held by the government. The corresponding intertemporal constraint is given by

$$b_0^G + \frac{\bar{\tau}}{r} = \int_0^\infty g_t^T e^{-rt} dt, \tag{136}$$

which says that the present discounted value of primary balances must match the initial government debt (given by $-b_0^G$).

In this context:

(a) Characterize the stationary equilibrium corresponding to constant paths of y^T, y^N, and g^T. To simplify the exercise assume that $b_0^G = 0$ and $pb_0 = 0$.

(b) Analyze the effects of an unanticipated and temporary increase in g^T. Does the twin deficits hypothesis holds? Discuss the intuition behind the results.

Answer

(a) From the consumer's first order condition for c^T and the resource constraint, it follows that the constant level of c^T is given by:

$$c_t^T = rb_0 + y^T - g^T. \tag{137}$$

Equilibrium in the non-tradable goods market implies that c^N is constant over time and given by:

$$c_t^N = y^N. \tag{138}$$

Combining the first-order conditions and using (137) and (138), we obtain the constant value of p:

$$p_t = \frac{rb_0 + y^T - g^T}{y^N} \tag{139}$$

Using (115) and (139), it follows that the constant level of the tax rate is given by:

$$\bar{\tau} = g^T.$$

(b) As usual when we analyze a temporary shock, we begin by establishing what happens to the endogenous variables at time $t = T$.

- From the first order condition corresponding to the consumption of tradable goods, we can see that c^T does not change at T. Since g^T falls at T, the trade balance will increase at T
- From the equilibrium condition in the market of non-tradable goods, we know that c^N does not change at T
- From the ratio of the first order conditions and the fact that neither c^T nor c^N change at T, we conclude that p does not change at T either.
- Since, by assumption, $\bar{\tau}$ does not change at T and g^T falls at time T, the primary balance will fall at time T.

Having established the behavior of the endogenous variables at time T, we can now infer the changes at $t = 0$ and hence determine the whole time path of the endogenous variables.

- Path of c^T. From the resource constraint, we know that the present discounted value of c^T will be lower than before the shock. Since we already know that c^T does not change at T, we infer that c^T must fall at $t = 0$. Hence, c^T falls at time 0 and stays that level forever.

- Path of the trade balance. Since c^T falls at $t = 0$ by the annuity value of the increase in the present discounted value of g^T, the trade balance will deteriorate as the increase in g^T at 0 is higher than the fall in c^T. Hence, the trade balance falls at $t = 0$ (i.e., goes into deficit if we assume that $b_0 = 0$) and then increases at time T (i.e., goes into surplus if $b_0 = 0$).
- Path of c^N. From the non-tradable goods market equilibrium, we know that c^N will not change at $t = 0$ either. Hence, the path of c^N remains unchanged.
- Path of p. Since c^T falls at 0 and c^N does not change, we infer that p falls at time 0. Hence, p falls at time 0 and remains at that lower level forever.
- By assumption, the level of $\bar{\tau}$ is constant over time. From the government's intertemporal constraint (136), we infer that $\bar{\tau}$ will rise at time 0 because the present discounted value of g^T has increased. Clearly, since $\bar{\tau}$ increases at time 0 by the annuity value of the increase in the present discounted value of g^T, the government will run a primary deficit between 0 and T and a primary surplus from T onwards.

The exercise's punchline is thus that temporary increase in g^T will lead on impact (i.e., at time 0) to both a primary deficit and a trade deficit. In other words, the temporary rise in g^T leads to twin deficits.

7. Endogenous supply revisited

In the context of the model in Section 5:

(a) Consider a more general version of this model in which the production of non-tradables is given by $y_t^N = Z_t^N(n_t^N)^\beta$, where β could be greater, equal, or smaller than α. Solve for the effects of an increase in b_0 and a temporary demand shock and show that the same results that we found in the text go through.

(b) Analyze in the more general version the effects of an unanticipated and permanent increase in Z^T. Do the results depend on whether

$\alpha \gtreqless \beta$? How do the results relate to the celebrated Balassa-Samuelson effect?

(c) Analyze in the more general version the effects of an unanticipated and permanent increase in n. Do the results depend on whether $\alpha \gtreqless \beta$?

(d) Suppose that production is linear in both sectors; that is, $y_t^T = Z_t^T n_t^T$ and $y_t^N = Z_t^N n_t^N$. Obtain a reduced form for all endogenous variables in the model. How is the real exchange rate determined in this model?

Answer

(a) The production efficiency condition now becomes

$$\alpha Z_t^T (n_t^T)^{\alpha-1} = \beta p_t Z_t^N (n_t^N)^{\beta-1}. \tag{140}$$

A stationary equilibrium will thus be caracterized by

$$c^T = rb_0 + Z^T (n^T)^\alpha, \tag{141}$$
$$c^N = Z^N \left(n^N\right)^\beta, \tag{142}$$
$$p = \frac{rb_0 + Z^T (n^T)^\alpha}{Z^N \left(n^N\right)^\beta}, \tag{143}$$
$$p = \frac{\alpha Z^T (n_t^T)^{\alpha-1}}{\beta Z^N (n - n_t^T)^{\beta-1}}, \tag{144}$$
$$n = n^T + n^N. \tag{145}$$

To solve for the wealth effect, proceed as in the text and obtain:

$$\frac{\alpha}{\beta} n (n_t^T)^{\alpha-1} - \frac{\alpha}{\beta}(n_t^T)^\alpha - (n_t^T)^\alpha - rb_0 (Z^T)^{-1} = 0$$

Totally differentiating the expression in brackets with respect to n^T and b_0, we obtain:

$$\frac{dn^T}{db_0} = -\frac{-\beta r / z^T}{\alpha\,(\alpha - 1)\,n(n_t^T)^{\alpha-2} - \alpha\,(\alpha+\beta)\,(n_t^T)^{\alpha-1}} < 0. \tag{146}$$

The same results as in the text follow.

For the temporary demand shock, proceed in the same way as in the text and all the same results will follow.

(b) The value of the five endogenous variables (c^T, c^N, p, n^T, and n^N) is fully determined by the following system of five equations:

$$\begin{aligned} c^T &= rb_0 + Z^T(n^T)^\alpha, && (147)\\ c^N &= Z^N\left(n^N\right)^\beta, && (148)\\ p &= \frac{rb_0 + Z^T(n^T)^\alpha}{Z^N\left(n^N\right)^\beta}, && (149)\\ p &= \frac{\alpha Z^T(n_t^T)^{\alpha-1}}{\beta Z^N(n-n_t^T)^{\beta-1}}, && (150)\\ n &= n^T + n^N. && (151) \end{aligned}$$

Substituting (150) and (151) into (23), we get n^T as an implicit function of Z^T, n, and b_0:

$$\frac{\alpha}{\beta}n(n_t^T)^{\alpha-1} - \frac{\alpha}{\beta}(n_t^T)^\alpha - (n_t^T)^\alpha = \frac{rb_0}{Z^T}. \tag{152}$$

Differentiating with respect to n^T and Z^T, we get:

$$\frac{dn^T}{dZ^T} = -\frac{rb_0}{(Z^T)^2\left[\frac{\alpha}{\beta}(\alpha-1)n(n_t^T)^{\alpha-2} - \left(\frac{\alpha^2}{\beta}+\alpha\right)(n_t^T)^{\alpha-1}\right]}\begin{cases} = 0 \text{ if } b_0 = 0\\ > 0 \text{.if } b_0 > 0\\ < 0 \text{ if } b_0 < 0\end{cases}$$

There are thus three cases that we need to distinguish based on the value of b_0:

i. $b_0 = 0$. In this case, n^T does not change. Since Z^T goes up and n^T does not change, it follows from (147) that c^T increases. Since n^T does not change, it follows from (151) that n^N does not change. By (148), c^N does not change either. Finally, since c^T increases but c^N does not change, condition (149) tells us that p increases.[45]

ii. $b_0 > 0$. In this case, n^T increases. Since both Z^T and n^T go up, it follows from (147) that c^T increases. Since n^T increases,

[45] In fact, for the $b_0 = 0$ case, we can obtain a reduced form for the model. In particular, it is easy to check that $n^T = (\alpha/(1+\alpha)n$ and $n^N = (1/(1+\alpha)n$. Hence, from (150), we can see that p will increase in the same proportion as Z^T and, from (147), the same is true of c^T.

it follows from (151) that n^N falls. Hence, by (148), c^N also falls. Finally, since c^T increases and c^N falls, condition (149) tells us that p increases.

iii. $b_0 < 0$. In this case, n^T falls. Since n^T falls, it follows from (151) that n^N increases. Hence, by (148), c^N also increases. Given that Z^T goes up and n^T falls, condition (150) tells us that p increases. Given that p increases and n^N increases, condition (149) says that $Z^T n^T$ must increase. Hence, from (147), c^T increases.

In all three cases, therefore, p increases as a result of the increase in Z^T. Intuitively, the increase in Z^T leads, all else equal, to an excess supply of tradable goods. To see this, notice that at an unchanged p and n^T, relative demand is of course unchanged, whereas the supply of tradables increases (recall (150)). The excess supply of tradable goods requires a fall in the relative price of tradables goods (i.e., an increase in p). In terms of Figure 2, the supply curve would now have a positive slope. An increase in Z^T would shift the supply curve to the left.

To understand intuitively why the change in n^T depends on b_0, let us proceed as follows. Suppose that Z^T goes up by 10 percent but that n^T does not change. This implies, from production efficiency (condition (150)), that p also goes up 10 percent. The question is then whether this is an equilibrium. To check this we need to see by how much the quantity demanded and supplied have changed in response. The quantity demanded (c^T/c^N) always goes up by 10 percent. The quantity supplied, however, depends on b_0; specifically:

$$\begin{aligned}
b_0 &= 0 \Longrightarrow \frac{b_0 + Z^T (n^T)^\alpha}{Z^N (n^N)^\beta} \text{ goes up by 10 percent} \\
&\Longrightarrow \Delta\text{Supply} = \Delta\text{Demand}, \\
b_0 &> 0 \Longrightarrow \frac{b_0 + Z^T (n^T)^\alpha}{Z^N (n^N)^\beta} \text{ goes up by less than percent} \\
&\Longrightarrow \Delta\text{Supply} < \Delta\text{Demand} \Longrightarrow n^T \text{ must increase}, \\
b_0 &< 0 \Longrightarrow \frac{b_0 + Z^T (n^T)^\alpha}{Z^N (n^N)^\beta} \text{ goes up by more than percent} \\
&\Longrightarrow \Delta\text{Supply} > \Delta\text{Demand} \Longrightarrow n^T \text{ must fall.}
\end{aligned}$$

We thus conclude that an increase in the productivity of tradable goods (i.e., an increase in Z^T) will result in a real appreciation. This is a simple version of the so-called Balassa-Samuelson effect that holds that the relative prices of non-tradable goods will be higher in richer countries (i.e., it is more expensive to get a hair cut in the United States than in India). Since richer countries are more productive and the higher productivity is mainly concentrated in the tradable goods sector, this should lead to an excess supply of tradable goods relative to non-tradable goods, which should result in an increase in the relative price of non-tradable goods, as predicted by our simple model.

(c) Differentiating (152) with respect to n^T and n, we get:

$$\frac{\partial n^T}{\partial n} = \frac{n_t^T}{(1-\alpha)\, n + (\alpha + \beta)\, n_t^T} > 0. \tag{153}$$

Notice that since we can write the denominator of this expression as $(1+\beta)n_t^T + (1-\alpha)n_t^N$, it follows that $\frac{\partial n^T}{\partial n} < 1$.
From (147), it then follows that

$$\frac{\partial c^T}{\partial n} > 0.$$

Since $0 < \frac{\partial n^T}{\partial n} < 1$, it then follows from (151) that

$$\frac{\partial n^N}{\partial n} > 0.$$

From (43), it follows that

$$\frac{\partial c^N}{\partial n} > 0.$$

What is the effect of n on p? As will become clear below, this effect depends on whether $\alpha \lesseqgtr \beta$. Two extreme cases are helpful to understand the intuition. Suppose $\beta = 1$ (production of non-tradables is linear) and hence $\alpha < \beta$. It then follows from (150) that

$$\frac{\partial p}{\partial n} < 0.$$

Intuitively, suppose that p did not change. Then, equation (150) – which equates the marginal productivity of labor across sectors of production – says that n^T would not change. In other words, at an unchanged relative price, all the increase in labor supply would go to the non-tradable goods sector (the labor intensive sector). But, at an unchanged p (and hence unchanged relative demand), this situation would lead to an excess supply of non-tradable goods. As result, p must fall to clear the market by increasing the demand for non-tradable goods and reducing some of the supply (relative to the case in which p would not change).

Conside now the case $\alpha = 1$ (i.e., production of tradables is linear) and hence $\alpha > \beta$. It then follows from (150) that

$$\frac{\partial p}{\partial n} > 0.$$

Intuuitively, at an unchanged p, all the increase in n would now go to the tradable sector. There would thus be an excess supply of non-tradables (i.e., excess demand for non-tradables) at an unchanged p. It follows that p needs to increase to bring about equilibrium.

Finally, consider the case $\alpha = \beta$ (and $b_0 = 0$ and $Z^T = Z^N$ to make the two sectors completely symmetric). It then follows from (149) and (150) (recalling that $n - n^T = n^N$) that $n^T = n^N$ and hence $p = 1$. Substituting $n^T = n^N$ into (151), it follows that $n^T = n^N = (1/2)n$. Note that these derivations are valid for any value of n and hence p does not change if we increase p. Intuitively, if n

increases, at an unchanged p, equal amounts of labor are allocated to both sectors and this is in fact an equilibrium so that no change in p is needed.

Coming back to the general case, totally differentiate (150) and use (153) to obtain

$$\frac{dp_t}{dn_t} = \frac{\alpha Z^T (n_t^T)^{\alpha-1}}{Z^N (n - n_t^T)^\beta} \left\{ \frac{(1-\beta)\, n^T - (1-\alpha)\, n^N}{[(1-\alpha)\, n + (\alpha+\beta)\, n_t^T]} \right\} \gtrless 0. \tag{154}$$

Notice, as particular cases, the cases discussed above:

- $\beta = 1$. Then:

$$\frac{dp_t}{dn_t} = -\frac{\alpha Z^T (n_t^T)^{\alpha-1}}{Z^N (n - n_t^T)^\beta} \left\{ \frac{(1-\alpha)\, n^N}{[(1-\alpha)\, n + (\alpha+\beta)\, n_t^T]} \right\} < 0.$$

- $\alpha = 1$. Then:

$$\frac{dp_t}{dn_t} = \frac{\alpha Z^T (n_t^T)^{\alpha-1}}{Z^N (n - n_t^T)^\beta} \left\{ \frac{(1-\beta)\, n_t^T}{[(1-\alpha)\, n + (\alpha+\beta)\, n_t^T]} \right\} > 0.$$

- $\alpha = \beta$ (and $b_0 = 0$ and $Z^T = Z^N$, which implies that $n^T = 2n$). Then

$$\frac{dp_t}{dn_t} = 0.$$

  In the general case, the sign of (154) will depend on the sign of $(1-\beta)\, n^T - (1-\alpha)\, n^N$. We leave it as a further exercise for the reader to show that

$$(1-\beta)\, n^T - (1-\alpha)\, n^N \gtreqless 0 \text{ as } \alpha \gtreqless \beta.$$

(d) Suppose that $y_t^T = Z_t^T n_t^T$. In this case, the system (147) through (151) reduces to:

$$
\begin{aligned}
c^T &= \frac{1}{2}\left(rb_0 + Z^T n\right), \\
c^N &= \frac{Z^N}{2}\left(n + \frac{rb_0}{Z^T}\right), \\
p &= \frac{Z^T}{Z^N}, \\
n^T &= \frac{1}{2}\left(n - \frac{rb_0}{Z^T}\right), \\
n^N &= \frac{1}{2}\left(n + \frac{rb_0}{Z^T}\right).
\end{aligned}
$$

Notice that the relative price of non-tradable goods, p, depends only on the technological parameters and is thus independent of demand considerations. Intuitively, the production possibilities frontier is linear and hence the equilibrium relative price is fully determined by the technology.