1

Introduction

Order and simplification are the first steps toward the mastery of a subject — the actual enemy is the unknown.
— Thomas Mann, The Magic Mountain

1.1 Programming Languages

Programming is a lot of fun. As you have no doubt experienced, clarity and simplicity are the keys to good programming. When you have a tangle of code that is difficult to understand, your confidence in its behavior wavers, and the code is no longer any fun to read or update.

Designing a new programming language is a kind of metalevel programming activity that is just as much fun as programming in a regular language (if not more so). You will discover that clarity and simplicity are even more important in language design than they are in ordinary programming. Today hundreds of programming languages are in use — whether they be scripting languages for Internet commerce, user interface programming tools, spreadsheet macros, or page format specification languages that when executed can produce formatted documents. Inspired application design often requires a programmer to provide a new programming language or to extend an existing one. This is because flexible and extensible applications need to provide some sort of programming capability to their end users.

Elements of programming language design are even found in “ordinary” programming. For instance, consider designing the interface to a collection data structure. What is a good way to encapsulate an iteration idiom over the elements of such a collection? The issues faced in this problem are similar to those in adding a looping construct to a programming language.

The goal of this book is to teach you the great ideas in programming languages in a simple framework that strips them of complexity. You will learn several ways to specify the meaning of programming language constructs and will see that small changes in these specifications can have dramatic consequences for program behavior. You will explore many dimensions of the programming
language design space, study decisions to be made along each dimension, and consider how decisions from different dimensions can interact. We will teach you about a wide variety of neat tricks for extending programming languages with interesting features like undoable state changes, exitable loops, and pattern matching. Our approach for teaching you this material is based on the premise that when language behaviors become incredibly complex, the descriptions of the behaviors must be incredibly simple. It is the only hope.

1.2 Syntax, Semantics, and Pragmatics

Programming languages are traditionally viewed in terms of three facets:

1. **Syntax** — the *form* of programming languages.
2. **Semantics** — the *meaning* of programming languages.
3. **Pragmatics** — the *implementation* of programming languages.

Here we briefly describe these facets.

**Syntax**

Syntax focuses on the concrete notations used to encode programming language phrases. Consider a phrase that indicates the sum of the product of \( v \) and \( w \) and the quotient of \( y \) and \( z \). Such a phrase can be written in many different notations — as a traditional mathematical expression:

\[ vw + y/z \]

or as a Lisp parenthesized prefix expression:

\[ (+ (* v w) (/ y z)) \]

or as a sequence of keystrokes on a postfix calculator:

\[
V \text{ ENTER} \ W \text{ ENTER} \times Y \text{ ENTER} \ Z \text{ ENTER} \div +
\]

or as a layout of cells and formulas in a spreadsheet:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( v = )</td>
<td>( v \times w = )</td>
<td>( A^2 \times B^2 )</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>( w = )</td>
<td>( y/z = )</td>
<td>( C^2 / D^2 )</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>( y = )</td>
<td>( \text{ans} = )</td>
<td>( A^4 + B^4 )</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>( z = )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
or as a graphical tree:

```
+  
* /  
\ /  
v w /y z
```

Although these concrete notations are superficially different, they all designate the same abstract phrase structure (the sum of a product and a quotient). The syntax of a programming language specifies which concrete notations (strings of characters, lines on a page) in the language are legal and which tree-shaped abstract phrase structure is denoted by each legal notation.

**Semantics**

Semantics specifies the mapping between the structure of a programming language phrase and what the phrase means. Such phrases have no inherent meaning: their meaning is determined only in the context of a system for interpreting their structure. For example, consider the following expression tree:

```
*  
+ 11  
10
```

Suppose we interpret the nodes labeled 1, 10, and 11 as the usual decimal notation for numbers, and the nodes labeled + and * as the sum and product of the values of their subnodes. Then the root of the tree stands for \((1+11) \cdot 10 = 120\). But there are many other possible meanings for this tree. If * stands for exponentiation rather than multiplication, the meaning of the tree could be \(12^{10}\). If the numerals are in binary notation rather than decimal notation, the tree could stand for (in decimal notation) \((1 + 3) \cdot 2 = 8\). Alternatively, suppose that odd integers stand for the truth value *true*, even integers stand for the truth value *false*, and + and * stand for, respectively, the logical disjunction (\(\lor\)) and conjunction (\(\land\)) operators on truth values; then the meaning of the tree is *false*. Perhaps the tree does not indicate an evaluation at all, and only stands for a property intrinsic to the tree, such as its height (3), its number of nodes (5), or its shape (perhaps it describes a simple corporate hierarchy). Or maybe the tree is an arbitrary encoding for a particular object of interest, such as a person or a book.
This example illustrates how a single program phrase can have many possible meanings. Semantics describes the relationship between the abstract structure of a phrase and its meaning.

Pragmatics

Whereas semantics deals with *what* a phrase means, pragmatics focuses on the details of *how* that meaning is computed. Of particular interest is the effective use of various resources, such as time, space, and access to shared physical devices (storage devices, network connections, video monitors, printers, speakers, etc.).

As a simple example of pragmatics, consider the evaluation of the following expression tree (under the first semantic interpretation described above):

```
/   
-    +
  / 
+  *
 a b
```

Suppose that a and b stand for particular numeric values. Because the phrase \((+ a b)\) appears twice, a naive evaluation strategy will compute the same sum twice. An alternative strategy is to compute the sum once, save the result, and use the saved result the next time the phrase is encountered. The alternative strategy does not change the meaning of the program, but does change its use of resources; it reduces the number of additions performed, but may require extra storage for the saved result. Is the alternative strategy better? The answer depends on the details of the evaluation model and the relative importance of time and space.

Another potential improvement in the example involves the phrase \((* 2 3)\), which always stands for the number 6. If the sample expression is to be evaluated many times (for different values of a and b), it may be worthwhile to replace \((* 2 3)\) by 6 to avoid unnecessary multiplications. Again, this is a purely pragmatic concern that does not change the meaning of the expression.

1.3 Goals

The goals of this book are to explore the semantics of a comprehensive set of programming language design idioms, show how they can be combined into complete
1.3 Goals

practical programming languages, and discuss the interplay between semantics and pragmatics.

Because syntactic issues are so well covered in standard compiler texts, we won’t say much about syntax except for establishing a few syntactic conventions at the outset. We will introduce a number of tools for describing the semantics of programming languages, and will use these tools to build intuitions about programming language features and study many of the dimensions along which languages can vary. Our coverage of pragmatics is mainly at a high level. We will study some simple programming language implementation techniques and program improvement strategies rather than focus on squeezing the last ounce of performance out of a particular computer architecture.

We will discuss programming language features in the context of several mini-languages. Each of these is a simple programming language that captures the essential features of a class of existing programming languages. In many cases, the mini-languages are so pared down that they are hardly suitable for serious programming activities. Nevertheless, these languages embody all of the key ideas in programming languages. Their simplicity saves us from getting bogged down in needless complexity in our explorations of semantics and pragmatics. And like good modular building blocks, the components of the mini-languages are designed to be “snapped together” to create practical languages.

Issues of semantics and pragmatics are important for reasoning about properties of programming languages and about particular programs in these languages. We will also discuss them in the context of two fundamental strategies for programming language implementation: interpretation and translation. In the interpretation approach, a program written in a source language $S$ is directly executed by an $S$-interpreter, which is a program written in an implementation language. In the translation approach, an $S$ program is translated to a program in the target language $T$, which can be executed by a $T$-interpreter. The translation itself is performed by a translator program written in an implementation language. A translator is also called a compiler, especially when it translates from a high-level language to a low-level one. We will use mini-languages for our source and target languages. For our implementation language, we will use the mathematical metalanguage described in Appendix A. However, we strongly encourage readers to build working interpreters and translators for the mini-languages in their favorite real-world programming languages. Metaprogramming — writing programs that manipulate other programs — is perhaps the most exciting form of programming!
1.4 PostFix: A Simple Stack Language

We will introduce the tools for syntax, semantics, and pragmatics in the context of a mini-language called PostFix. PostFix is a simple stack-based language inspired by the PostScript graphics language, the Forth programming language, and Hewlett Packard calculators. Here we give an informal introduction to PostFix in order to build some intuitions about the language. In subsequent chapters, we will introduce tools that allow us to study PostFix in more depth.

1.4.1 Syntax

The basic syntactic unit of a PostFix program is the command. Commands are of the following form:

- Any integer numeral. E.g., 17, 0, -3.
- One of the following special command tokens: add, div, eq, exec, gt, lt, mul, nget, pop, rem, sel, sub, swap.
- An executable sequence — a single command that serves as a subroutine. It is written as a parenthesized list of subcommands separated by whitespace (any contiguous sequence of characters that leave no mark on the page, such as spaces, tabs, and newlines). E.g., (7 add 3 swap) or (2 (5 mul) exec add).

Since executable sequences contain other commands (including other executable sequences), they can be arbitrarily nested. An executable sequence counts as a single command despite its hierarchical structure.

A PostFix program is a parenthesized sequence consisting of (1) the token postfix followed by (2) a natural number (i.e., nonnegative integer) indicating the number of program parameters followed by (3) zero or more PostFix commands. Here are some sample PostFix programs:

(postfix 0 4 7 sub)
(postfix 2 add 2 div)
(postfix 4 4 nget 5 nget mul mul swap 4 nget mul add add)
(postfix 1 ((3 nget swap exec) (2 mul swap exec) swap)
       (5 sub) swap exec exec)

In PostFix, as in all the languages we’ll be studying, all parentheses are required and none are optional. Moving parentheses around changes the structure of the program and most likely changes its behavior. Thus, while the following
PostFix executable sequences use the same numerals and command tokens in the same order, they are distinguished by their parenthesization, which, as we shall see below, makes them behave differently.

\[
(1) \quad (2 \ 3 \ 4) \quad \text{swap exec}
\]
\[
(1 \ 2) \quad (3 \ 4) \quad \text{swap exec}
\]
\[
(1 \ 2) \quad (3 \ 4 \ \text{swap}) \quad \text{exec}
\]

### 1.4.2 Semantics

The meaning of a PostFix program is determined by executing its commands in left-to-right order. Each command manipulates an implicit stack of values that initially contains the integer arguments of the program (where the first argument is at the top of the stack and the last argument is at the bottom). A value on the stack is either (1) an integer numeral or (2) an executable sequence. The result of a program is the integer value at the top of the stack after its command sequence has been completely executed. A program signals an error if (1) the final stack is empty, (2) the value at the top of the final stack is not an integer, or (3) an inappropriate stack of values is encountered when one of its commands is executed.

The behavior of PostFix commands is summarized in Figure 1.1. Each command is specified in terms of how it manipulates the implicit stack. We use the notation \(P \args v\) to mean that executing the PostFix program \(P\) on the integer argument sequence \(\args\) returns the value \(v\). The notation \(P \args \text{error}\) means that executing the PostFix program \(P\) on the arguments \(\args\) signals an error. Errors are caused by inappropriate stack values or an insufficient number of stack values. In practice, it is desirable for an implementation to indicate the type of error. We will use comments (delimited by braces) to explain errors and other situations.

To illustrate the meanings of various commands, we show the results of some simple program executions. For example, numerals are pushed onto the stack, while \texttt{pop} and \texttt{swap} are the usual stack operations.

\[
\begin{align*}
\text{(postfix 0 1 2 3)} & \quad \downarrow 3 \quad \{\text{Only the top stack value is returned.}\} \\
\text{(postfix 0 1 2 3 pop)} & \quad \downarrow 2 \\
\text{(postfix 0 1 2 swap 3 pop)} & \quad \downarrow 1 \\
\text{(postfix 0 1 swap)} & \quad \downarrow \text{error} \quad \{\text{Not enough values to swap.}\} \\
\text{(postfix 0 1 pop pop)} & \quad \downarrow \text{error} \quad \{\text{Empty stack on second pop.}\}
\end{align*}
\]

Program arguments are pushed onto the stack (from last to first) before the execution of the program commands.
\( N \): Push the numeral \( N \) onto the stack.

\( \text{sub} \): Call the top stack value \( v_1 \) and the next-to-top stack value \( v_2 \). Pop these two values off the stack and push the result of \( v_2 - v_1 \) onto the stack. If there are fewer than two values on the stack or the top two values aren’t both numerals, signal an error. The other binary arithmetic operators — \( \text{add} \) (addition), \( \text{mul} \) (multiplication), \( \text{div} \) (integer division\(^a\)), and \( \text{rem} \) (remainder of integer division) — behave similarly. Both \( \text{div} \) and \( \text{rem} \) signal an error if \( v_1 \) is zero.

\( \text{lt} \): Call the top stack value \( v_1 \) and the next-to-top stack value \( v_2 \). Pop these two values off the stack. If \( v_2 < v_1 \), then push a 1 (a true value) on the stack, otherwise push a 0 (false). The other binary comparison operators — \( \text{eq} \) (equals) and \( \text{gt} \) (greater than) — behave similarly. If there are fewer than two values on the stack or the top two values aren’t both numerals, signal an error.

\( \text{pop} \): Pop the top element off the stack and discard it. Signal an error if the stack is empty.

\( \text{swap} \): Swap the top two elements of the stack. Signal an error if the stack has fewer than two values.

\( \text{sel} \): Call the top three stack values (from top down) \( v_1, v_2, \) and \( v_3 \). Pop these three values off the stack. If \( v_3 \) is the numeral 0, push \( v_1 \) onto the stack; if \( v_3 \) is a nonzero numeral, push \( v_2 \) onto the stack. Signal an error if the stack does not contain three values, or if \( v_3 \) is not a numeral.

\( \text{nget} \): Call the top stack value \( v_{\text{index}} \) and the remaining stack values (from top down) \( v_1, v_2, \ldots, v_n \). Pop \( v_{\text{index}} \) off the stack. If \( v_{\text{index}} \) is a numeral \( i \) such that \( 1 \leq i \leq n \) and \( v_i \) is a numeral, push \( v_i \) onto the stack. Signal an error if the stack does not contain at least one value, if \( v_{\text{index}} \) is not a numeral, if \( i \) is not in the range \([1..n]\), or if \( v_i \) is not a numeral.

\( (C_1 \ldots C_n) \): Push the executable sequence \((C_1 \ldots C_n)\) as a single value onto the stack. Executable sequences are used in conjunction with \( \text{exec} \).

\( \text{exec} \): Pop the executable sequence from the top of the stack, and prepend its component commands onto the sequence of currently executing commands. Signal an error if the stack is empty or the top stack value isn’t an executable sequence.

\(^a\)The integer division of \( n \) and \( d \) returns the integer quotient \( q \) such that \( n = qd + r \), where \( r \) (the remainder) is such that \( 0 \leq r < |d| \) if \( n \geq 0 \) and \(-|d| < r \leq 0 \) if \( n < 0 \).

**Figure 1.1** English semantics of PostFix commands.

\[
\begin{align*}
\text{(postfix 2)} & \quad [3,4] \rightarrow 3 \quad \{ \text{Initial stack has 3 on top with 4 below.} \} \\
\text{(postfix 2 swap)} & \quad [3,4] \rightarrow 4 \\
\text{(postfix 3 pop swap)} & \quad [3,4,5] \rightarrow 5
\end{align*}
\]
1.4.2 Semantics

It is an error if the actual number of arguments does not match the number of parameters specified in the program.

(postfix 2 swap) $\rightarrow$ error \{Wrong number of arguments.\}
(postfix 1 pop) $^{[4,5]} \rightarrow$ error \{Wrong number of arguments.\}

Note that program arguments must be integers — they cannot be executable sequences.

Numerical operations are expressed in postfix notation, in which each operator comes after the commands that compute its operands. \texttt{add}, \texttt{sub}, \texttt{mul}, and \texttt{div} are binary integer operators. \texttt{lt}, \texttt{eq}, and \texttt{gt} are binary integer predicates returning either 1 (true) or 0 (false).

(postfix 1 4 sub) $\rightarrow$ -1
(postfix 1 4 add 5 mul 6 sub 7 div) $\rightarrow$ 4
(postfix 5 add mul sub swap div) $^{[7,6,5,4,3]} \rightarrow$ -20
(postfix 3 4000 swap pop add) $^{[300,20,1]} \rightarrow$ 4020
(postfix 2 add 2 div) $^{[3,7]} \rightarrow$ 5 \{An averaging program.\}
(postfix 1 3 div) $^{[17]} \rightarrow$ 5
(postfix 1 3 rem) $^{[17]} \rightarrow$ 2
(postfix 1 4 lt) $^{[3]} \rightarrow$ 1
(postfix 1 4 lt) $^{[5]} \rightarrow$ 0
(postfix 1 4 lt 10 add) $^{[3]} \rightarrow$ 11
(postfix 1 4 mul add) $^{[3]} \rightarrow$ error \{Not enough numbers to add.\}
(postfix 2 4 sub div) $^{[4,5]} \rightarrow$ error \{Divide by zero.\}

In all the above examples, each stack value is used at most once. Sometimes it is desirable to use a number two or more times or to access a number that is not near the top of the stack. The \texttt{nget} command is useful in these situations; it puts at the top of the stack a copy of a number located on the stack at a specified index. The index is 1-based, from the top of the stack down, not counting the index value itself.

(postfix 2 1 nget) $^{[4,5]} \rightarrow$ 4 \{4 is at index 1, 5 at index 2.\}
(postfix 2 2 nget) $^{[4,5]} \rightarrow$ 5

It is an error to use an index that is out of bounds or to access a nonnumeric stack value (i.e., an executable sequence) with \texttt{nget}.

(postfix 2 3 nget) $^{[4,5]} \rightarrow$ error \{Index 3 is too large.\}
(postfix 2 0 nget) $^{[4,5]} \rightarrow$ error \{Index 0 is too small.\}
(postfix 1 (2 mul) 1 nget) $^{[3]} \rightarrow$ error
\{Value at index 1 is not a number but an executable sequence.\}
The nget command is particularly useful for numerical programs, where it is common to reference arbitrary parameter values and use them multiple times.

\[ (postfix \ 1 \ 1 \ nget \ mul) \rightarrow 25 \quad \text{\{A squaring program.\}} \]
\[ (postfix \ 4 \ 4 \ nget \ 5 \ nget \ mul \ mul \ swap \ 4 \ nget \ mul \ add \ add) \rightarrow 25 \]
\{Given \ a, \ b, \ c, \ x, \ calculates \ ax^2 + bx + c.\} 

As illustrated in the last example, the index of a given value increases every time a new value is pushed onto the stack. The final stack in this example contains (from top down) 25 and 2, showing that the program may end with more than one value on the stack.

Executable sequences are compound commands like (2 mul) that are pushed onto the stack as a single value. They can be executed later by the exec command. Executable sequences act like subroutines in other languages; execution of an executable sequence is similar to a subroutine call, except that transmission of arguments and results is accomplished via the stack.

\[ (postfix \ 1 \ (2 \ mul) \ exec) \rightarrow 14 \quad \text{\{(2 mul) \ is \ a \ doubling \ subroutine.\}} \]
\[ (postfix \ 0 \ (0 \ swap \ sub) \ 7 \ swap \ exec) \rightarrow -7 \]
\{0 swap sub \ is \ a \ negation \ subroutine.\}
\[ (postfix \ 0 \ (2 \ mul)) \rightarrow \text{error} \quad \text{\{Final top of stack is not an integer.\}} \]
\[ (postfix \ 0 \ 3 \ (2 \ mul) \ gt) \rightarrow \text{error} \]
\{Executable sequence where number expected.\}
\[ (postfix \ 0 \ 3 \ exec) \rightarrow \text{error} \quad \text{\{Number \ where \ executable \ sequence \ expected.\}} \]
\[ (postfix \ 0 \ (7 \ swap \ exec) \ (0 \ swap \ sub) \ swap \ exec) \rightarrow -7 \]
\[ (postfix \ 2 \ (mul \ sub) \ (1 \ nget \ mul) \ 4 \ nget \ swap \ exec \ swap \ exec) \rightarrow -10, 2 \]
\{Given \ a \ and \ b, \ calculates \ b - a*b^2.\} 

The last two examples illustrate that evaluations involving executable sequences can be rather contorted.

The sel command selects between two values based on a test value, where zero is treated as false and any nonzero integer is treated as true. It can be used in conjunction with exec to conditionally execute one of two executable sequences.

\[ (postfix \ 1 \ 2 \ 3 \ sel) \rightarrow 2 \]
\[ (postfix \ 1 \ 2 \ 3 \ sel) \rightarrow 3 \]
\[ (postfix \ 1 \ 2 \ 3 \ sel) \rightarrow 2 \quad \text{\{Any nonzero number is “true.”\}} \]
\[ (postfix \ 0 \ (2 \ mul) \ 3 \ 4 \ sel) \rightarrow \text{error} \quad \text{\{Test not a number.\}} \]
\[ (postfix \ 4 \ lt \ (add) \ (mul) \ sel \ exec) \rightarrow 30 \]
\[ (postfix \ 4 \ lt \ (add) \ (mul) \ sel \ exec) \rightarrow 11 \]
\[ (postfix \ 1 \ 1 \ nget \ 0 \ lt \ (0 \ swap \ sub) \ () \ sel \ exec) \rightarrow 7 \quad \text{\{An absolute value program.\}} \]
\[ (postfix \ 1 \ 1 \ nget \ 0 \ lt \ (0 \ swap \ sub) \ () \ sel \ exec) \rightarrow 6 \]
Exercise 1.1 Determine the value of the following PostFix programs on an empty stack.

a. (postfix 0 10 (swap 2 mul sub) 1 swap exec)
b. (postfix 0 (5 (2 mul) exec) 3 swap)
c. (postfix 0 () exec) exec
d. (postfix 0 2 3 1 add mul sel)
e. (postfix 0 2 3 1 (add) (mul) sel)
f. (postfix 0 2 3 1 (add) (mul) sel exec)
g. (postfix 0 0 (2 3 add) 4 sel exec)
h. (postfix 0 1 (2 3 add) 4 sel exec)
i. (postfix 0 (5 6 lt) (2 3 add) 4 sel exec)
j. (postfix 0 (swap exec swap exec) (1 sub) swap (2 mul) swap 3 swap exec)

Exercise 1.2

a. What function of its argument does the following PostFix program calculate?
   (postfix 1 ((3 nget swap exec) (2 mul swap exec) swap) (5 sub) swap exec exec)

b. Write a simpler PostFix program that performs the same calculation.

Exercise 1.3 Recall that executable sequences are effectively subroutines that, when invoked (by the exec command), take their arguments from the top of the stack. Write executable sequences that compute the following logical operations. Recall that 0 stands for false and all other numerals are treated as true.

a. not: return the logical negation of a single argument.

b. and: given two numeric arguments, return 1 if their logical conjunction is true, and 0 otherwise.

c. short-circuit-and: return 0 if the first argument is false; otherwise return the second argument.

d. Demonstrate the difference between and and short-circuit-and by writing a PostFix program with zero arguments that has a different result if and is replaced by short-circuit-and.

Exercise 1.4

a. Without nget, is it possible to write a PostFix program that squares its single argument? If so, write it; if not, explain.
b. Is it possible to write a PostFix program that takes three integers and returns the smallest of the three? If so, write it; if not, explain.

c. Is it possible to write a PostFix program that calculates the factorial of its single argument (assume it’s nonnegative)? If so, write it; if not, explain.

1.4.3 The Pitfalls of Informal Descriptions

The “by-example” and English descriptions of PostFix given above are typical of the way that programming languages are described in manuals, textbooks, courses, and conversations. That is, a syntax for the language is presented, and the semantics of each of the language constructs is specified using English prose and examples. The utility of this method for specifying semantics is apparent from the fact that the vast majority of programmers learn to read and write programs via this approach.

But there are many situations in which informal descriptions of programming languages are inadequate. Suppose that we want to improve a program by transforming complex phrases into phrases that are simpler and more efficient. How can we be sure that the transformation process preserves the meaning of the program?

Or suppose that we want to prove that the language as a whole has a particular property. For instance, it turns out that every PostFix program is guaranteed to terminate (i.e., a PostFix program cannot enter an infinite loop). How would we go about proving this property based on the informal description? Natural language does not provide any rigorous framework for reasoning about programs or programming languages. Without the aid of some formal reasoning tools, we can only give hand-waving arguments that are not likely to be very convincing.

Or suppose that we wish to extend PostFix with features that make it easier to use. For example, it would be nice to name values, to collect values into arrays, to query the user for input, and to loop over sequences of values. With each new feature, the specification of the language becomes more complex, and it becomes more difficult to reason about the interaction between various features. We’d like techniques that help to highlight which features are orthogonal and which can interact in subtle ways.

Or suppose that a software vendor wants to develop PostFix into a product that runs on several different machines. The vendor wants any given PostFix program to have exactly the same behavior on all of the supported machines. But how do the development teams for the different machines guarantee that they’re all implementing the “same” language? If there are any ambiguities in the PostFix specification that they’re implementing, different development
teams might resolve the ambiguity in incompatible ways. What’s needed in this case is an unambiguous specification of the language as well as a means of proving that an implementation meets that specification.

The problem with informal descriptions of a programming language is that they’re neither concise nor precise enough for these kinds of situations. English is often verbose, and even relatively simple ideas can be unduly complicated to explain. Moreover, it’s easy for the writer of an informal specification to underspecify a language by forgetting to cover all the special cases (e.g., error situations in PostFix). It isn’t that covering all the special cases is impossible; it’s just that the natural-language framework doesn’t help much in pointing out what the special cases are.

It is possible to overspecify a language in English as well. Consider the PostFix programming model introduced above. The current state of a program is captured in two entities: the stack and the current command sequence. To programmers and implementers alike, this might imply that a language implementation must have explicit stack and command sequence elements in it. Although these would indeed appear in a straightforward implementation, they are not in any way required; there are alternative models and implementations for PostFix (e.g., see Exercise 3.12 on page 70). It would be desirable to have a more abstract definition of what constitutes a legal PostFix implementation so that a would-be implementer could be sure that an implementation was faithful to the language definition regardless of the representations and algorithms employed.

1.5 Overview of the Book

The remainder of Part I introduces a number of tools that address the inadequacies outlined above and that form an essential foundation for the study of programming language design. Chapter 2 presents s-expression grammars, a simple specification for syntax that we will use to describe the structure of all of the mini-languages we will explore. Then, using PostFix and a simple expression language as our objects of study, we introduce two approaches to formal semantics:

- An operational semantics (Chapter 3) explains the meaning of programming language constructs in terms of the step-by-step process of an abstract machine.

- A denotational semantics (Chapter 4) explains the meaning of programming language constructs in terms of the meaning of their subparts.
These approaches support the unambiguous specification of programming languages and provide a framework in which to reason about properties of programs and languages. Our discussion of tools concludes in Chapter 5 with a presentation of a technique for determining the meaning of recursive specifications. Throughout the book, and especially in these early chapters, we formalize concepts in terms of a mathematical **metalanguage** described in Appendix A. Readers are encouraged to familiarize themselves with this language by skimming this appendix early on and later referring to it in more detail on an “as needed” basis.

Part II focuses on **dynamic semantics**, the meaning of programming language constructs and the run-time behavior of programs. In Chapter 6, we introduce FL, a mini-language we use as a basis for investigating dimensions of programming language design. By extending FL in various ways, we then explore programming language features along key dimensions: naming (Chapter 7), state (Chapter 8), control (Chapter 9), and data (Chapter 10). Along the way, we will encounter several **programming paradigms**, high-level approaches for viewing computation: function-oriented programming, imperative programming, and object-oriented programming.

In Part III, we shift our focus to **static semantics**, properties of programs that can be determined without executing them. In Chapter 11, we introduce the notion of type — a description of what an expression computes — and develop a simple type-checking system for a dialect of FL such that “well-typed” programs cannot encounter certain kinds of run-time errors. In Chapter 12, we study some more advanced features of typed languages: subtyping, universal polymorphism, bounded quantification, and kind systems. A major drawback to many of our typed mini-languages is that programmers are required to annotate programs with significant amounts of explicit type information. In some languages, many of these annotations can be eliminated via type reconstruction, a technique we study in Chapter 13. Types can be used as a mechanism for enforcing data abstraction, a notion that we explore in Chapter 14. In Chapter 15, we show how many of the dynamic and static semantics features we have studied can be combined to yield a mini-language in which program modules with both value and type components can be independently type-checked and then linked together in a type-safe way. We wrap up our discussion of static semantics in Chapter 16 with a study of effect systems, which describe how expressions compute rather than what they compute.

The book culminates, in Part IV, in a pragmatics segment that illustrates how concepts from dynamic and static semantics play an important role in the implementation of a programming language. Chapter 17 presents a compiler that translates from a typed dialect of FL to a low-level language that resembles
assembly code. The compiler is organized as a sequence of meaning-preserving translation steps that construct explicit representations for the naming, state, control, and data aspects of programs. In order to automatically reclaim memory in a type-safe way, the run-time system for executing the low-level code generated by the compiler uses garbage collection, a topic that is explored in Chapter 18.

While we will emphasize formal tools throughout this book, we do not imply that formal tools are a panacea or that formal approaches are superior to informal ones in an absolute sense. In fact, informal explanations of language features are usually the simplest way to learn about a language. In addition, it’s very easy for formal approaches to get out of control, to the point where they are overly obscure, or require too much mathematical machinery to be of any practical use on a day-to-day basis. For this reason, we won’t cover material as a dry sequence of definitions, theorems, and proofs. Instead, our goal is to show that the concepts underlying the formal approaches are indispensable for understanding particular programming languages as well as the dimensions of language design. The tools, techniques, and features introduced in this book should be in any serious computer scientist’s bag of tricks.