Preface

The last several years have seen a tremendous surge of activity at the interface of computer science and economics. This book is a brief introduction to two intertwined facets of this emerging research area, *the price of anarchy* and *selfish routing*.

The price of anarchy measures the extent to which competition approximates cooperation. It is a rendezvous between the idea of an equilibrium, an idea fundamental to game theory, and the concept of approximation, which is ubiquitous in theoretical computer science. Selfish routing refers to a mathematical model of traffic in a congested network, and it is the price of anarchy's biggest success story so far. This model is interesting in its own right and has a long history and a wide array of applications. In addition to serving as an introduction, this book provides sufficient preparation for original research in these two areas.

**Intended Audience**

This is a research monograph, not a textbook. Nevertheless, I have endeavored to keep the discussion at a level accessible to an eager first- or second-year doctoral student in a mathematical discipline. Some parts of the book may fall short of this goal, but I hope they are few and far between.

Realistically, I expect that researchers and graduate students in theoretical computer science or in optimization will have the easiest time understanding and appreciating this book. On the other hand, I earnestly hope that readers coming from other parts of computer science and operations research, and from economics, electrical engineering, mathematics, and transportation science will find something that interests them. When I betray my roots as a theoretical computer scientist, be it with jargon, interminable discussions of NP-completeness, or an untreatable addiction to worst-case analysis, I have tried to be honest about it. I hope that readers from other fields will have little trouble looking past these biases.

**Overview**

Chapter 1 provides a nontechnical introduction to selfish routing. It includes an informal overview of the seminal examples of Pigou and Braess to motivate the analyses that follow. Chapter 2 is a clearinghouse for technical results of a preliminary nature, and it primarily serves as a reference for the other chapters. Chapter 3 is the heart of the book and is devoted to the price of anarchy of selfish routing, with Pigou's example playing a central role. This chapter covers only the most basic material on the topic. Chapter 4 surveys some of the many generalizations and variants of the work described in Chapter 3. Chapter 5 shifts

the discussion to Braess's Paradox. This chapter studies both the worst-case severity of the paradox and the computational complexity of algorithmically detecting it. Chapter 6 concludes by describing Stackelberg routing, which is one way to reduce the price of anarchy of selfish routing using a modest degree of centralized control.

**Chapter Notes**

This book is an updated and slightly expanded revision of my doctoral thesis. As such, it is inevitably slanted toward my own research. I have, however, included work by other researchers when I felt that it was important and blended in well with the rest of the book. At the same time, I wanted the book to have a brisk pace, a goal that seemed to preclude careful discussions of who did what. As a compromise, in all of the chapters I have deferred all bibliographic references to a final section entitled "Notes." While not encyclopedic and written at a higher level than the rest of the book, I think these sections give a thorough and accessible review of the relevant literature. Indeed, one of the purposes of the chapter notes is to summarize the work of different research communities addressing a common problem. I hope that my efforts catalyze further cross-fertilization between these different fields.

I also have relegated to the chapter notes discussions that are of secondary importance or that do not fit naturally into the text proper. Examples include historical notes, references to topics that I view as prerequisite material or as outside the scope of this book, and extensions and refinements of the models and results in the preceding sections. The chapter notes can be skipped without interfering with the use of the rest of the book.

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